Fast Fading

Electromagnetics
and
Remote Sensing Lab
(ERSLab)

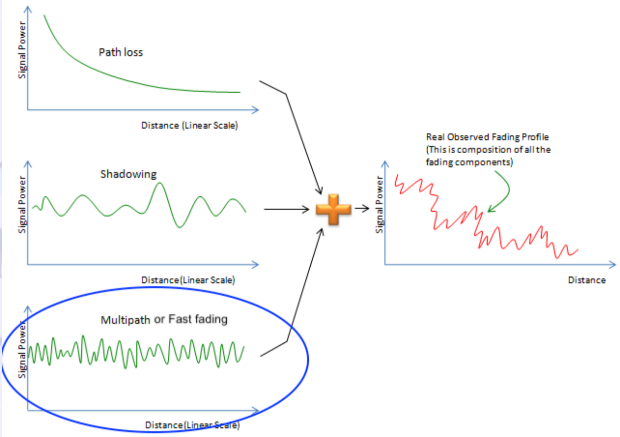
Università degli Studi di Napoli Parthenope
Dipartimento di Ingegneria
Centro Direzionale, isola C4 - 80143 - Napoli, Italy

ferdinando.nunziata@uniparthenope.it

Outline

Introduction

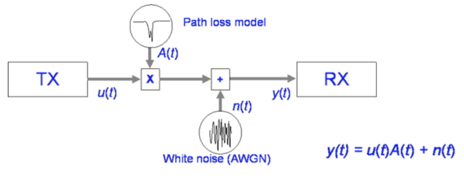
It manifests itself as large variations in the signal strength due to small changes in the distance between TX and RX.



Outline

AWGN channel

The simplest practical case of mobile radio channel is the one affected by additive white Gaussian noise (AWGN).



- Complex base-band notation is understood.
- This channel applies when the mobile and the surrounding scenario are not in motion.
- The received signal is perturbed by additive noise and by a fixed path loss that includes shadowing.

AWGN channel - SNR

A key parameter to evaluate the performance of a TLC system is the Signal to Noise Ratio (SNR).

- It is the ratio between the signal and the noise powers and is given by:

$$\gamma = \frac{E(A^2 u^2(t))}{2P_n} = \frac{A^2}{2\sigma_n^2} \qquad (1)$$

- where σ_n^2 is the variance of the zero-mean Gaussian processes that characterize the real and imaginary parts of the complex base-band noise components.
- a unit variance is assumed for the modulator output.

AWGN channel - BER

When dealing with the performance of a digital TLC systems it is more convenient to deal with the Bit Error Rate (BER).

- In case of a symbol with a finite duration T, whose energy is E_s (it consists of m bits with energy E_b), the SNR can be expressed as follows:

$$\gamma \propto \frac{E_s}{N_o} = \frac{mE_b}{N_o} \tag{2}$$

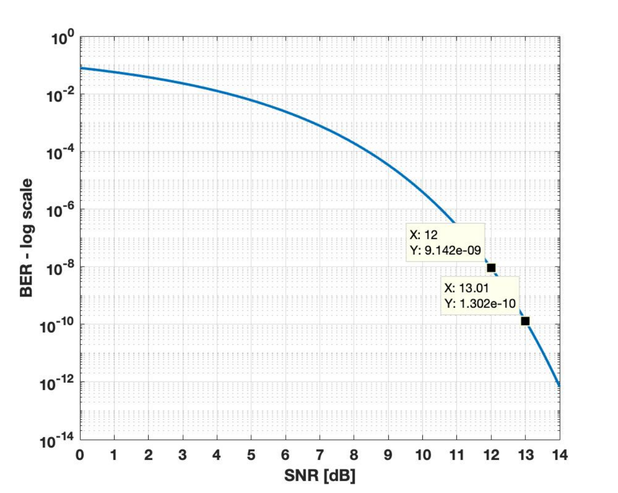
BER for a digital modulation scheme

It can be shown that the BER is given by:

$$BER = Q\left(\sqrt{\frac{A^2 d^2}{2N_o}}\right) \tag{3}$$

with d being the Euclidean distance between the TX symbols.

AWGN channel - BER in the BPSK case



In the BPSK case, eq.(3), becomes: $Q(\sqrt{2\gamma})$.

Outline

The fast fading channel

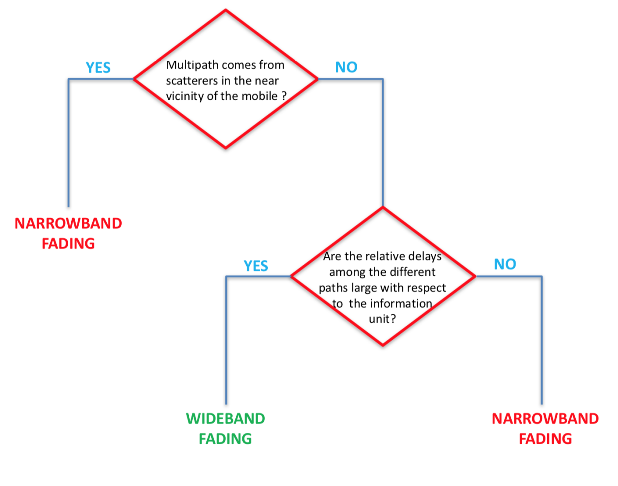
Mobile radio performance - in general - is worse than the AWGN case since propagation is affected by fast fading, i.e.; a multiplicative, time-variant process.

Fast fading

Fast fading indicates the fluctuations in the received signal as a result of multipath components. Several replicas of the signal arrive at the receiver through different propagation paths, adding constructively and destructively.

- Fast fading can be further divided in:
  - Flat or Narrowband fast fading.
  - Frequency selective or Wideband fast fading.

Narrowband vs Wideband fading channel



The definition of the wideband fading channel includes characteristics of both the signal and the channel.

Narrowband vs Wideband fading channel

Narrowband fading channel

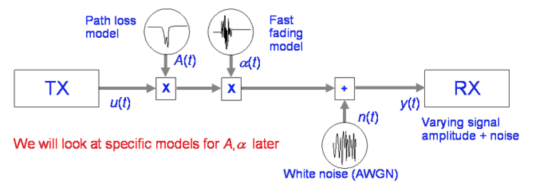
Multipath fading due to small path length differences between rays coming from scatterers in the near vicinity of the mobile. Hence, although significant phase differences are experienced, the rays all arrive at essentially the same time, so all frequencies within a wide bandwidth are affected in the same way.

Wideband fading channel

Strong scatterers are present well-off of the great circle path between the base and mobile. Time difference between rays may be large compared to the basic unit of information transmitted on the channel (usually a symbol or a bit), the signal will then experience significant distortion, which varies across the channel bandwidth.

Narrowband fading channel

It affects all the frequencies of the signal equally. This means that it can be modeled as a single multiplicative process.



$$y(t) = A\alpha(t)u(t) + n(t) \quad (4)$$

where $\alpha(\cdot)$ is the complex fading coefficient and baseband notation is used, i.e.; $u(\cdot)$ and $n(\cdot)$ are phasors.

Fading: SNR

Fading varies with time; hence, SNR also will vary with time. This implies, that, despite the AWGN case, the instantaneous $\gamma(t)$ and the mean Γ SNR must be considered.

$$\gamma(t) = \frac{A^2|\alpha(t)|^2 E(|u(t)|^2)}{2P_n} = \frac{A^2|\alpha(t)|^2}{2P_n} \quad (5)$$

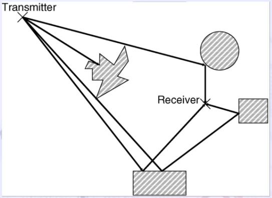
$$\Gamma = E(\gamma(t)) \quad (6)$$

The performance of a fading channel is fully characterized by:

- the mean SNR;
- variations of the instantaneous SNR around the mean SNR.

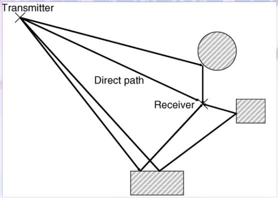
Fading, i.e.; multipath propagation

Fading means that, due to the presence of objects, several waves reach RX by different routes: multipath propagation. Two scenarios must be distinguished:



NLOS

LOS



Multipath propagation

Multipath means that the propagation medium contains several distinguishable “paths” that connect TX and RX. Hence, some fraction of the total energy unavoidably arrives over each path. Two main scenarios must be distinguished:

LOS

A single stronger direct path that connects TX and RX exists, along with multipath links due to local scatterers.

NLOS

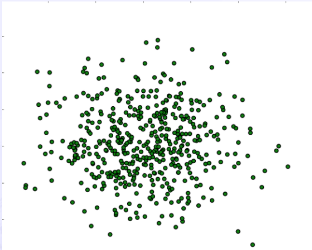
The direct link between TX and RX is blocked. Waves reaching the receiver call for a uniformly distributed phase value. This means that all the multi-paths are equally probable.

ERSLab

F. Nunziata

Outline

Random walk



Multipath signals, due to the differential electrical-path delays, combine at receiver with their relative phases.

- They may add either constructively or destructively according to the value of the relative phase shift.

Random walk

The problem of determining the resultant of a set of random phasors is familiar as that of a 2D random walk

Random walk: the Rayleigh distribution

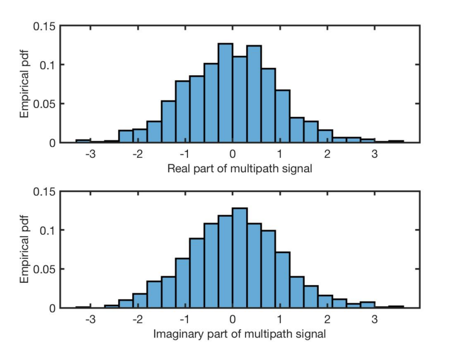
- The real and imaginary parts of the multipath components at the receiver consist of a sum of large number of independent random variables.
- The central limit theorem shows that a sum of enough independent random variables approaches to a normal distribution.
- The variable r, i.e.; the measure of each point from the origin is given by:

$$r = |\alpha| = \sqrt{x^2 + y^2} \tag{7}$$

- r is Rayleigh distributed:

$$p_R(r) = \frac{r}{\sigma^2} e^{\frac{-r^2}{2\sigma^2}} \tag{8}$$

Multipath distribution

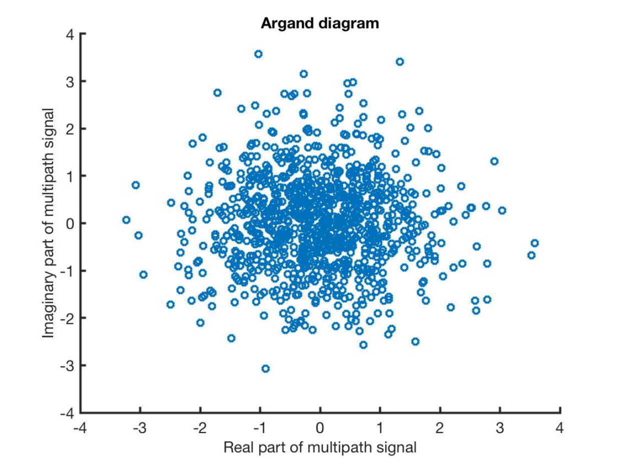


1- $N = 1000$ independent normally distributed random number for the real and imaginary parts of the complex multipath: $\alpha = x + jy$.

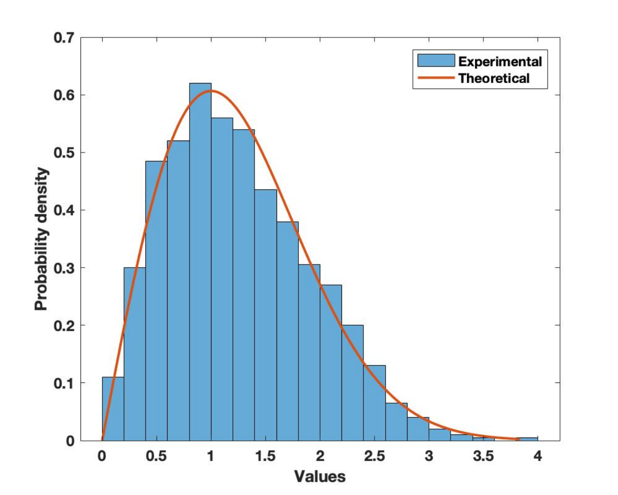
2- x and y are well-described by a normal distribution.

Multipath distribution

The previous points can be also plotted according to the Argand diagram

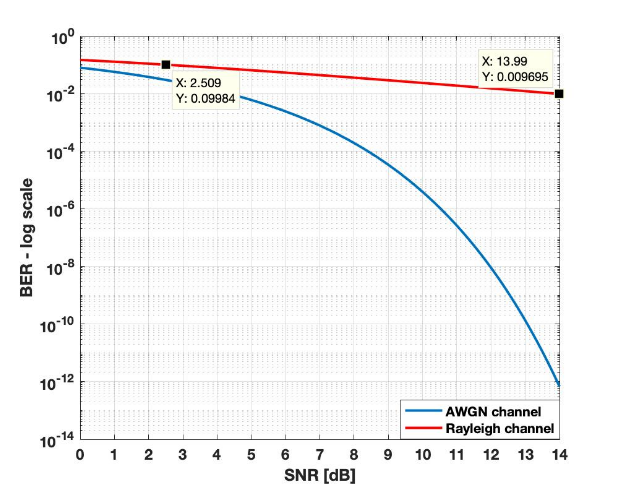


Multipath distribution



The received signal is Rayleigh-distributed

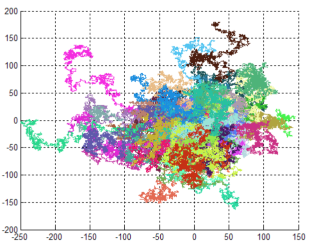
BER when a BPSK modulation is considered



$$BER = 0.5\left(1 - \sqrt{\frac{\Gamma}{1+\Gamma}}\right)$$

Outline

Biased random walk



The received signal consists of a multipath random (Rayleigh-distributed) component plus a coherent LOS component.

- LOS component has approximately constant power (within the bounds set by path loss and shadowing).
- It affects the Rayleigh distribution when it has strong power.

Random walk: the Rice distribution

- The real and imaginary part of the multipath components at the receiver consist of a sum of large number of independent random variables plus a LOS constant component.
- The variable r, i.e.; the measure of each point from the origin is given by:

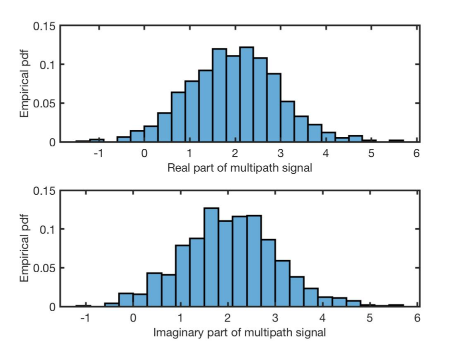
$$r = |\alpha| = \sqrt{x^2 + y^2} \tag{9}$$

- r is Rice distributed:

$$p_R(r) = \frac{r}{\sigma^2} e^{\frac{-(r^2+s^2)}{2\sigma^2}} I_0\left(\frac{rs}{\sigma^2}\right) \tag{10}$$

- with σ and s being the std of the multipath components and the magnitude of the LOS component, respectively.

Received signal distribution

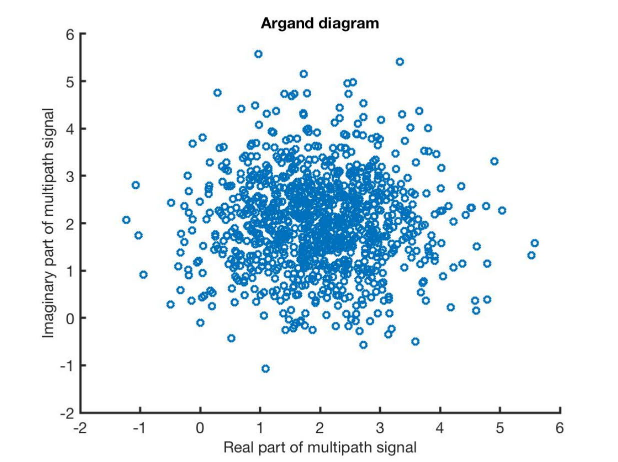


1- $N = 1000$ independent Gaussian distributed random number (mean =LOS, std = 1) for the real and imaginary parts of the complex multipath: $\alpha = x + jy$.

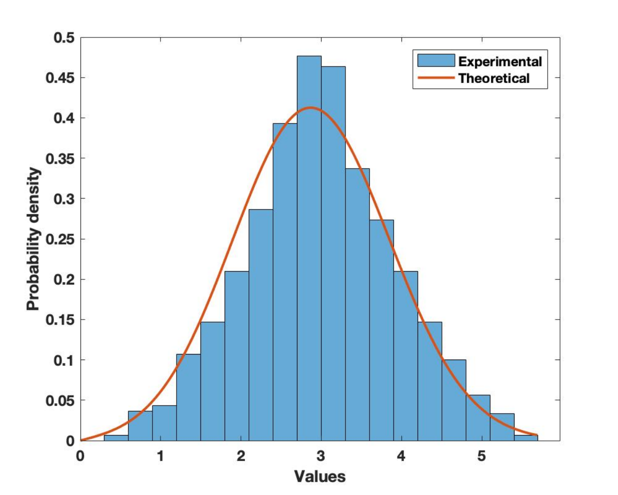
2- x and y are well-described by a Gaussian distribution.

Received signal distribution

The previous points can be also plotted according to the Argand diagram



Received signal distribution



The received signal is Rice-distributed

The Rice factor k

The Rice distribution is often expressed in terms of the Rice factor k.

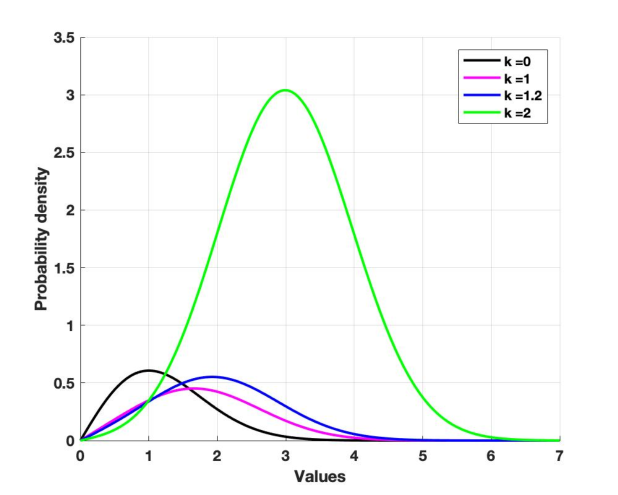
Rice factor

$$k = \frac{\text{Power of the LOS part}}{\text{Power of the multipath part}} = \frac{s^2}{2\sigma^2} \tag{11}$$

- Hence, the Rice distribution can be written as:

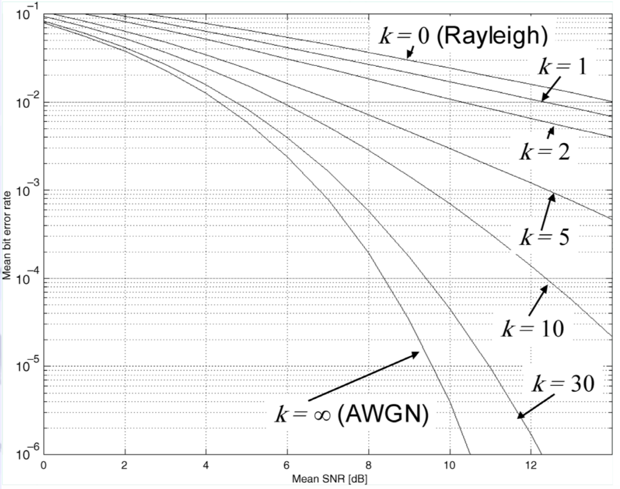
$$p_R(r) = \frac{r}{\sigma^2} e^{\left(\frac{-r^2}{2\sigma^2}\right)} e^{-k} I_0\left(\frac{\sqrt{2rk}}{\sigma}\right) \tag{12}$$

Received signal distribution



The larger is k, the stronger is the LOS component.

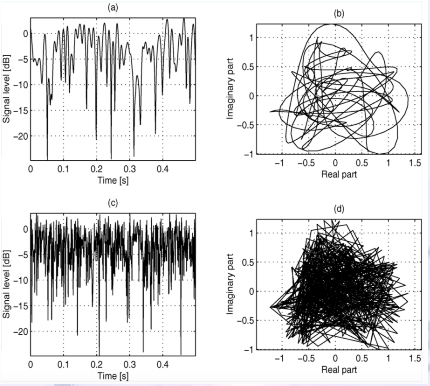
BER when a BPSK modulation is considered



Rice channel is more “friendly” than the Rayleigh one

Outline

What it is: signal's spectrum



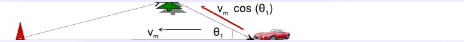
Correlated (non-flat spectrum)

Uncorrelated (flat spectrum)

Second-order statistics tell how rapidly the signal magnitude changes between different levels. This info is commonly specified in terms of spectrum of the signal.

Signal's spectrum: The Doppler effect

The specific shape of the signal's spectrum can be explained in terms of Doppler spreading.



- A mobile moves at a velocity v_m in straight line.
- The mobile direction makes an angle θ_1 with the incoming em wave.

The Doppler effect

It results in a shift of the frequency of the incoming wave by a factor that is proportional to the component of the mobile speed along with the em wave direction.

The frequency increases (decreases) when the mobile moves towards (away from) the source.

The Doppler shift

The Doppler shift

It is associated with the rate at which the mobile crosses wavefronts of the incoming em wave.

- The Doppler shift f_d is given by:

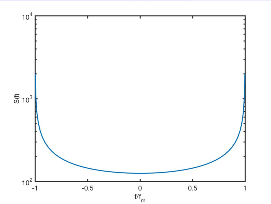
$$f_d = f_m cos\theta \quad \text{with } f_m = f_c \frac{v}{c} \tag{13}$$

- f_m is the maximum Doppler shift and it is associated to $\theta = 0$.
- The Doppler shift f_d associated to the em wave can have apparent frequency in the range:

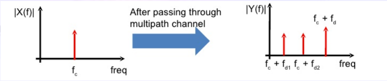
$$f_c - f_m \leq f \leq f_c + f_m \tag{14}$$

The Doppler bandwidth

- In case of multipath, waves arrive with several directions.



- Each wave has its own Doppler shift.
- The bandwidth of the received signal is therefore spread wrt the transmitted one: Doppler spread.



The overall spectral width associated to the received signal is termed as Doppler bandwidth.

The Doppler spectrum

The Doppler spectrum affects significantly second-order fading statistics.

- To model the Doppler spectrum, assumptions on the arriving angle of the multipath signals must be done.
- To obtain a reference model, one can assume arriving angles being uniformly distributed within $[-\pi, \pi]$. Hence, $p(\theta)\ U(-\pi, \pi)$.
- If the mobile antenna is pointing in the direction θ, with a gain $G(\theta)$, the mean power arriving from an elementary angle $d\theta$ is given by:

$$P(\theta) = G(\theta)p(\theta)d\theta \tag{15}$$

The classical Doppler spectrum

- The power spectrum of the received signal $S(f)$ is simply given by:

$$P(f) = S(f)df \tag{16}$$

- Hence, noting that two angles $\pm\theta$ call for the same Doppler shift:

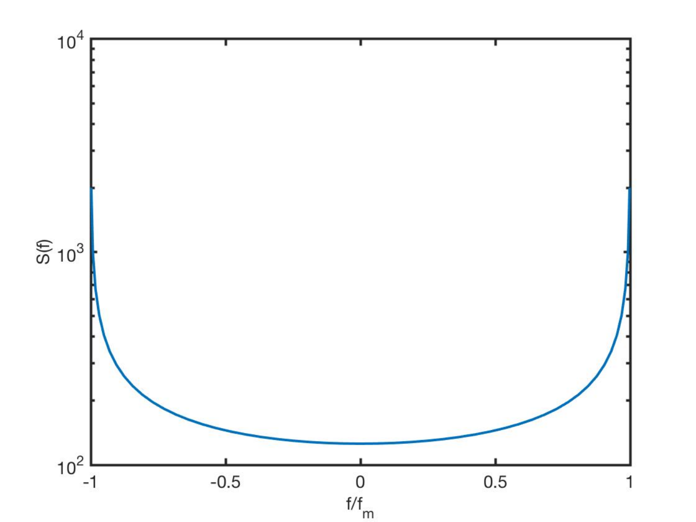
$$|S(f)| = \frac{G(\theta)p(\theta) + G(-\theta)p(-\theta)}{|df/d\theta|} \tag{17}$$

- Since, $|df/d\theta| = f_m| - sin\theta|$ and assuming $G(\theta) = 1.5$:

$$|S(f)| = \frac{1.5/2\pi + 1.5/2\pi}{f_m| - sin\theta|} = \frac{1.5}{\pi f_m \sqrt{1 - (f/f_m)^2}} \tag{18}$$

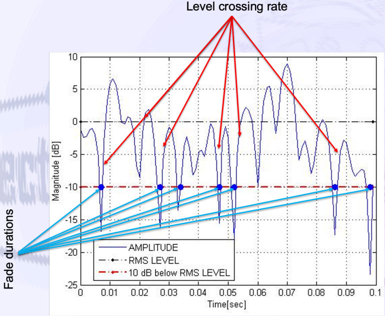
- In the last step, $sin\theta = \sqrt{1 - cos^2\theta} = \sqrt{1 - (f/f_m)^2}$

The classical Doppler spectrum



Synthetic parameters

Sometimes is difficult measuring Doppler spectra due to their limited fractional bandwidth.



Parameters, which are directly related to the Doppler spectrum, and can be measured more directly are:

- the level of crossing rate;
- the average fade duration.

Outline

Autocorrelation function

The effects of the Doppler spread can be analyzed in the time domain using the autocorrelation function that deals with the correlation of a signal with its delayed version.

$$\rho(\tau) = \frac{E[\alpha(\tau)\alpha^*(t+\tau)]}{E[|\alpha|^2]} \quad (19)$$

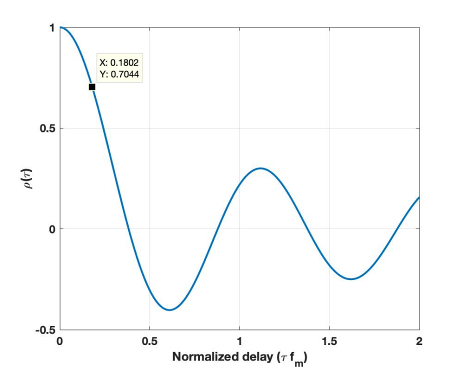
- When a classical Dppler spectrum with Raylaigh fading is assumed, the autocorrelation function becomes:

$$\rho(\tau) = J_o(2\pi f_m \tau) \quad (20)$$

- with $J_o(\cdot)$ being the Bessel function of the first kind and zero order.

Autocorrelation function - coherence time

The coherence time T_c is the time over which the channel response can be considered constant. This implies a normalized autocorrelation function close to unity within T_c seconds.



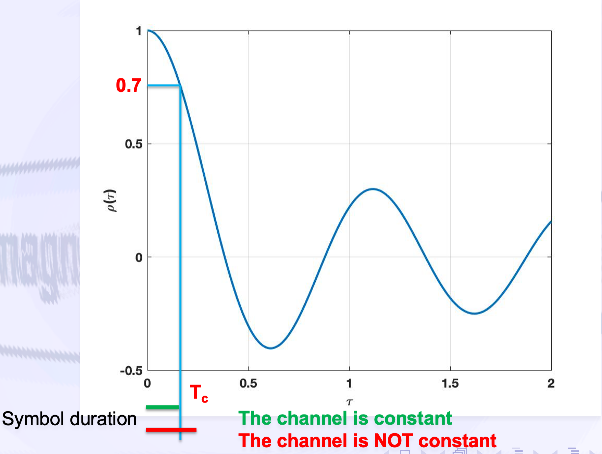
$$T_c \propto \frac{1}{f_m} \qquad (21)$$

- Typically a threshold equal to 0.7 is assumed:

$$T_c \approx \frac{9}{16\pi f_m} \qquad (22)$$

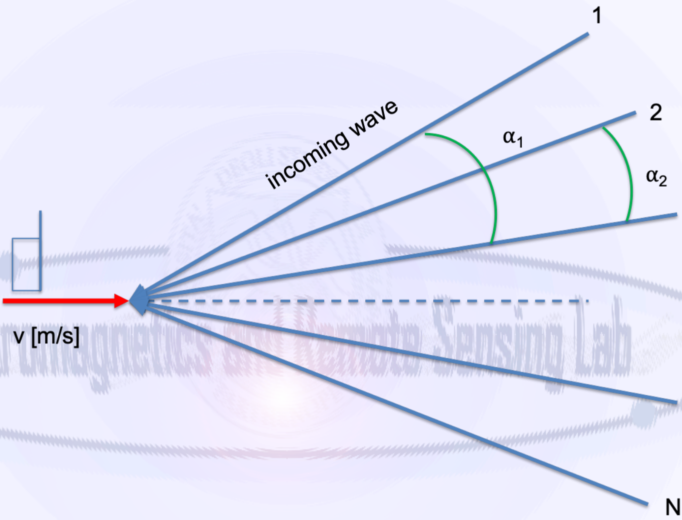
Channel vs symbol rate

The channel can be considered constant when the maximum symbol duration is smaller than T_c.



Outline

Sketch of the scenario



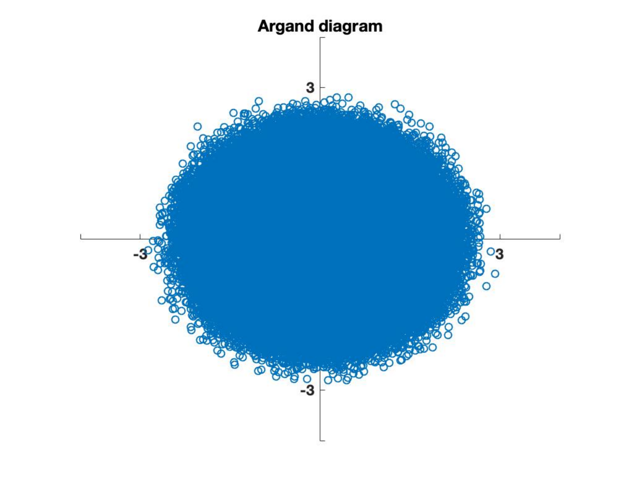
Simulation rationale

The baseband received signal is assumed to be made of N waves coming from the N paths.

$$
\begin{aligned}
y(t) &= y_c(t) + j y_s(t) \\
y_c(t) &= \frac{1}{\sqrt{N}} \sum_{n=1}^{N} cos(\omega_d t cos\alpha_n + \phi_n) \\
y_s(t) &= \frac{1}{\sqrt{N}} \sum_{n=1}^{N} sin(\omega_d t cos\alpha_n + \phi_n)
\end{aligned}
\tag{23}
$$

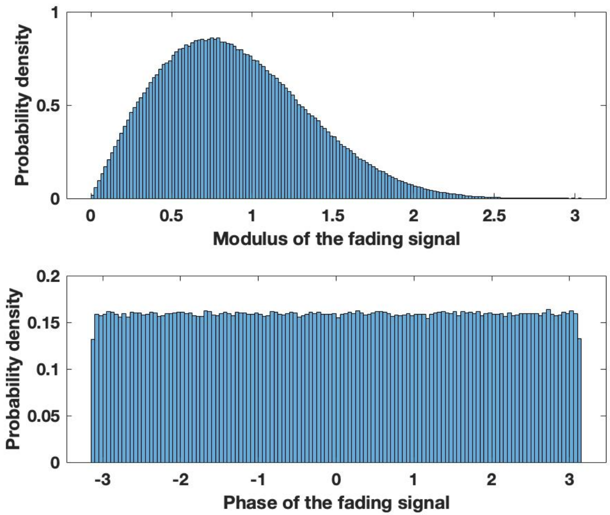
- ω_d is the maximum radian Doppler frequency;
- $\alpha_n = \frac{2\pi n + \theta_n}{N}$ is the arriving angle;
- ϕ_n and θ_n are statistically independent and uniformly distributed over $[-\pi, \pi)$ for all n.

Do it yourself - Argand

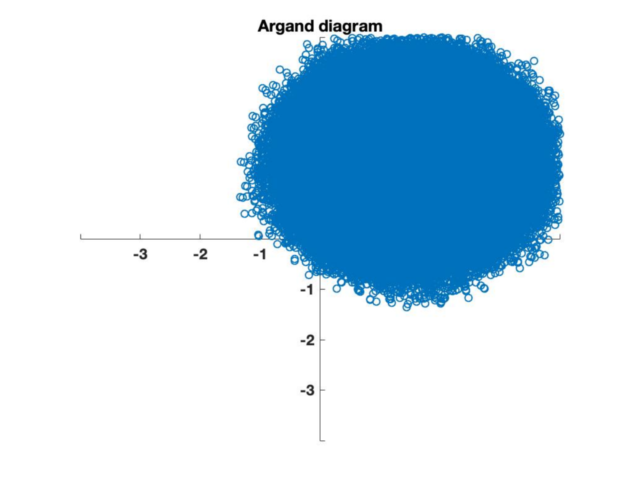


Sampling frequency = $1e6$Hz, mobile velocity $50km/h$, N=10

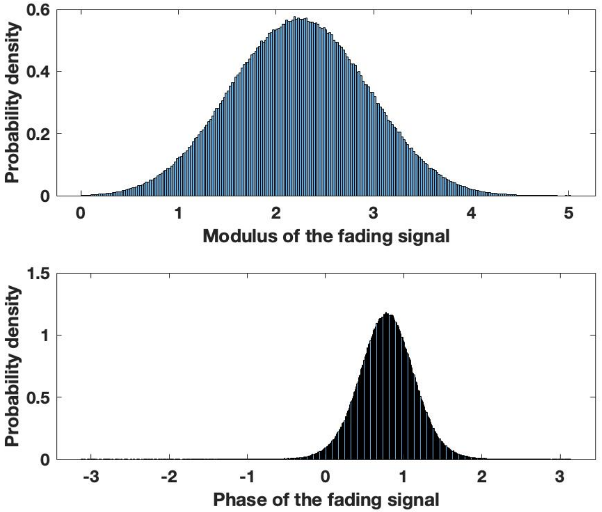
Do it yourself - Density function



Do it yourself - Argand - LOS=1.5



Do it yourself - Density function

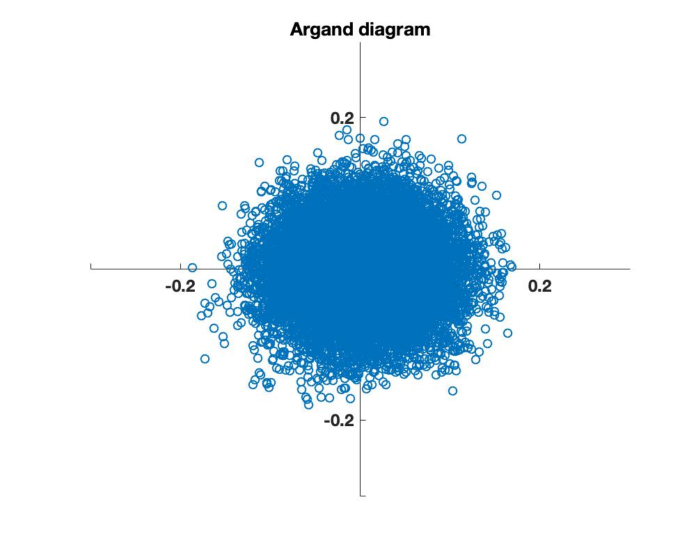


Outline

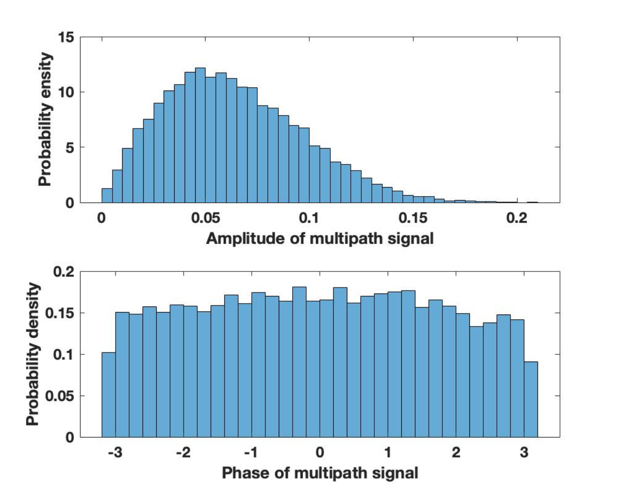
Reverberating chamber

A microwave electrically-large metallic chamber where the field is made random and on the average uniform and isotropic by means of a proper stirring.

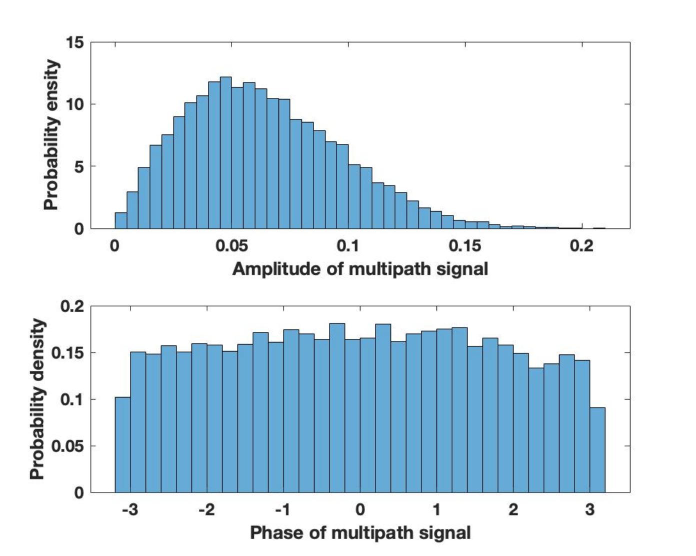
Argand



Multipath components

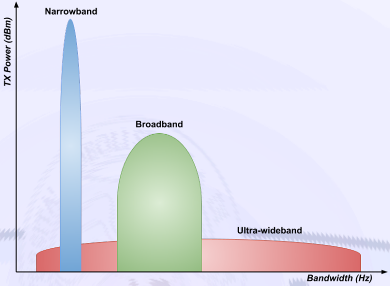


Probability density



Outline

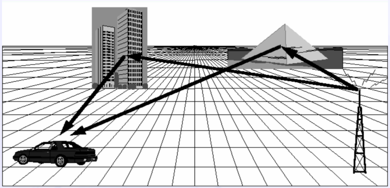
Wideband fading



Broad-band communication

Ultra-wideband communications use channels that have a bandwidth of 500 MHz or more, with transmissions at a low power. Wide-band communications refer to a spectral bandwidth of 20 MHz or more.

Wideband fading



The signal that arrives at the receiver consists of multiple beams.

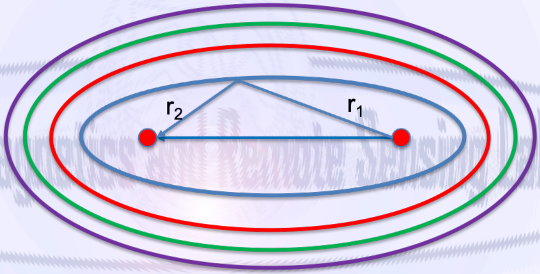
- Each beam consists of several waves that, due to the scattering properties of the obstacles, may be affected by narrowband fading.
- If the delay of the beams is comparable or even larger than the symbol duration, wideband fading occurs.

Outline

A simple model

Equal-delay ellipses

Let the mobile and the base station be the foci of an ellipse. All the scatterers located on the ellipse contribute to the received energy with the same delay τ.



$$\tau = \frac{r_1 + r_2}{c} \tag{24}$$

A simple model

- The signal received by the mobile terminal consists of the sum of the waves (echoes) resulting from each scatterer.
- Each wave will call for a phase θ and an amplitude a that are related to the electrical and geometrical properties of the scatterer.
- The time delay that characterizes each wave is given by eq.(24).

The signal r received by the mobile is given by:

$$r = a_1 e^{j(\omega\tau_1 + \theta_1)} + a_2 e^{j(\omega\tau_2 + \theta_2)} + \dots \tag{25}$$

A simple model

In the narrowband channel, the time delays are approximately the same.

$$r = e^{j\omega\tau}\left(a_1 e^{j\theta_1} + a_2 e^{j\theta_2} + \dots\right). \qquad (26)$$

- The amplitude does not depend on the carrier frequency.
- The channel can be onsidered as a single multiplicative process since all the frequencies of the received signal are affected in the same way by the channel.
- No signal distortion occurs.

A simple model

Channel transfer function

It depicts the power of the received signal with respect to the frequency

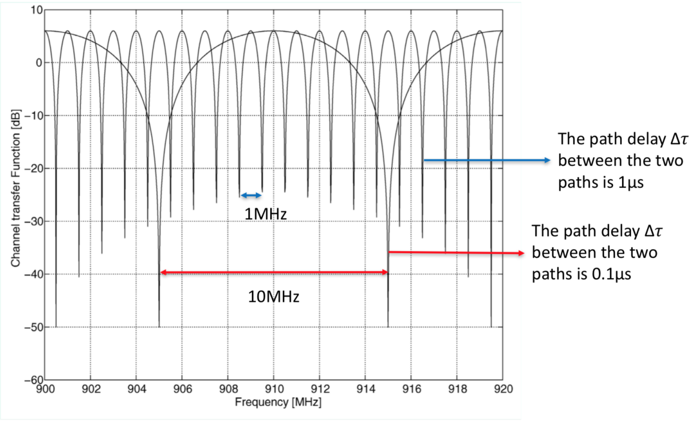
- By specializing eq.(25) in the case of only two paths received with the same amplitude *a*:

$$\begin{aligned} |r| &= \left| a e^{j(\omega\tau_1+\theta_1)} \left(1 + e^{j(\omega(\tau_2-\tau_1)+(\theta_2-\theta_1))}\right) \right| \\ &= a\sqrt{(1+cosx)^2 + sin^2 x} \\ &= a\sqrt{2(1+cosx)} \end{aligned} \qquad (27)$$

- with $x = \omega\delta_\tau + \delta_\theta$, $\delta_\tau = \tau_2 - \tau_1$ and $\delta_\theta = \theta_2 - \theta_1$.

Why we do care

Transfer function of a two-path channel for two relative delays



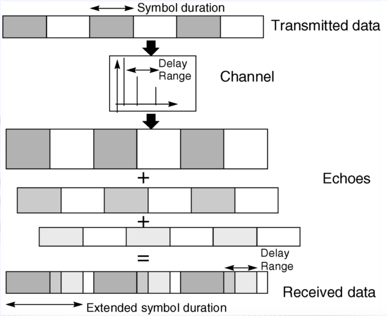
Why we do care

Let's suppose to transmit a signal whose bandwidth is 1MHz

- If $\Delta\tau = 0.1\mu s$, the transfer function cancels at multiple of 10MHz. Hence, the TX signal would experience a constant attenuation and the channel is a narrowband one.
- If $\Delta\tau = 1\mu s$, the transfer function cancels at multiple of 1MHz. Hence, the channel amplitudes vary significantly across the signal bandwidth and it must be considered wideband.

Intersymbol interference (ISI)

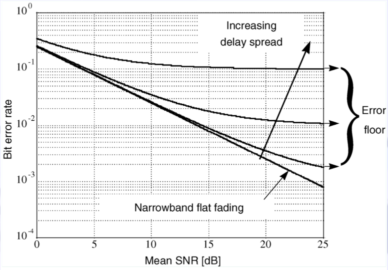
The energy of the TX symbol is spread in time. Hence, the symbol reaches the receiver with a delay that consists of a constant transmission delay plus a delay spreading



The symbol is still arriving at the receiver when the energy associated to the next symbol starts arriving: ISI.

Impact of delay spread on BER

The ISI makes the BER reaches a floor (aka “error floor”) at larger SNR.

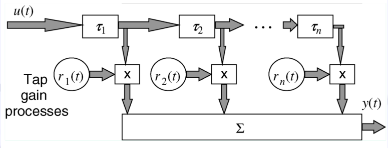


Despite the case of narrowband fading where BER decreases without any limit, in this case ISI dominates at higher signal levels flattening the BER wrt SNR.

Outline

Wideband channel model

The effects of scatterers at discrete delays ranges are concentrated in individual taps each representing single beam whose amplitude varies in time according to the narrowband fading statistics.



Wideband channel

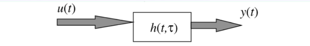
Wideband channel is therefore a combination of several paths subjected to narrowband fading, combined together with appropriate delays

Input delay spread function

- The beams are assumed to be uncorrelated from each other. This is justified by the fact that they arise from physically distinct scatterers that are separated by many wavelengths.
- Wideband channels is characterized by a time-variant impulse response function, aka input delay spread function:

Input delay spread function

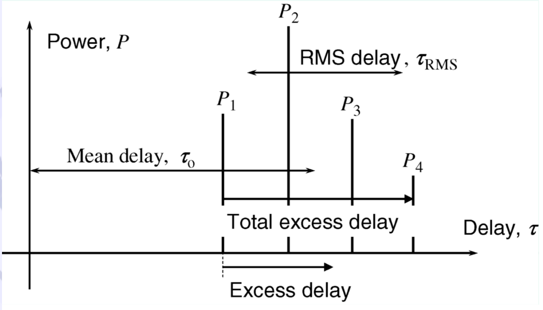
$$y(t) = u(t) ** h(t,\tau) = \int_{-\infty}^{+\infty} h(t,\tau)u(t-\tau)d\tau \qquad (28)$$



Power delay profile (PDP)

It represents the variation of the mean power in the channel with delay

$$P(\tau) = \frac{E[|h(t,\tau)|^2]}{2} \qquad (29)$$



PDP is typically discretized along with the delay to provide n individual beams of power $P_1 \ldots P_n$.

PDP descriptors

Few synthetic parameters may be used to characterize PDP

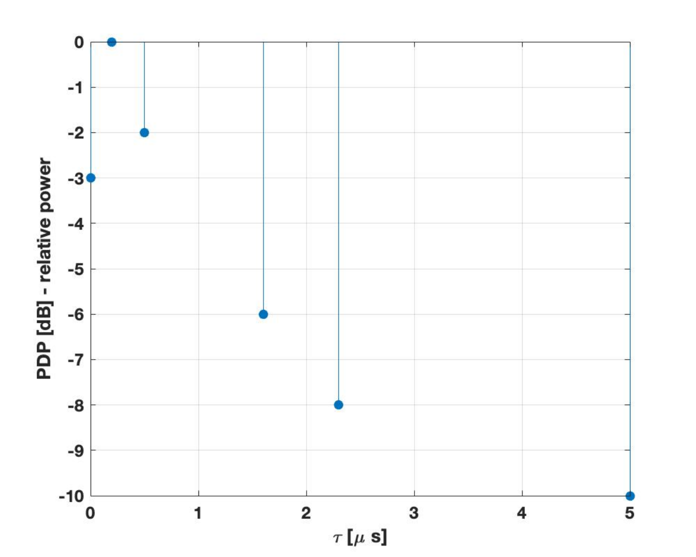
- Total excess delay: It describes the spreading of the symbol when transmitted through the channel. It is given by the difference between the delays of the first and the last arrived beam.
- Mean delay

$$\tau_o = \frac{1}{P_T}\sum_{i=1}^{N} P_i\tau_i \quad P_T = \sum_{i=1}^{N} P_i \tag{30}$$

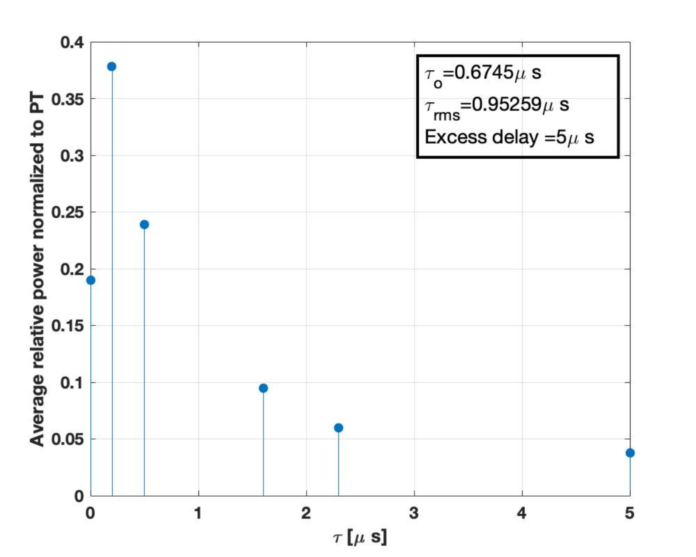
- RMS delay spread τ_{rms}. It described the spreading of the delay with respect to the mean delay.

$$\tau_{RMS} = \sqrt{\frac{1}{P_T}\sum_{i=1}^{N} P_i\tau_i^2 - \tau_o^2} \tag{31}$$

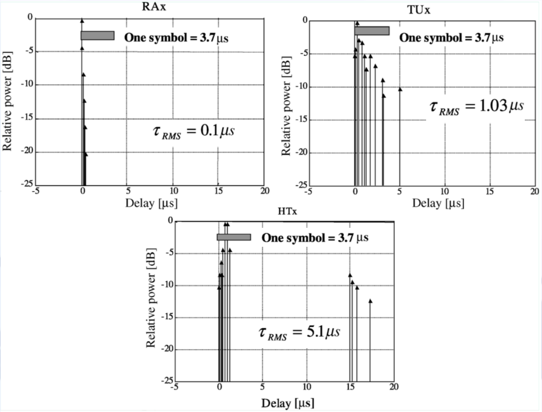
PDP descriptors - Example



PDP descriptors - Example



Examples of PDP in the GSM system



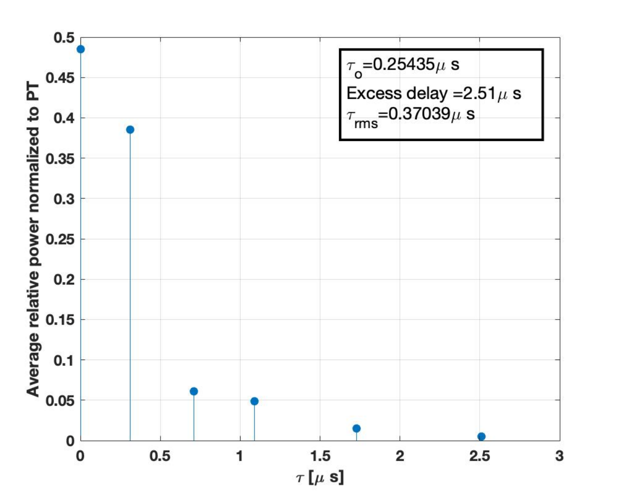
Example of measured GSM PDPs profiles related to macrocells that refer to Rural Area (RA), Typical Urban (TU) and Hilly Terrain (HT).

Typical reference values of RMS delay spreads

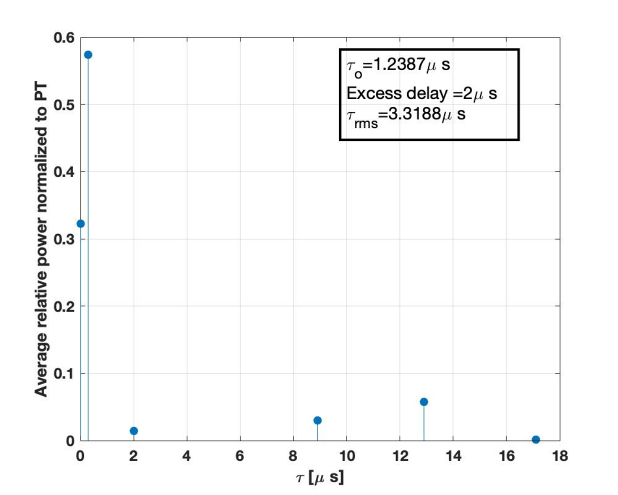
Environment	Approximate RMS delay spread [μs]
Indoor cells	0.01-0.05
Mobile satellite	0.04-0.05
Open area	<0.2
Suburban macrocell	<1
Urban macrocell	1-3
Hilly area macrocell	3-10

These values indicate that, for instance, indoor cells may admit (nominal) data rates larger than hilly area macrocells. Equalization techniques must be applied to improve data rates in hilly areas reducing ISI.

Examples of PDP in the UMTS system - low τ_{RMS}



Examples of PDP in the UMTS system - high τ_{RMS}



Outline

Time-variant transfer function (TVT)

The channel may be analyzed in the frequency domain where it is fully characterized by its TVT. Hence the channel may be considered as a filter with a time-variant frequency response.

TVT

$$T(f,t) = F[h(t,\tau)] = \int_{-\infty}^{+\infty} h(t,\tau) e^{-j2\pi f} d\tau \tag{32}$$

- The spectrum of the output signal at the time t is given by:

$$Y(f,t) = U(f)T(f,t) \tag{33}$$

Coherence bandwidth

- In practical cases, TVT is unknown a priori. The channel can be hence characterized in terms of the correlation between frequency components of the output spectrum separated by a given shift.
- The correlation between two components of the TVT function separated by Δf and Δt is denoted by $\rho(\Delta f, \Delta t)$.

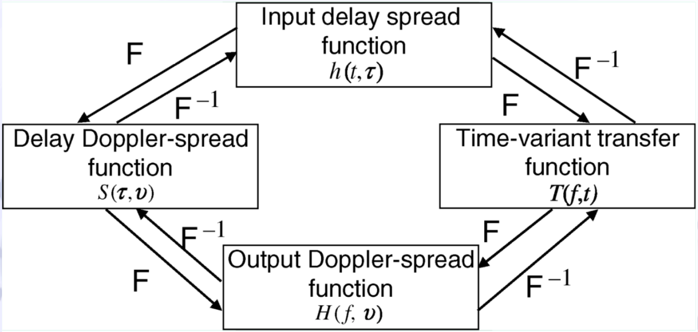
Coherence bandwidth B_c

It is the Δf that makes $\rho(\Delta f, \Delta t)$, evaluated at $\Delta t = 0$, equals 0.5.

A wideband channel is such that the signal bandwidth is larger than B_c or, equivalently, the symbol duration is shorter than τ_{rms}

$$B_c \propto \frac{1}{\tau_{rms}} \tag{34}$$

The Bello functions



Outline

Countermeasures to overcome impairments due to wideband fading

- Directional antennas: to reduce the energy transmitted towards scatterers resulting in far-out echoes.
- Small cells: they allow a reduced maximum differential delay.
- Diversity: It allows reducing the level of deep fades therefore reducing the SNR for a given BER and, hence, the error floor is reduced
- Equalizers: It consists of applying an adaptive filter to flatten the channel frequency response or by making constructive use of the energy in the delayed taps.
- Data rate: By transmitting the required data simultaneously on a large number of carriers, each with a narrow bandwidth, the data throughput can be maintained (OFDM) and wideband fading is attenuated.