UNIVERSITÀ PARTHENOPE
DiST
Dipartimento di Scienze e Tecnologie

Programmazione II e Laboratorio di P2

Merge Sort

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Introduzione

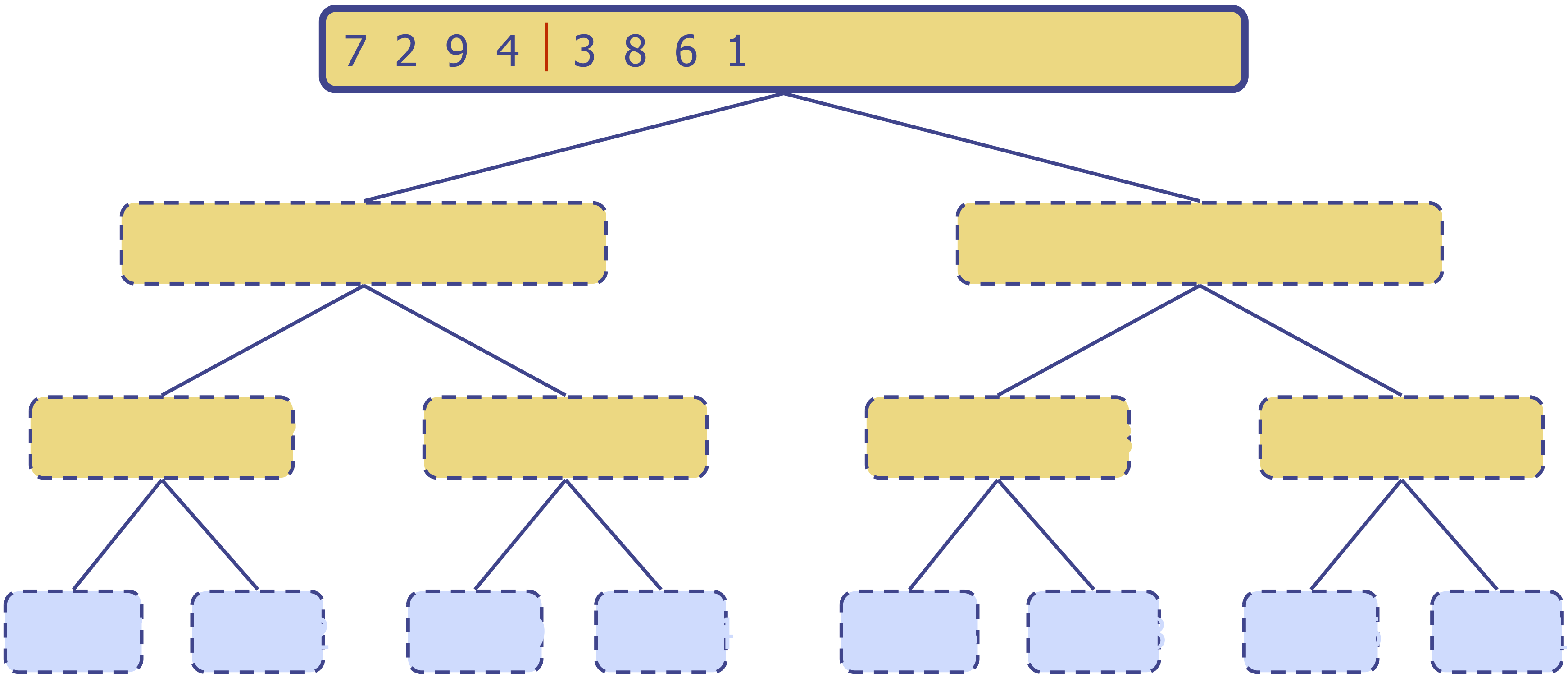
- Inventato da von Neumann nel 1945
- Esempio del paradigma algoritmico del divide et impera
- Richiede spazio ausiliario
  - O(*N*)

- Si divide il vettore dei dati in due parti ordinate separatamente, quindi si fondono le parti per ottenere un vettore ordinato globalmente

Complessità

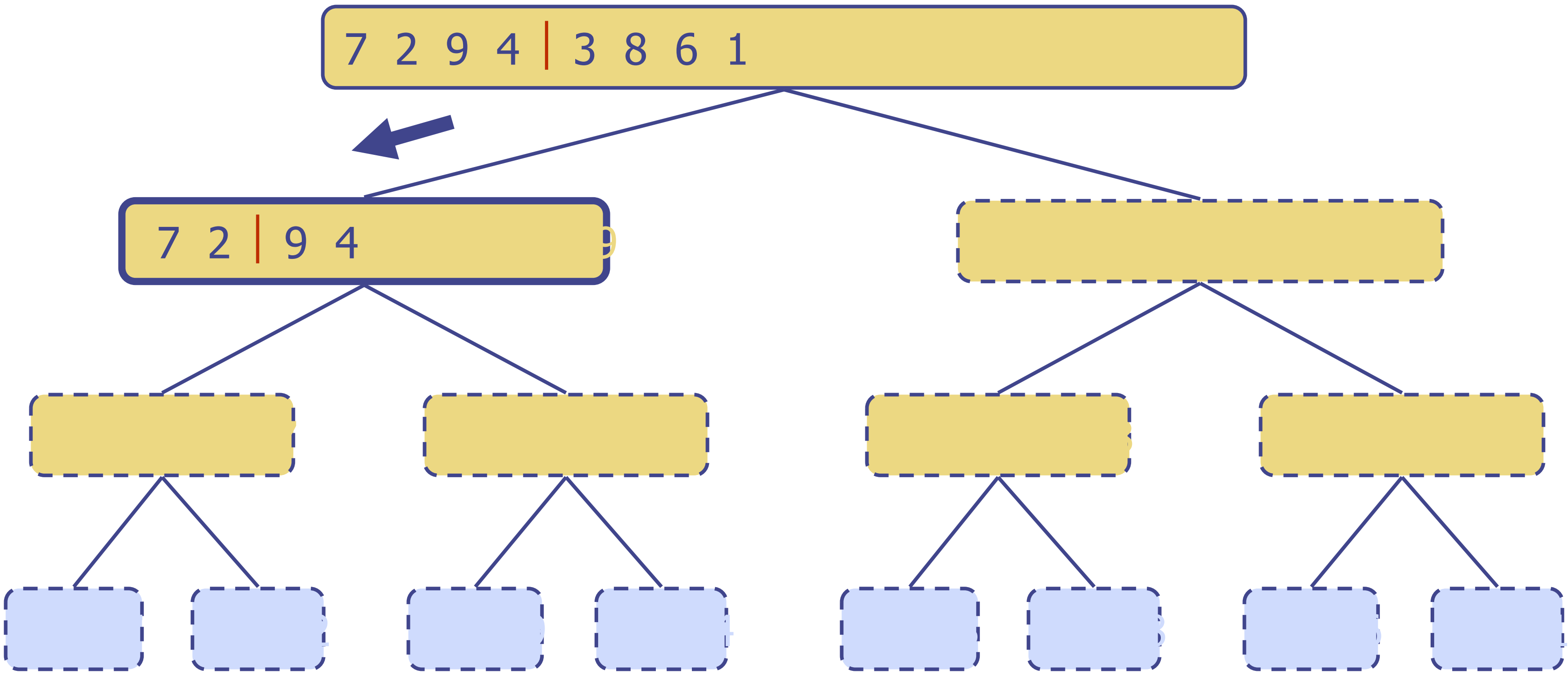
Algorithm	Time	Notes
selection-sort	$O(n^2)$	▪ slow ▪ in-place ▪ for small data sets (< 1K)
insertion-sort	$O(n^2)$	▪ slow ▪ in-place ▪ for small data sets (< 1K)
heap-sort	$O(n \log n)$	▪ fast ▪ in-place ▪ for large data sets (1K — 1M)
merge-sort	$O(n \log n)$	▪ fast ▪ sequential data access ▪ for huge data sets (> 1M)

Esempio di esecuzione

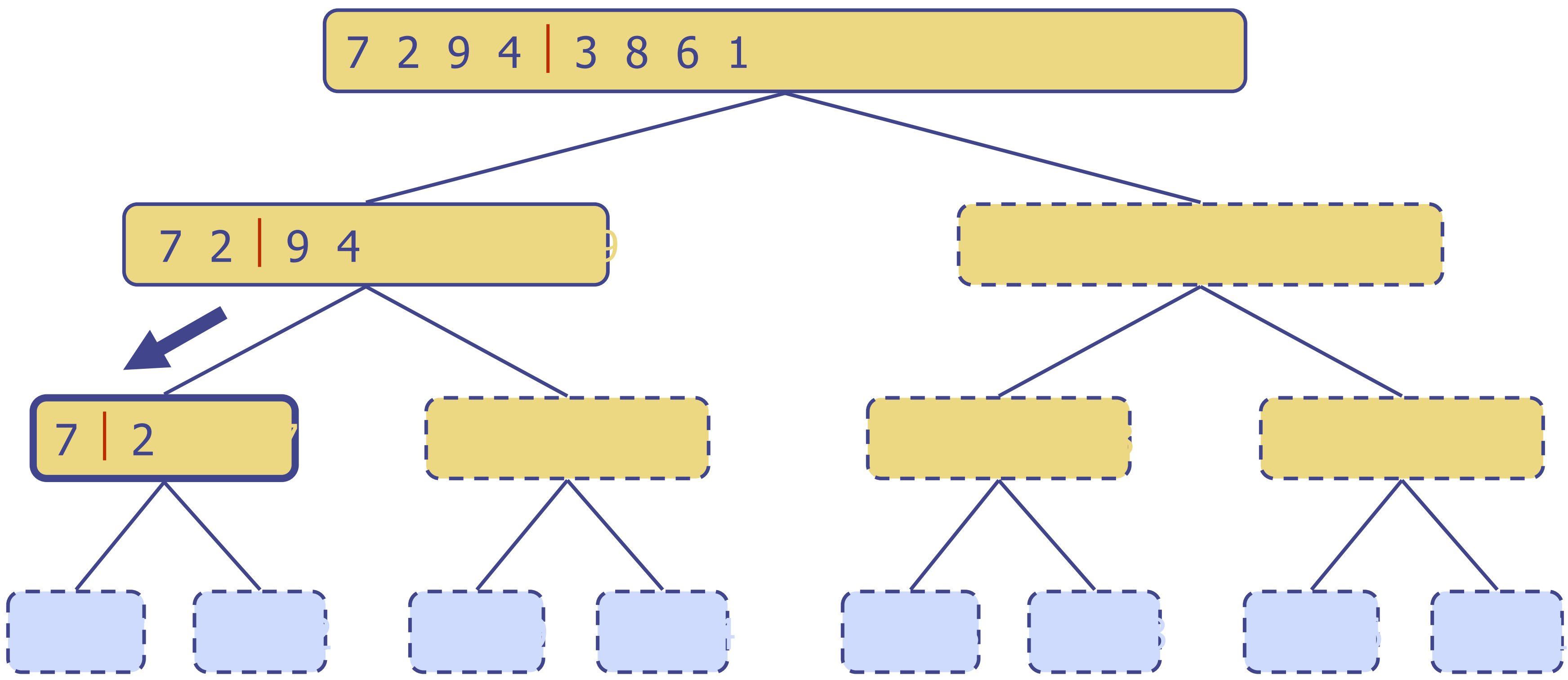


Mergesort

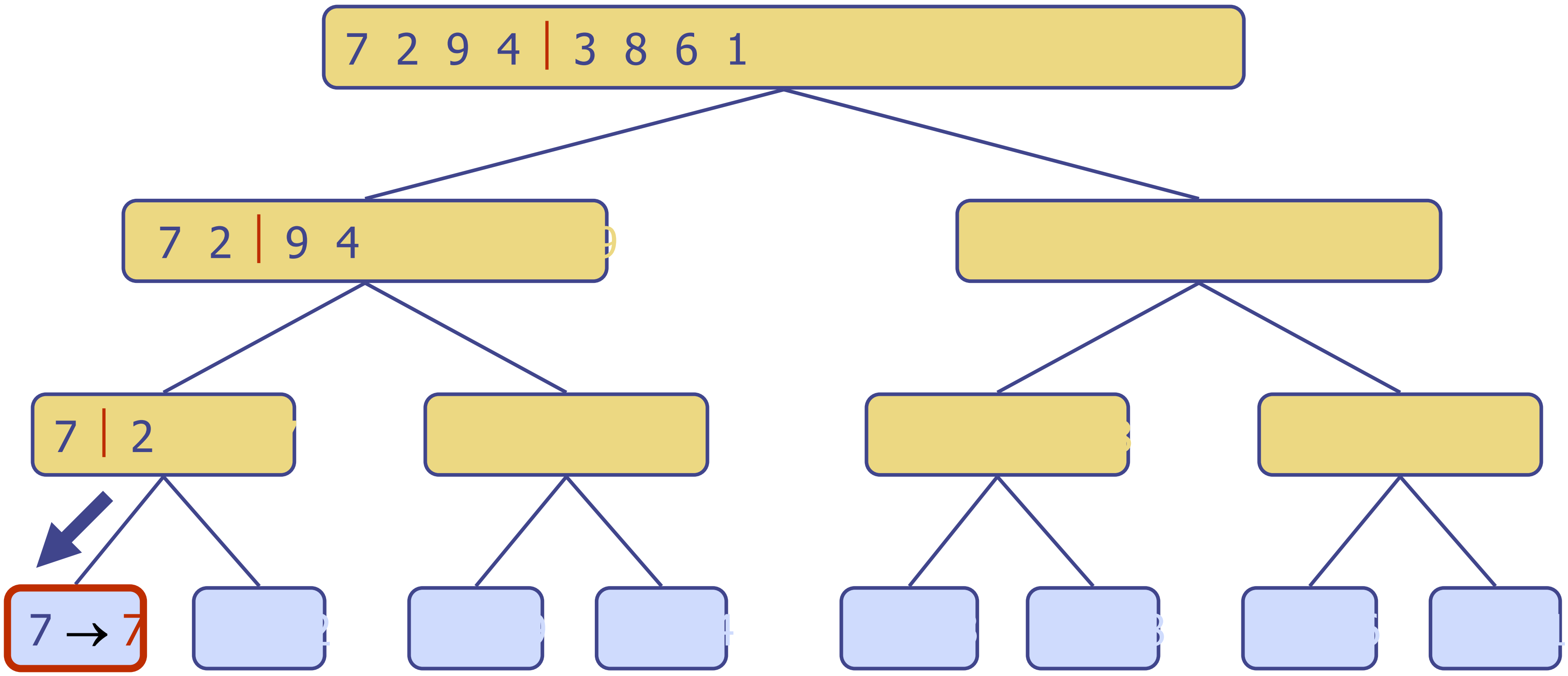
Esempio di esecuzione



Esempio di esecuzione



Esempio di esecuzione



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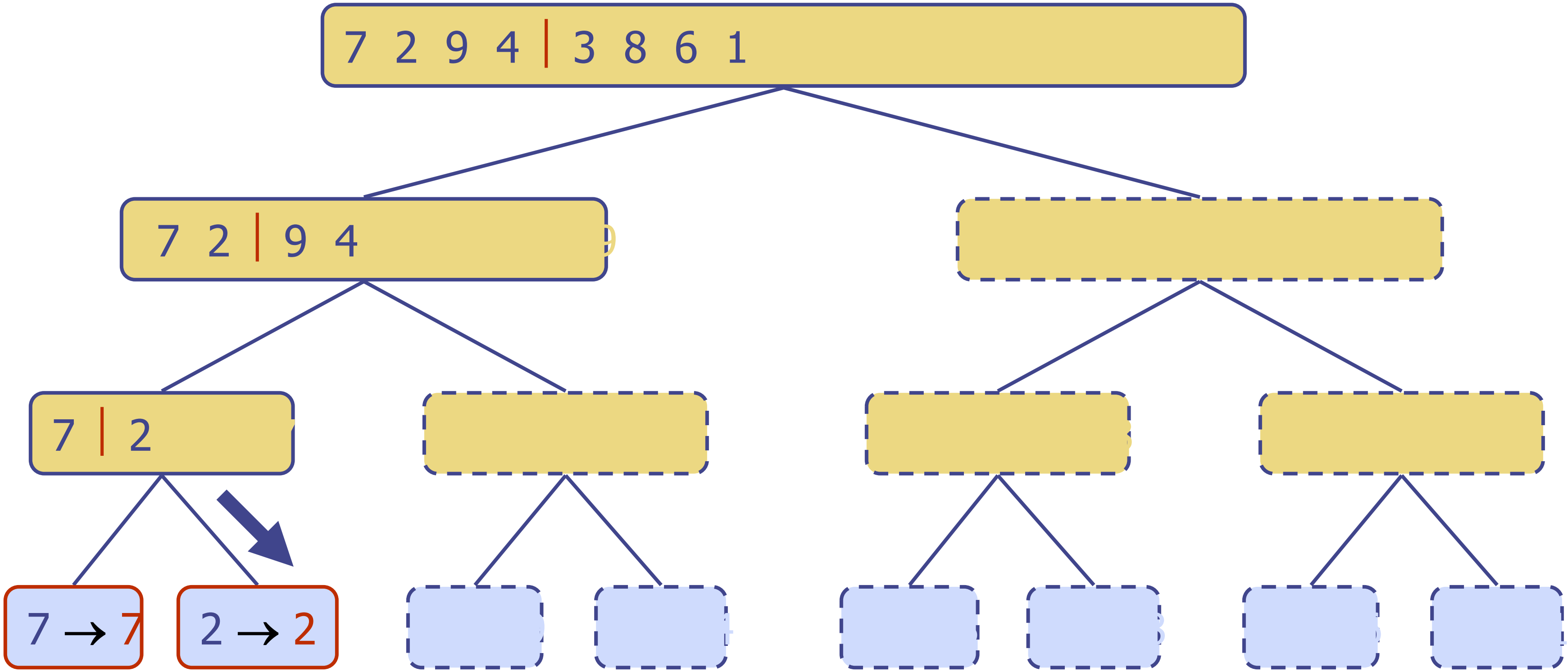
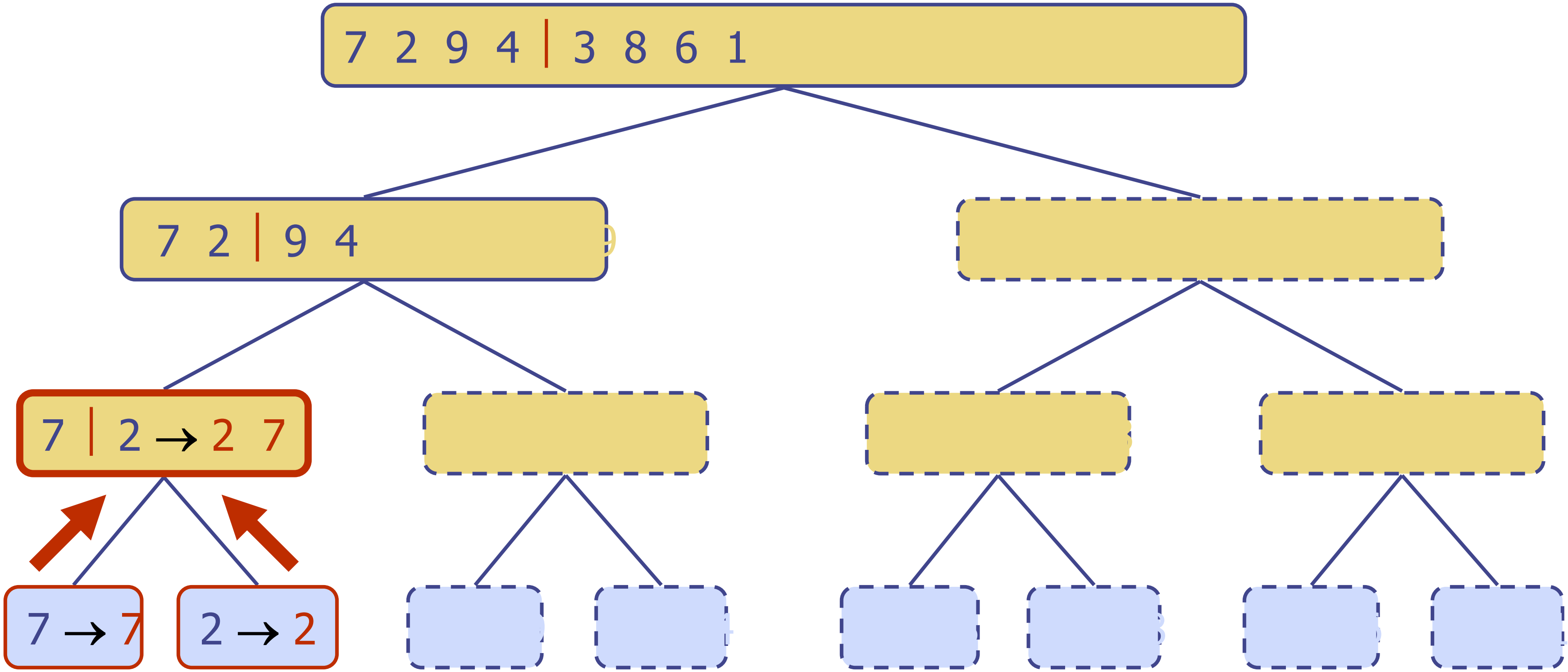
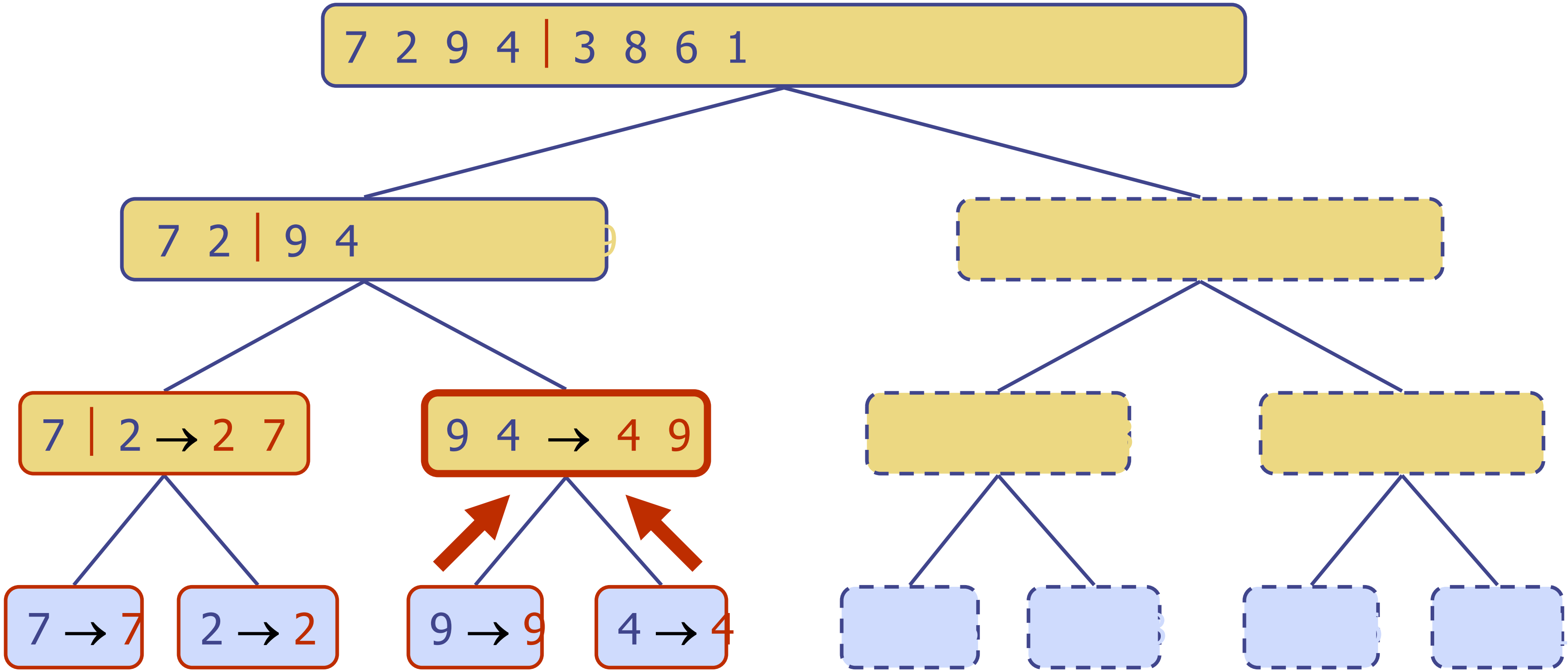


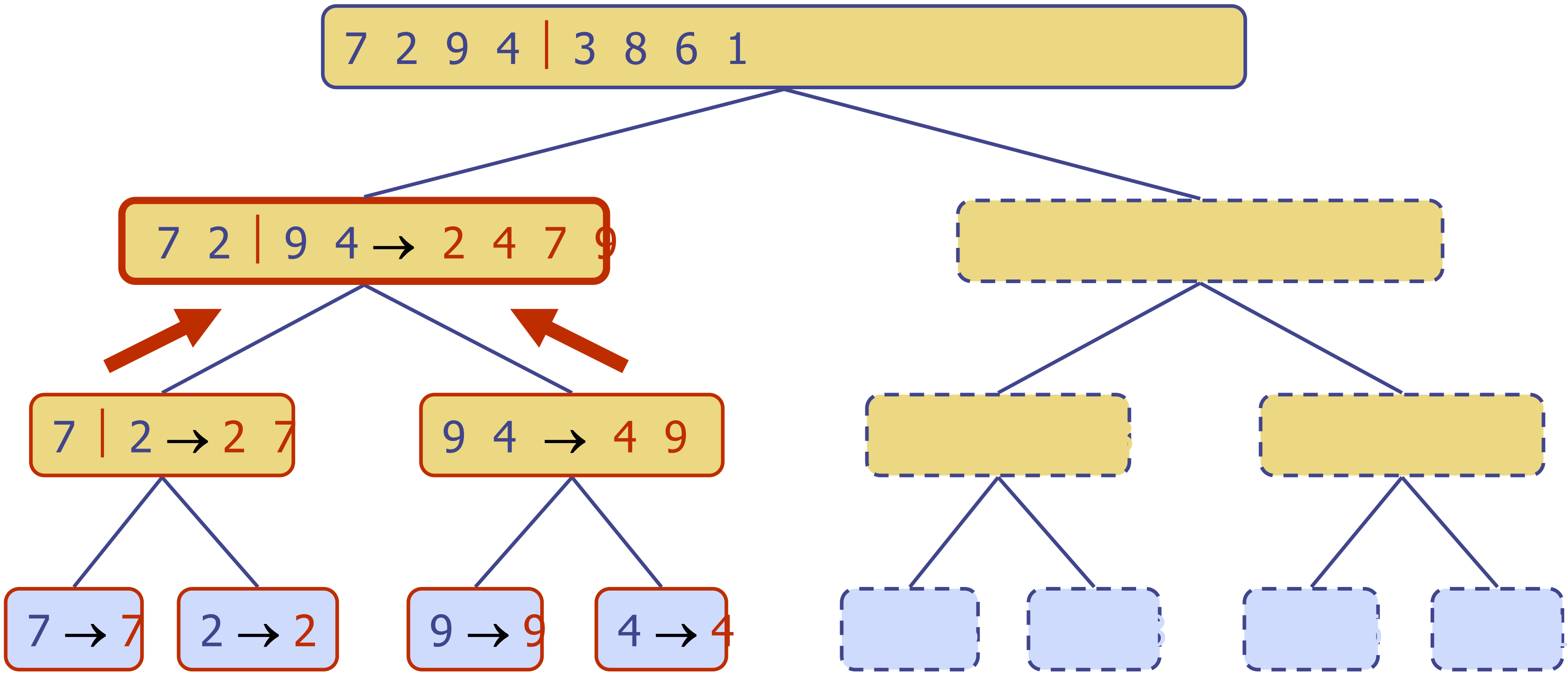
Esempio di esecuzione



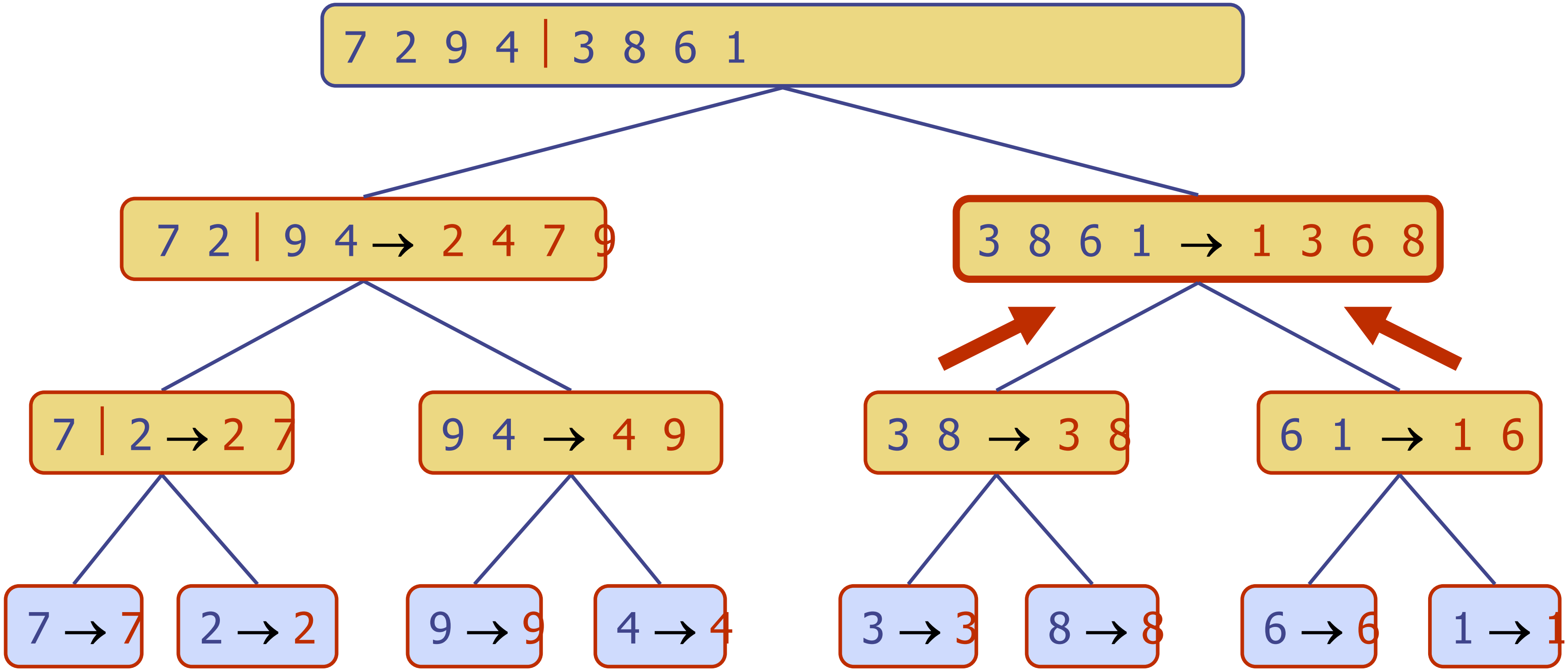
Esempio di esecuzione



Esempio di esecuzione

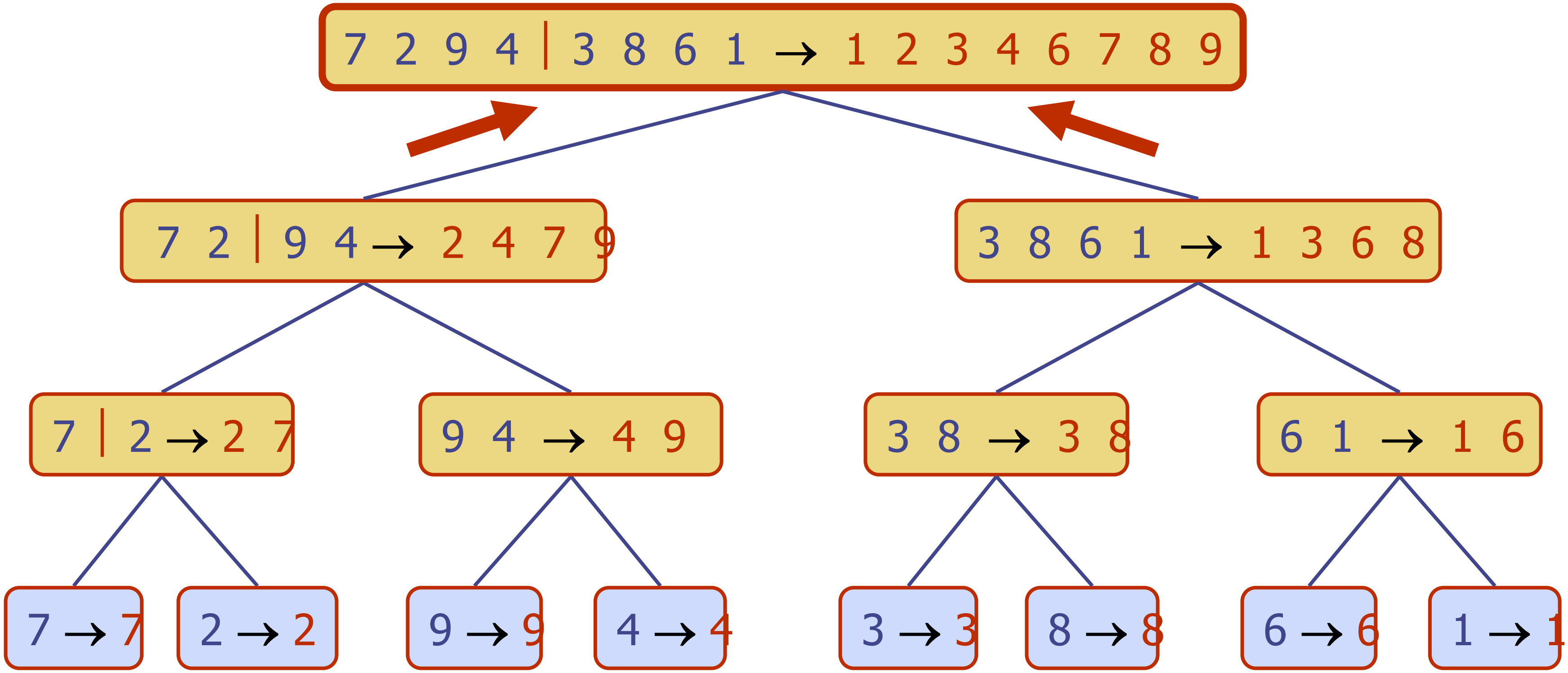


Esempio di esecuzione



Mergesort

Esempio di esecuzione



Algoritmo

```
Algorithm mergeSort(S, C)
        Input sequence S with n
              elements, comparator C
        Output sequence S sorted
        according to C
    if S.size() > 1
        (S1, S2) ← partition(S, n/2)
        mergeSort(S1, C)
        mergeSort(S2, C)
        S ← merge(S1, S2)
```

Approccio divide-et-impera

Algoritmo

```
class MergeSort
{
public:
   MergeSort( int ); // constructor initializes
vector
   void sort(); // sort vector using merge sort
   void displayElements() const; // display vector
elements
private:
   int size; // vector size
   vector< int > data; // vector of ints
   void sortSubVector( int, int ); // sort subvector
   void merge( int, int, int, int ); // merge two
sorted vectors
   void displaySubVector( int, int ) const; //
display subvector
};
```

Mergesort

Algoritmo

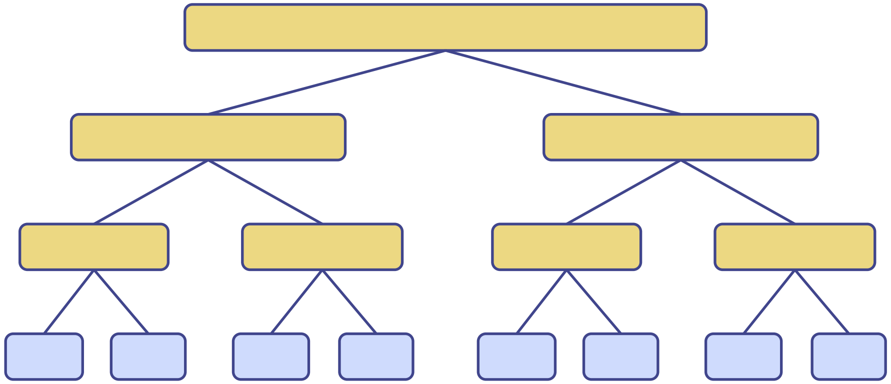
```
Algorithm merge(A, B)
        Input sequences A and B with
                 n/2 elements each
        Output sorted sequence of A ∪ B

    S ← empty sequence
    while ¬A.empty()  ∧ ¬B.empty()
        if A.front() < B.front()
                S.addBack(A.front()); A.eraseFront();
        else
                 S.addBack(B.front()); B.eraseFront();
    while ¬A.empty()
        S.addBack(A.front()); A.eraseFront();
    while ¬B.empty()
        S.addBack(B.front()); B.eraseFront();
    return S
```

Approccio divide-et-impera

Analisi dell'algoritmo

depth	#seqs	size
0	1	n
1	2	$n/2$
i	2^i	$n/2^i$
…	…	…



Complessità

Analisi dell'algoritmo

Ricorrenza

$$t(n) = \begin{cases} b & \text{if } n \leq 1 \\ 2t(n/2) + cn & \text{otherwise.} \end{cases}$$

$$\begin{aligned} t(n) &= 2(2t(n/2^2) + (cn/2)) + cn \\ &= 2^2 t(n/2^2) + 2(cn/2) + cn = 2^2 t(n/2^2) + 2cn. \end{aligned}$$

$$t(n) = 2^i t(n/2^i) + icn.$$

$n/2^i = 1$, i.e., $i = \log n$

$$\begin{aligned} t(n) &= 2^{\log n} t(n/2^{\log n}) + (\log n)cn \\ &= nt(1) + cn \log n \\ &= nb + cn \log n. \end{aligned}$$