Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2021-2022 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Antenna Parameters

Introduction

- To describe the performance of an antenna, definitions of various parameters are necessary.
- Some of the parameters are interrelated and not all of them need be specified for complete description of the antenna performance.

Antenna Parameters

Parameters of the Tx Antenna

Parameters of the Rx Antenna

Parameters of the Tx Antenna

- Effective length
  - Radiation pattern
  - Radiation pattern lobes
  - Beamwidth
- Directivity
- Gain
- Radiation Resistance
- Equivalent circuit of the tx antenna
- Input Impedance and Input Resistance

Effective Length

$r >> D$

$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r, \vartheta, \varphi) = \frac{j\zeta I}{2\lambda} \frac{e^{-j\beta r}}{r} \mathbf{l}(\vartheta, \varphi)$$

$$\zeta \mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{l}(\vartheta, \varphi) = l_\vartheta(\vartheta, \varphi)\hat{i}_\vartheta + l_\varphi(\vartheta, \varphi)\hat{i}_\varphi$$

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}|\vec{\mathbf{E}}|^2 \hat{i}_r = \frac{\zeta}{2}|\vec{\mathbf{H}}|^2 \hat{i}_r$$

$\mathbf{l}(\vartheta, \varphi) = l_\vartheta(\vartheta, \varphi)\hat{i}_\vartheta + l_\varphi(\vartheta, \varphi)\hat{i}_\varphi$ **effective length** of the antenna

Effective Length

$r >> D$

$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\mathbf{l}(\vartheta,\varphi)$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{l}(\vartheta,\varphi) = l_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + l_\varphi(\vartheta,\varphi)\hat{i}_\varphi$$

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{i}_r$$

Elementary electrical dipole

$$\mathbf{E}(\vec{\mathbf{r}}) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\Delta z\sin\vartheta\,\hat{i}_\vartheta$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{l}(\vartheta,\varphi) = \Delta z\sin\vartheta\,\hat{i}_\vartheta$$

Small loop antenna

$$\mathbf{E}(\vec{\mathbf{r}}) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}(-j\beta\Delta S)\sin\vartheta\,\hat{i}_\varphi$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{l}(\vartheta,\varphi) = -j\beta\Delta S\sin\vartheta\,\hat{i}_\varphi$$

Parameters of the Tx Antenna

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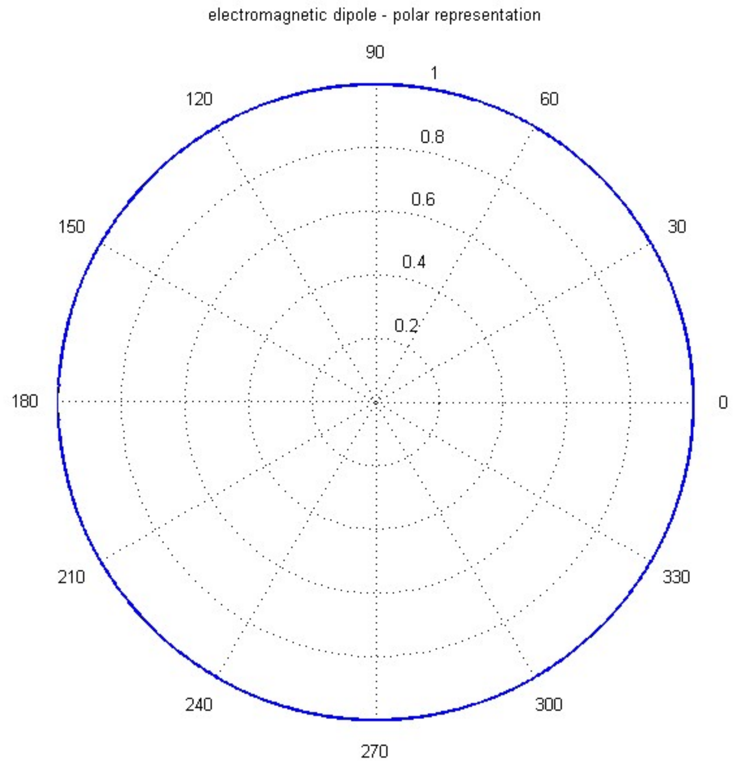
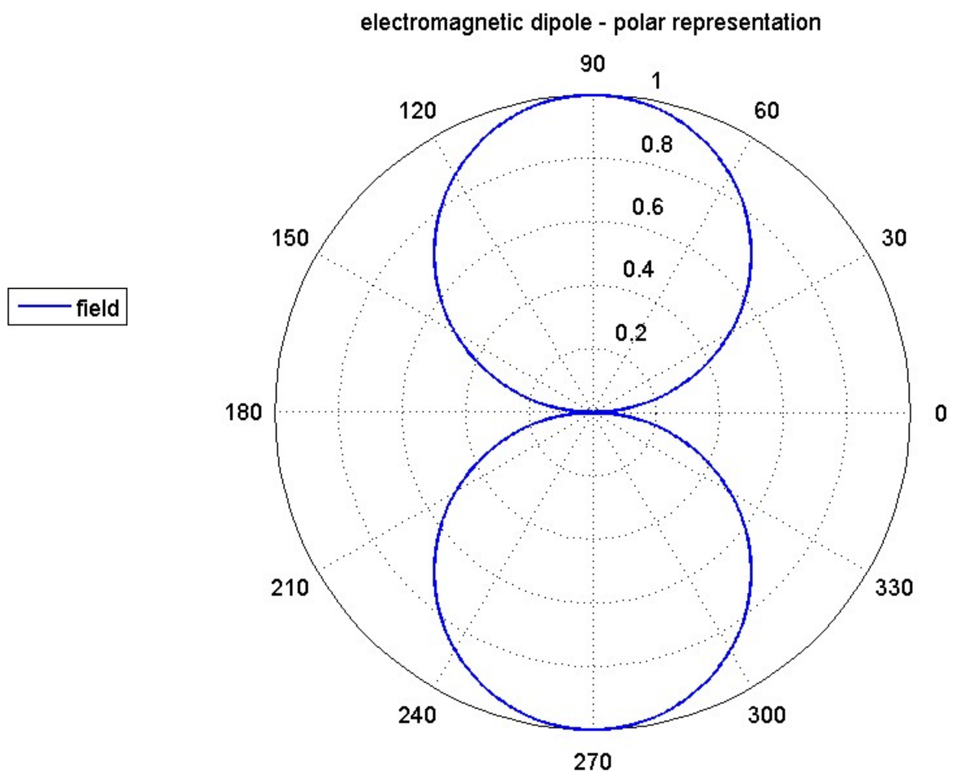
Radiation pattern

an example: the electrical elementary dipole

$$\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta\, \hat{i}_\vartheta$$





Parameters of the Tx Antenna

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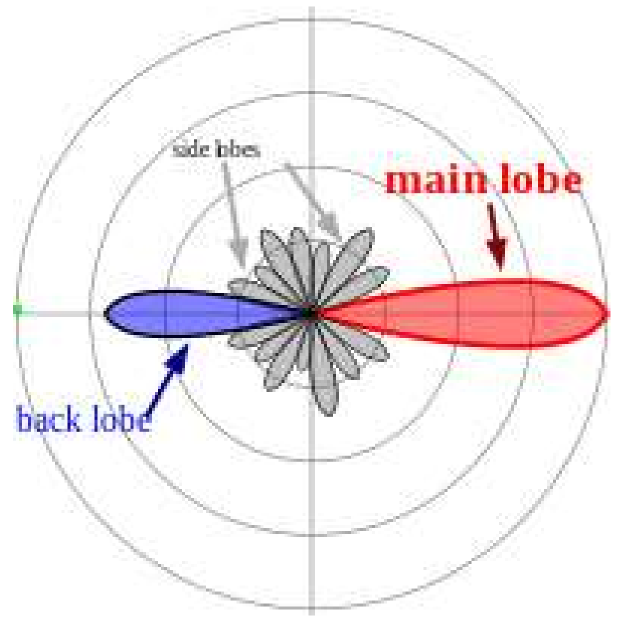
Radiation pattern lobes

- In some very specific directions there are zeros, or *nulls*, in the pattern indicating no radiation.
- The protuberances between the nulls are referred to as *lobes*, and the main, or major, lobe is in the direction of maximum radiation.

- There are also *side lobes* and *back lobes*.

  - A *back lobe* is "a radiation lobe whose axis makes an angle of approximately 180◦ with respect to the beam of an antenna." Usually it refers to a minor lobe that occupies the hemisphere in a direction opposite to that of the major (main) lobe.

  - *Side lobes* and *back lobes* divert power away from the main beam and are desired as small as possible.

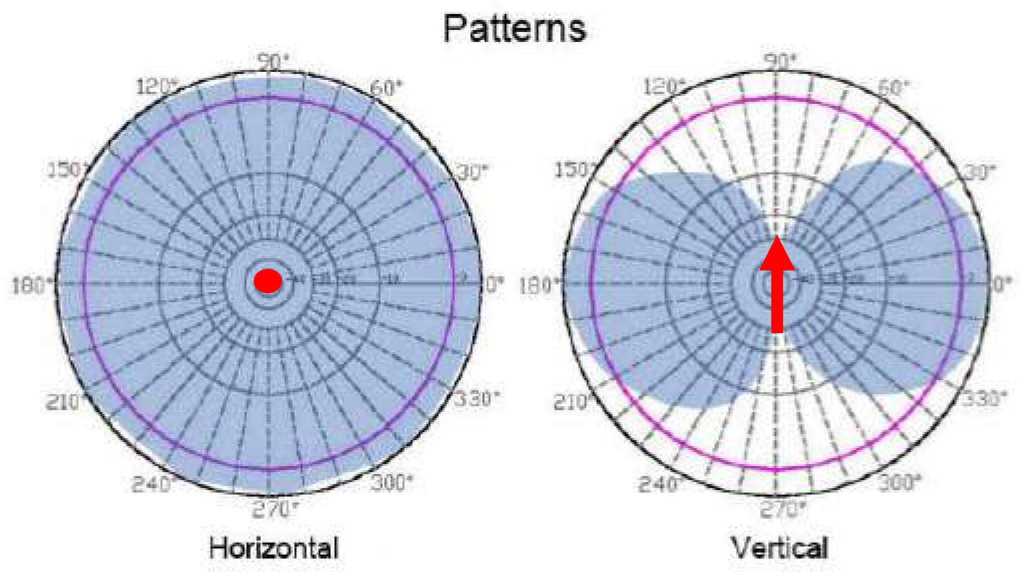


Radiation pattern

three examples from the real life

Radiation pattern

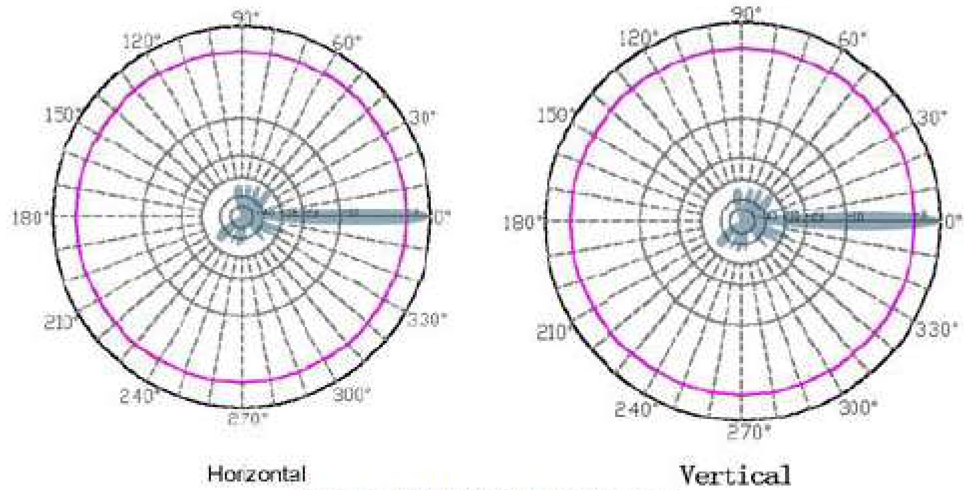
three examples from the real life



ZDAEW1900-3 **Patterns**

Radiation pattern

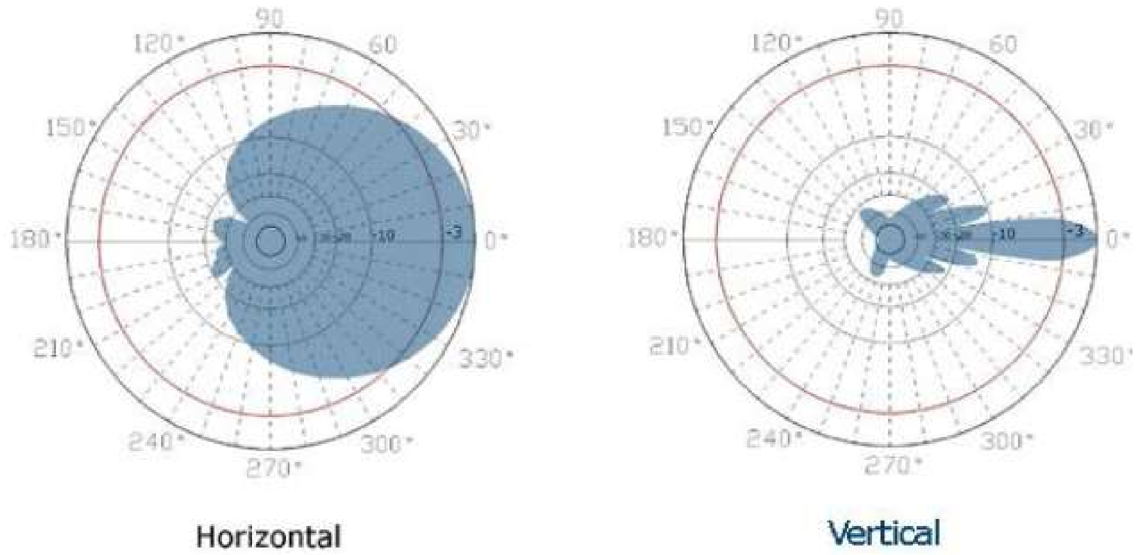
three examples from the real life



ZDASP5400-29-6 **Patterns**

Radiation pattern

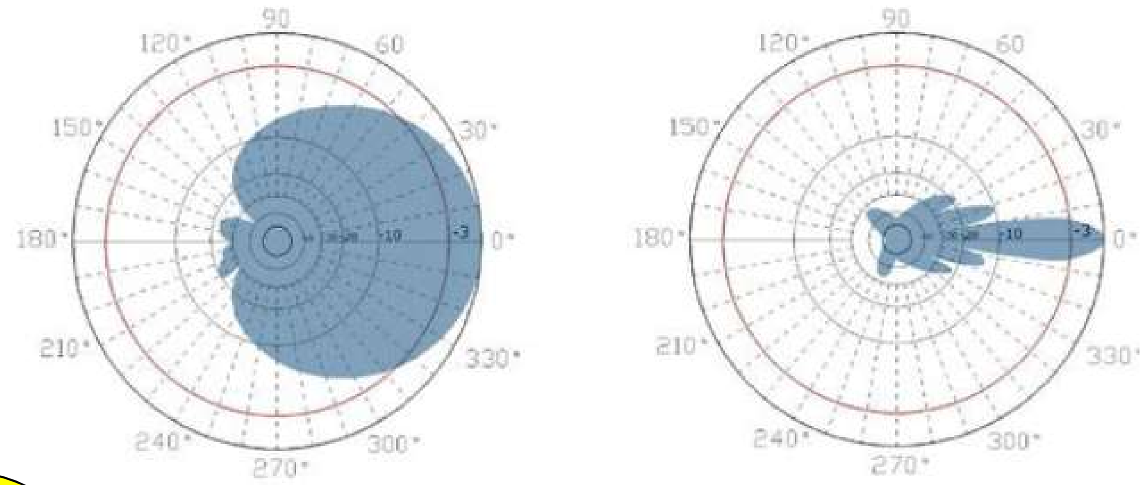
three examples from the real life



ZDADJ800-13-90 Patterns

Radiation pattern

three examples from the real life



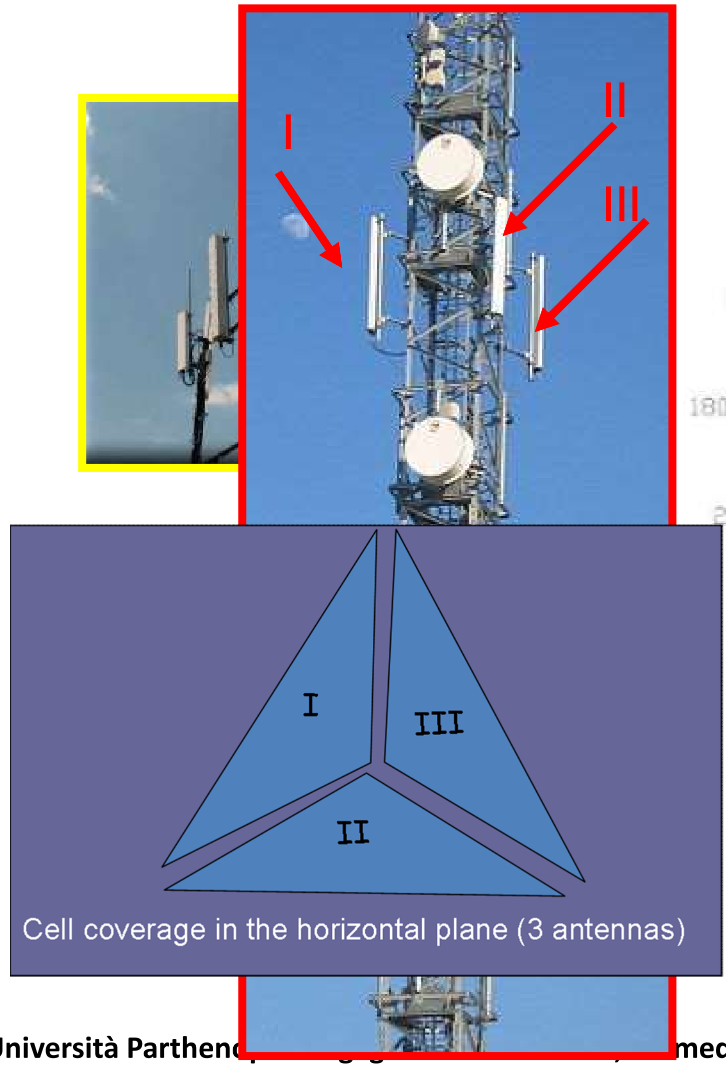
Horizontal   Vertical

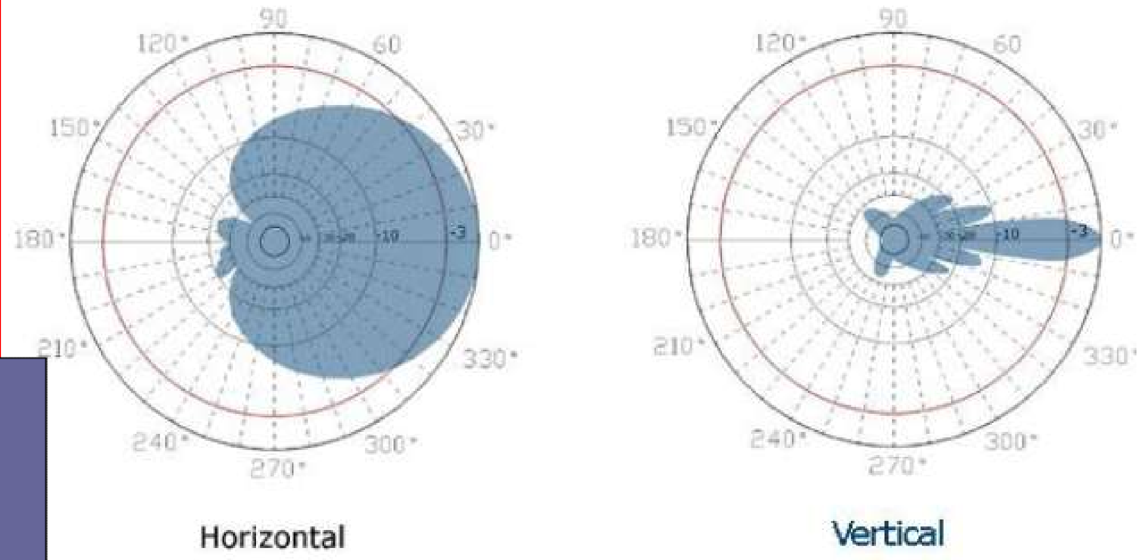
ZDADJ800-13-90 Patterns

….they should be isotropic in the horizontal plane!!

Radiation pattern

three examples from the real life

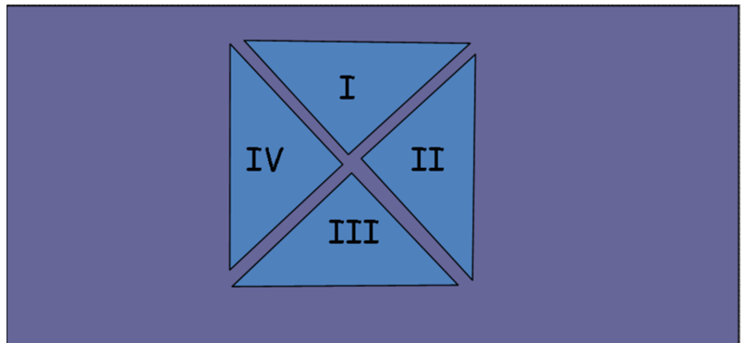




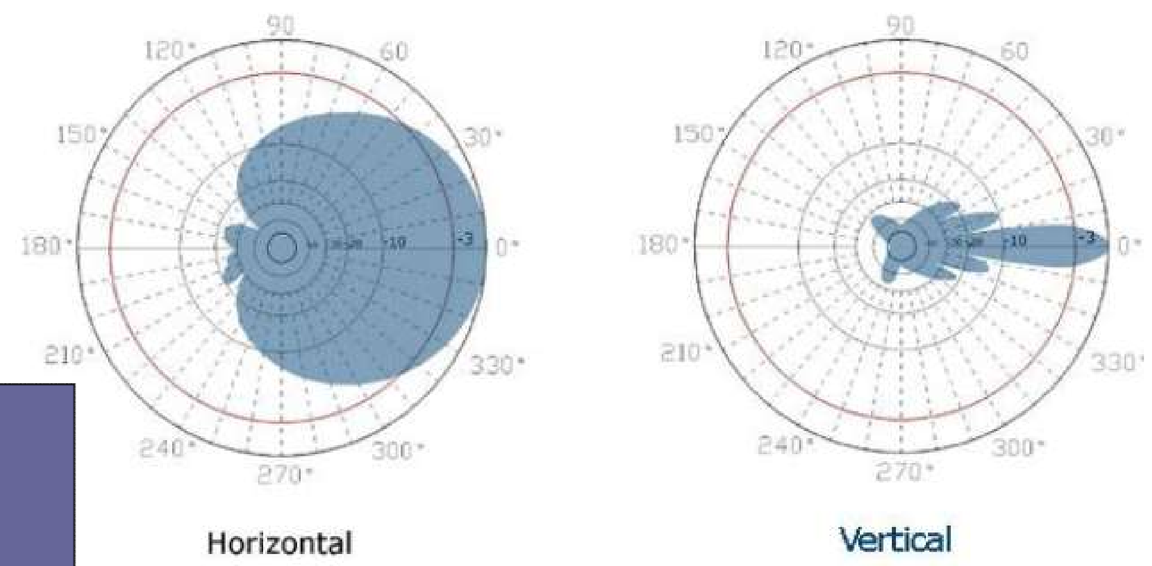
ZDADJ800-13-90 Patterns

Radiation pattern

three examples from the real life



Cell coverage in the horizontal plane (4 antennas)



ZDADJ800-13-90 Patterns

Parameters of the Tx Antenna

- Effective length
  - Radiation pattern
  - Radiation pattern lobes
  - Beamwidth
- Directivity
- Gain
- Radiation Resistance
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- Input Impedance and Input Resistance

Beamwidth

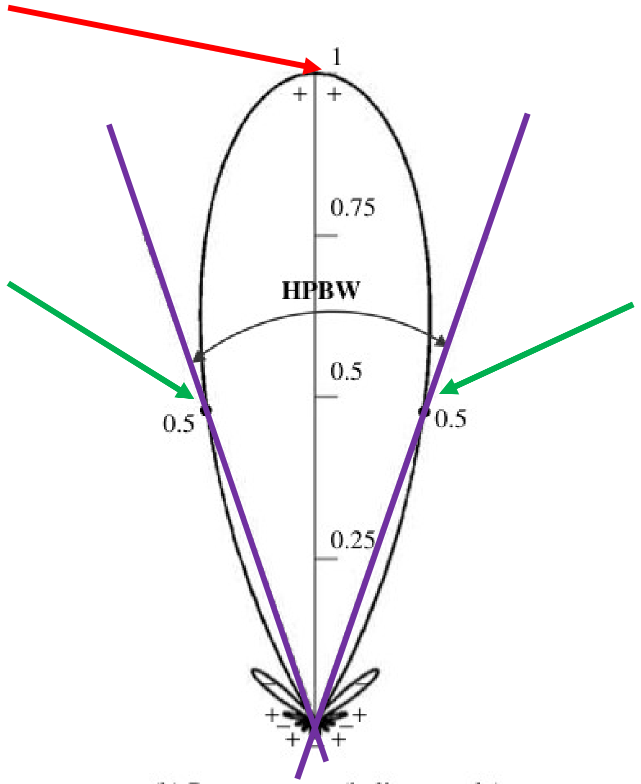
- Associated with the pattern of an antenna is a parameter designated as *beamwidth*.
- The *beamwidth* of a pattern is defined as the angular separation between two identical points on opposite side of the pattern maximum.

A number of different definitions for the beamwidth exist

- One of the most widely used is the *Half-Power Beamwidth* (*HPBW*), or 3-dB beamwidth.
- Another one is the angular separation between the two nulls, and it is referred to as the *First-Null Beamwidth* (*FNBW*).

Beamwidth

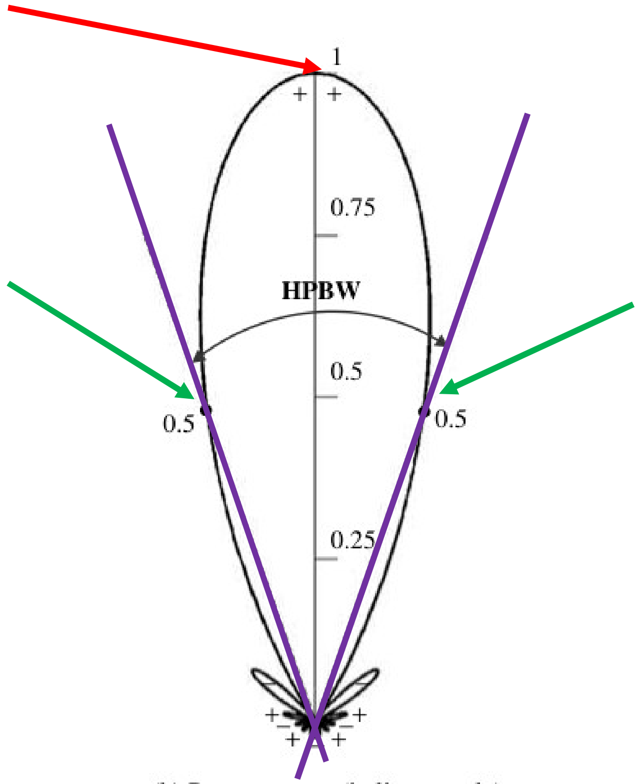
Two-dimensional normalized *power* pattern (*linear scale*)



(b) Power pattern (in linear scale)

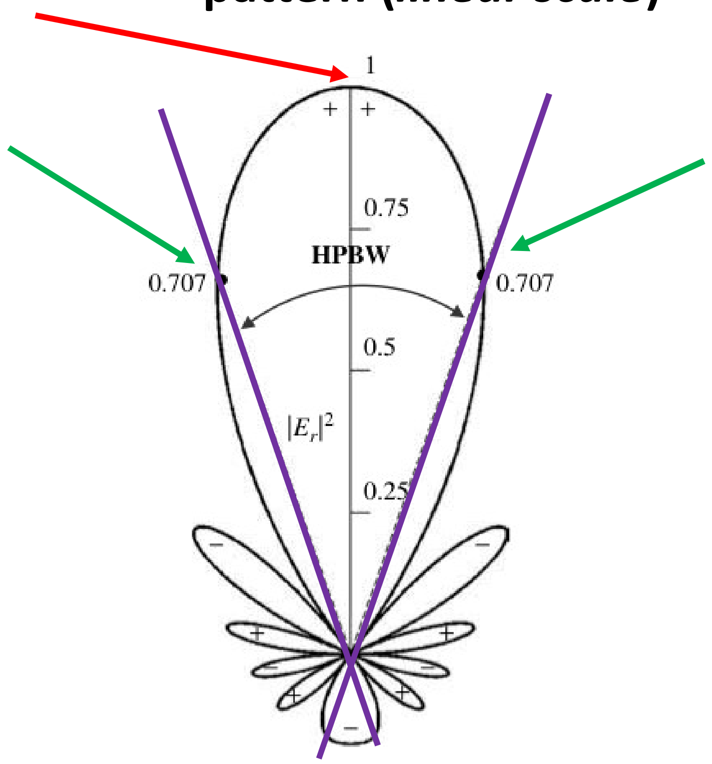
Beamwidth

Two-dimensional normalized *power* pattern (*linear scale*)



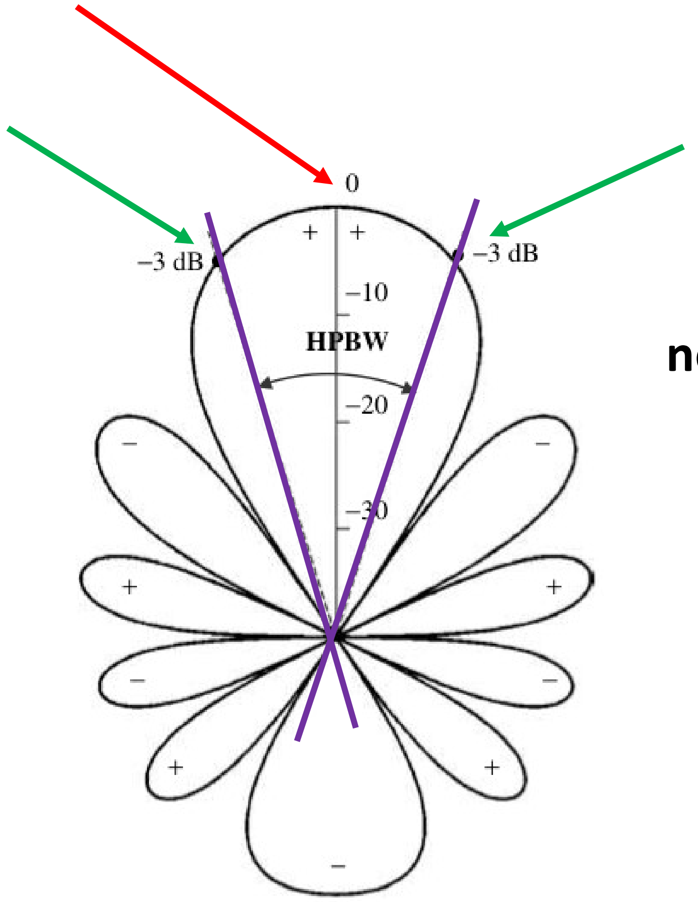
(b) Power pattern (in linear scale)

Two-dimensional normalized *field* pattern (*linear scale*)



(a) Field pattern (in linear scale)

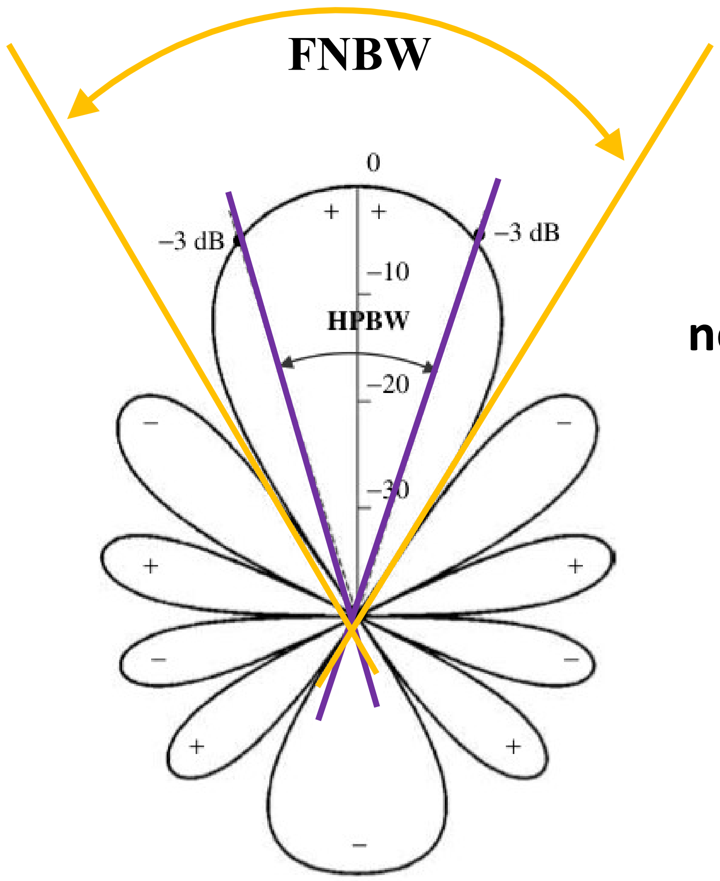
Beamwidth



(c) Power pattern (in dB)

Two-dimensional normalized *power* pattern (*in dB*)

Beamwidth



(c) Power pattern (in dB)

Two-dimensional normalized *power* pattern (*in dB*)

Radiation pattern

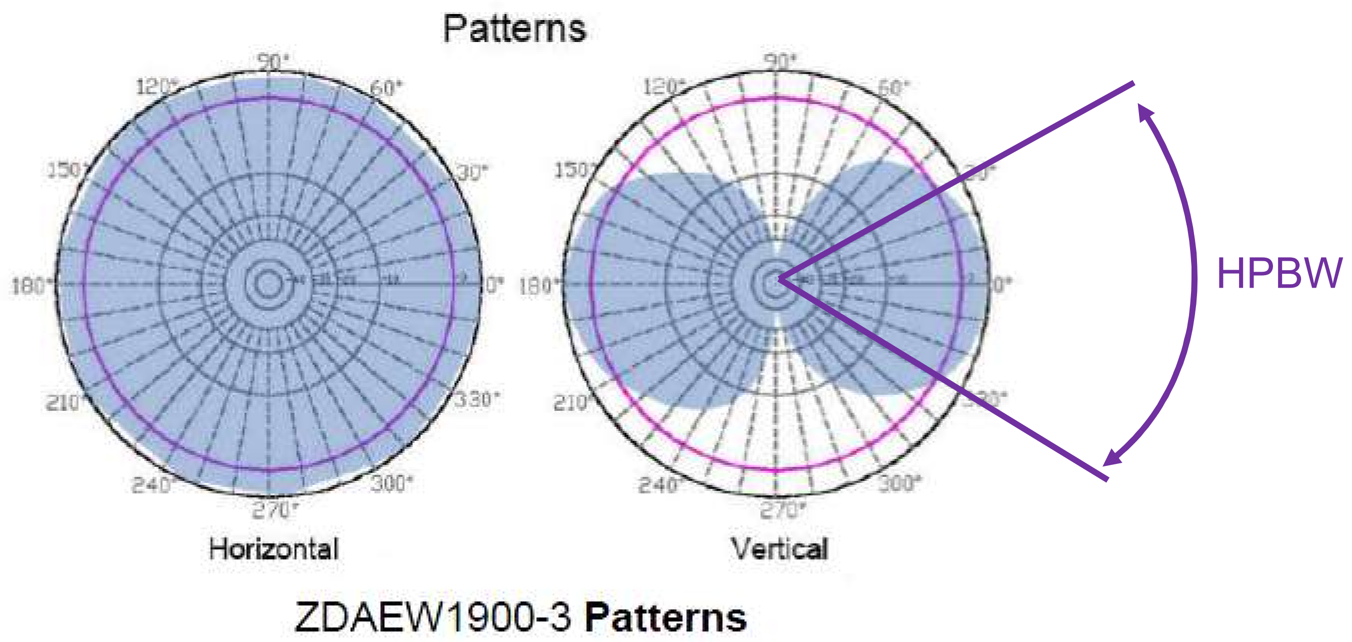
three examples from the real life

Radiation pattern

three examples from the real life

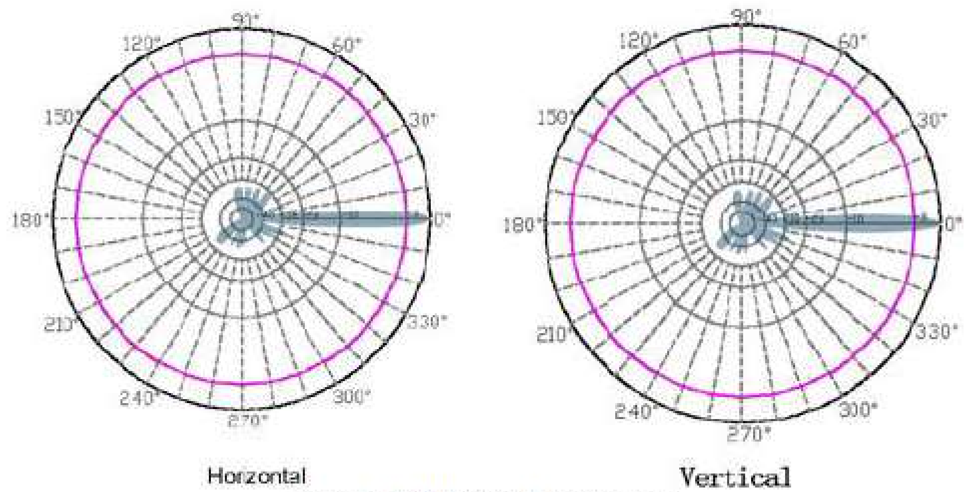
HPBW (vertical) =60°

HPBW (horizontal) =360°



Radiation pattern

three examples from the real life



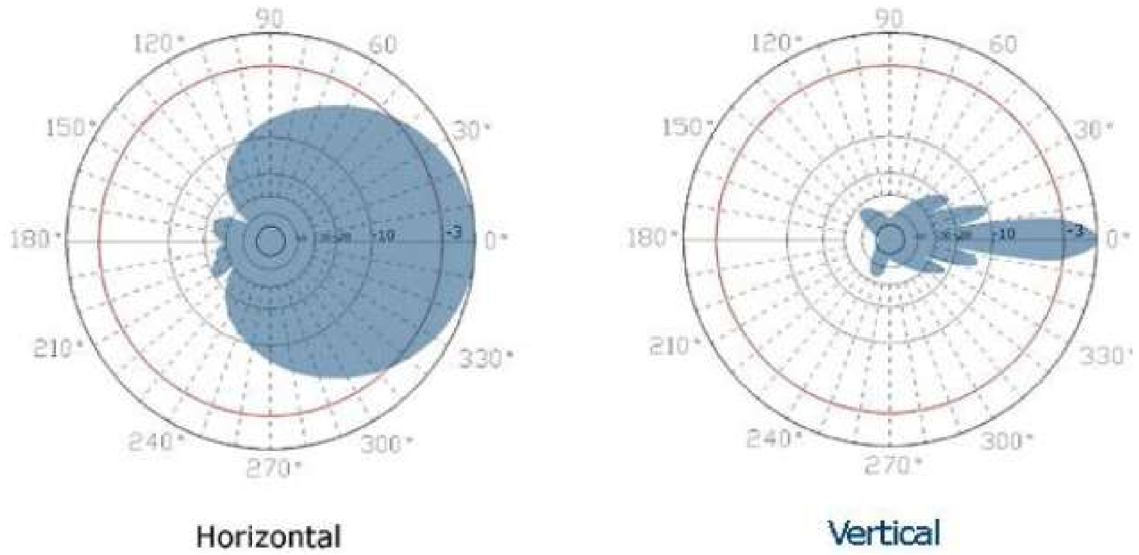
ZDASP5400-29-6 **Patterns**

HPBW (vertical) = 6°

HPBW (horizontal) = 6°

Radiation pattern

three examples from the real life



HPBW (vertical) = 14°

HPBW (horizontal) =90°

ZDADJ800-13-90 Patterns

Parameters of the Tx Antenna

- Effective length
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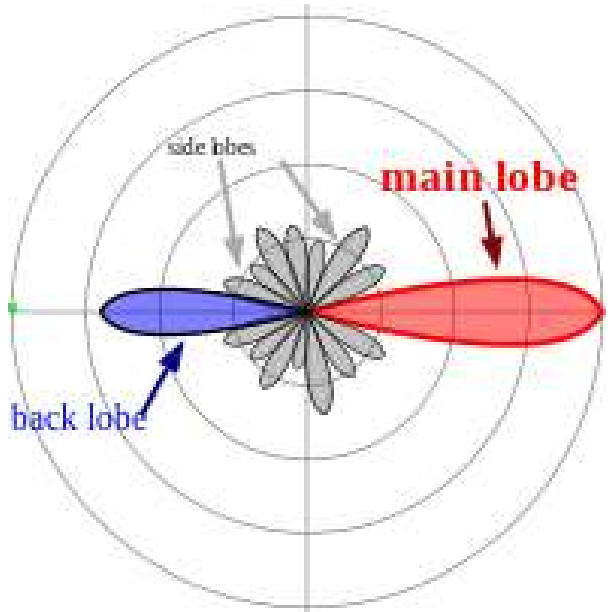
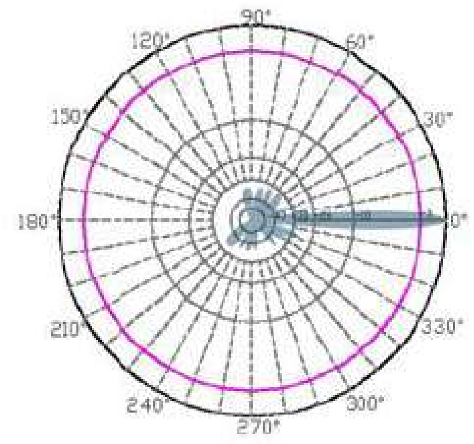
- Radiation Resistance
- Equivalent circuit of the tx antenna
- Input Impedance and Input Resistance

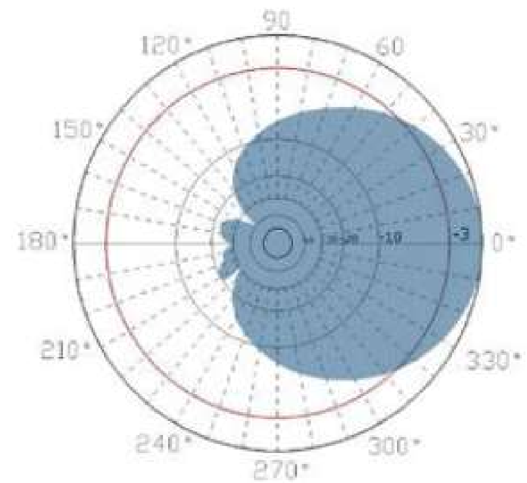
Directivity



Directivity

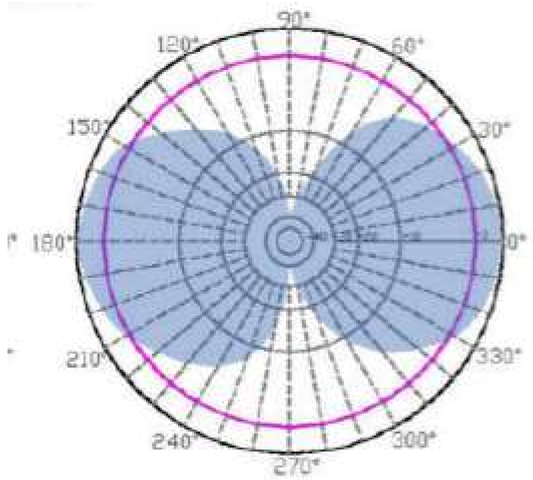


HPBW = 6°

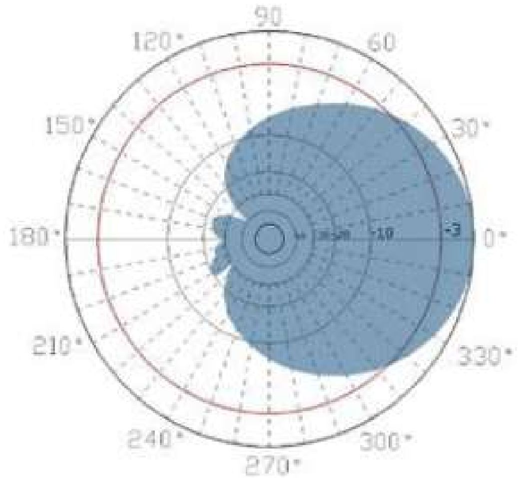


HPBW = 90°

Directivity



HPBW = 60°



HPBW = 90°

Directivity

The directivity of an antenna is :

$$D(\vartheta,\varphi) = \lim_{r\to\infty} \frac{\frac{1}{2\zeta}\left|\mathbf{E}(r,\vartheta,\varphi)\right|^2}{\frac{1}{4\pi r^2} P_{rad}}$$

Directivity

$r >> D$

$r > \dfrac{2D^2}{\lambda}$

$r >> \lambda$

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\mathbf{l}(\vartheta,\varphi)$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{l}(\vartheta,\varphi) = l_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + l_\varphi(\vartheta,\varphi)\hat{i}_\varphi$$

$$\boxed{\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2 \hat{i}_r} = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2 \hat{i}_r$$

The directivity of an antenna is :

$$D(\vartheta,\varphi) = \lim_{r\to\infty}\frac{\dfrac{1}{2\zeta}\left|\mathbf{E}(r,\vartheta,\varphi)\right|^2}{\dfrac{1}{4\pi r^2}P_{rad}}$$

The directivity of an isotropic source is equal to 1 (that is, 0 dB)

$r \to \infty$

$$P_{rad} = P_1 = \oiint_A dA\frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2$$

$$\oiint_A dA\vec{\mathbf{S}}\cdot\hat{\mathbf{n}} = P_1 + jP_2$$

Directivity of the elementary electrical dipole

The directivity of an antenna is :

$$D(\vartheta,\varphi) = \lim_{r\to\infty} \frac{\frac{1}{2\zeta}\left|\mathbf{E}\left(r,\vartheta,\varphi\right)\right|^2}{\frac{1}{4\pi r^2} P_{rad}}$$

Directivity of the elementary electrical dipole

Far field expression

$$\boxed{\mathbf{E}(\mathbf{r}) = \mathrm{E}_\vartheta(r,\vartheta)\hat{i}_\vartheta = j\zeta\frac{I\Delta z}{2\lambda r}\sin\vartheta\exp(-j\beta r)\hat{i}_\vartheta}$$

$$\boxed{P_1 = P_{rad} = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2|I|^2}$$

The directivity of an antenna is :

$$D(\vartheta,\varphi) = \lim_{r\to\infty}\frac{\boxed{\frac{1}{2\zeta}|\mathbf{E}(r,\vartheta,\varphi)|^2}}{\frac{1}{4\pi r^2}\boxed{P_{rad}}}$$

$$|\mathbf{E}|^2 = \zeta^2\frac{|I|^2\Delta z^2}{4\lambda^2 r^2}\sin^2\vartheta \qquad \boxed{\frac{1}{2\zeta}|\mathbf{E}|^2 = \frac{\zeta}{2}\frac{|I|^2\Delta z^2}{4\lambda^2 r^2}\sin^2\vartheta}$$

$$D = D(\vartheta) = \frac{\boxed{\frac{\zeta}{2}\frac{|I|^2\Delta z^2}{4\lambda^2 r^2}\sin^2\vartheta}}{\frac{1}{4\pi r^2}\boxed{\frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2|I|^2}} = \frac{3}{2}\sin^2\vartheta$$

$$D_{\max} = 10\log_{10}1.5 = 1.76\text{ dB}$$

Directivity of the small loop antenna

Far field expression

$$\mathbf{E}(\mathbf{r}) = \mathrm{E}_\varphi(r,\vartheta)\hat{i}_\varphi = \frac{\zeta\beta\Delta sI}{2\lambda r}\sin\vartheta\exp(-j\beta r)\hat{i}_\varphi$$

$$\boxed{P_1 = P_{rad} = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\beta\Delta S}{\lambda}\right)^2 |I|^2}$$

$$|\mathbf{E}|^2 = \zeta^2\frac{|I|^2(\beta\Delta s)^2}{4\lambda^2 r^2}\sin^2\vartheta \qquad \boxed{\frac{1}{2\zeta}|\mathbf{E}|^2 = \frac{\zeta}{2}\frac{|I|^2(\beta\Delta s)^2}{4\lambda^2 r^2}\sin^2\vartheta}$$

The directivity of an antenna is :

$$D(\vartheta,\varphi) = \lim_{r\to\infty}\frac{\boxed{\frac{1}{2\zeta}|\mathbf{E}(r,\vartheta,\varphi)|^2}}{\frac{1}{4\pi r^2}\boxed{P_{rad}}}$$

Elementary electric dipole

$$\boxed{P_1 = P_{rad} = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2 |I|^2} \qquad \boxed{\frac{1}{2\zeta}|\mathbf{E}|^2 = \frac{\zeta}{2}\frac{|I|^2\Delta z^2}{4\lambda^2 r^2}\sin^2\vartheta}$$

$$D = D(\vartheta) = \frac{3}{2}\sin^2\vartheta$$

Directivity of the small loop antenna

Far field expression

$$\mathbf{E}(\mathbf{r}) = \mathrm{E}_{\varphi}(r,\vartheta)\hat{i}_{\varphi} = \frac{\zeta\beta\Delta sI}{2\lambda r}\sin\vartheta \exp(-j\beta r)\hat{i}_{\varphi}$$

$$P_1 = P_{rad} = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\beta\Delta S}{\lambda}\right)^2 |I|^2$$

The directivity of an antenna is :

$$D(\vartheta,\varphi) = \lim_{r\to\infty}\frac{\frac{1}{2\zeta}\left|\mathbf{E}(r,\vartheta,\varphi)\right|^2}{\frac{1}{4\pi r^2}P_{rad}}$$

$$|\mathbf{E}|^2 = \zeta^2\frac{|I|^2(\beta\Delta s)^2}{4\lambda^2 r^2}\sin^2\vartheta \qquad \frac{1}{2\zeta}|\mathbf{E}|^2 = \frac{\zeta}{2}\frac{|I|^2(\beta\Delta s)^2}{4\lambda^2 r^2}\sin^2\vartheta$$

$$D = D(\vartheta) = \frac{3}{2}\sin^2\vartheta$$

$$D_{\max} = 10\log_{10}1.5 = 1.76 \text{ dB}$$

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- Effective length
  - Radiation pattern
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- Gain
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Gain

Directivity

$$D(\vartheta,\varphi) = \lim_{r\to\infty} \frac{\frac{1}{2}\frac{|\mathbf{E}|^2}{\zeta}}{\frac{1}{4\pi r^2}P_{rad}}$$

Gain

$$G(\vartheta,\varphi) = \lim_{r\to\infty} \frac{\frac{1}{2}\frac{|\mathbf{E}|^2}{\zeta}}{\frac{1}{4\pi r^2}P_{in}}$$

***P_{rad}*: radiated power**

***P_{in}*: input power**

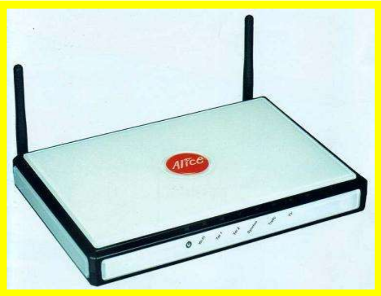
If one replace P_{rad} with the input real power to the antenna P_{in} one finds the definition of the *Gain*.

For a lossless antenna, $P_{\mathrm{in}}=P_{\mathrm{rad}}$ and *G=D.* If losses are present $P_{\mathrm{ing}}>P_{\mathrm{rad}}$ and *G<D.*

Note that both D and G are dimensionless.

Gain

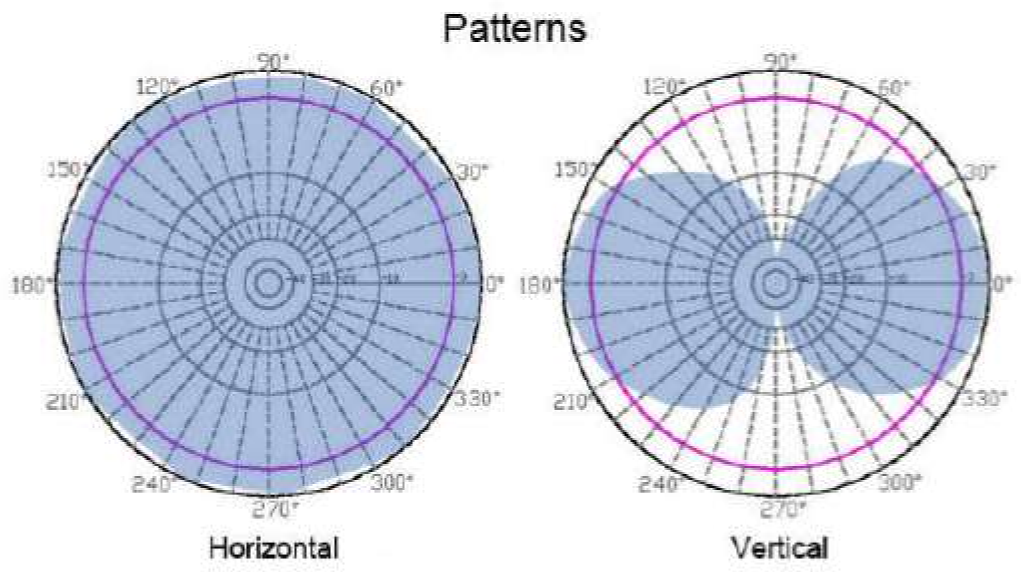
three examples from the real life



Gain

three examples from the real life

(maximum) Gain = 3 dB

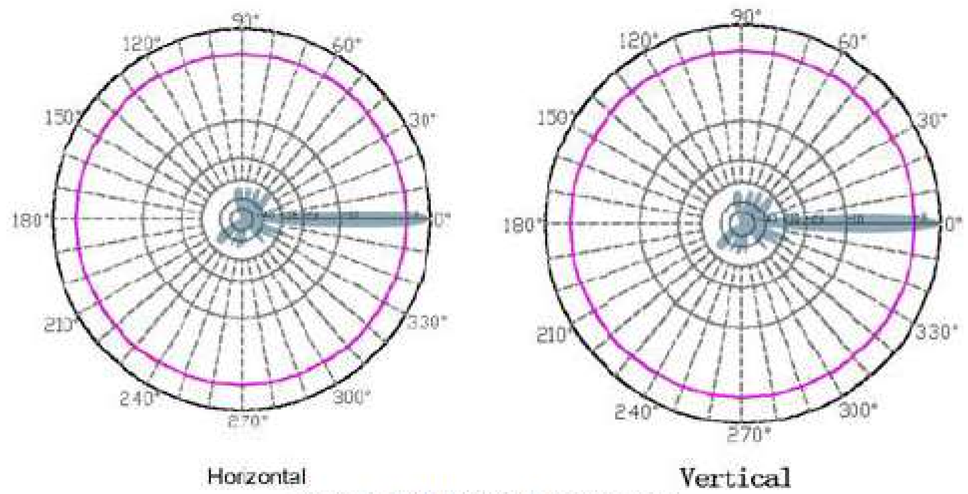


ZDAEW1900-3 **Patterns**

Gain

three examples from the real life

(maximum) Gain = 29 dB

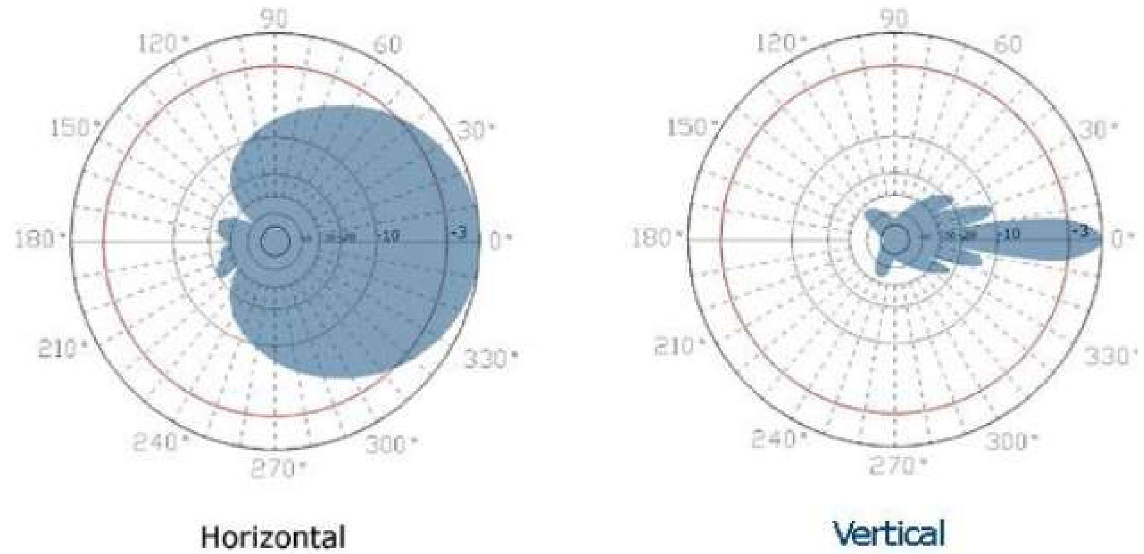


ZDASP5400-29-6 **Patterns**

Gain

three examples from the real life

(maximum) Gain = 13 dB



ZDADJ800-13-90 Patterns

Parameters of the Tx Antenna

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- Radiation Resistance
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Radiation resistance

Associated to the far-field radiated power one can define the radiation Resistance R_{rad}:

$$P_{rad} = \frac{1}{2} R_{rad} \left| I \right|^2$$

Radiation resistance

Associated to the far-field radiated power one can define the radiation Resistance R_{rad}:

$$P_{rad} = \frac{1}{2} R_{rad} \left| I \right|^2$$

Elementary electrical dipole

$$P_1 = P_{rad} = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2 \left| I \right|^2 \Rightarrow R_{rad} = \frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2$$

Radiation resistance

Associated to the far-field radiated power one can define the radiation Resistance R_{rad}:

$$P_{rad} = \frac{1}{2} R_{rad} |I|^2$$

Elementary electrical dipole

$$P_1 = P_{rad} = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2 |I|^2 \quad \Rightarrow \quad R_{rad} = \frac{2\pi}{3}\zeta\left(\frac{\Delta z}{\lambda}\right)^2$$

Small loop antenna

$$P_1 = P_{rad} = \frac{1}{2}\frac{2\pi}{3}\zeta\left(\frac{\beta\Delta S}{\lambda}\right)^2 |I|^2 \quad \Rightarrow \quad R_{rad} = \frac{2\pi}{3}\zeta\left(\frac{\beta\Delta S}{\lambda}\right)^2$$

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Input impedance

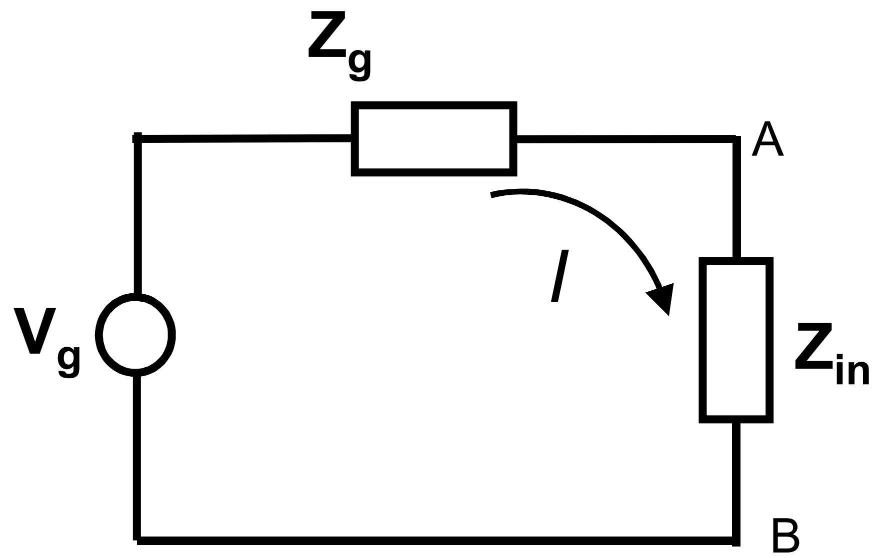
Input impedance is defined as “the impedance presented by an antenna at its terminals or the ratio of the voltage to current at a pair of terminals or the ratio of the appropriate components of the electric to magnetic fields at a point.”

The input impedance of an antenna is generally a function of frequency.

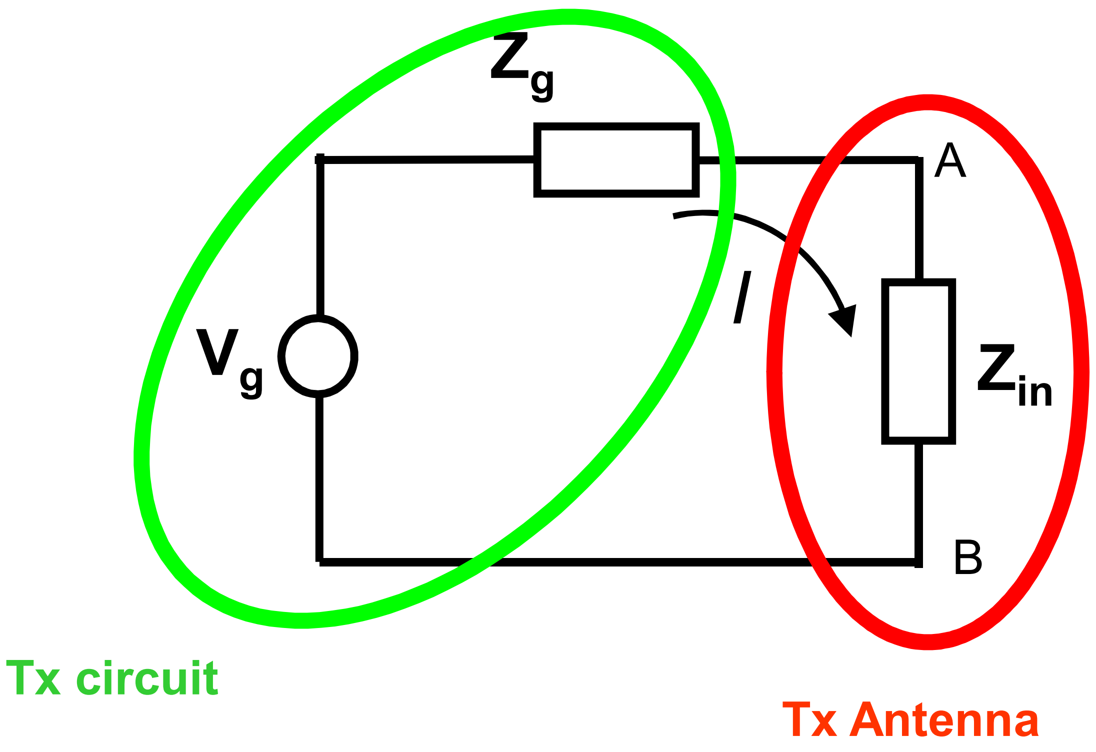
The input impedance of the antenna depends on many factors including its geometry, its method of excitation, and its proximity to surrounding objects.

Because of their complex geometries, only a limited number of practical antennas have been investigated analytically. For many others, the input impedance has been determined experimentally.

Equivalent circuit of the Tx antenna



Equivalent circuit of the Tx antenna



$$Z_{in} = R_{in} + jX_{in}$$

$$P_{in} = \frac{1}{2} R_{in} |I|^2$$

$$P_{rad} = \frac{1}{2} R_{rad} |I|^2$$

$$P_{rad} \leq P_{in}$$

$\Rightarrow$ $R_{rad} \leq R_{in}$ $\Rightarrow$ $R_{in} = R_{rad} + R_{\Omega}$

Radiation efficiency

Directivity

$$D(\vartheta,\phi)=\lim_{r\to\infty}\frac{\frac{1}{2}\frac{|\mathbf{E}|^2}{\zeta}}{\frac{1}{4\pi r^2}P_{rad}}$$

Gain

$$G(\vartheta,\phi)=\lim_{r\to\infty}\frac{\frac{1}{2}\frac{|\mathbf{E}|^2}{\zeta}}{\frac{1}{4\pi r^2}P_{in}}$$

$$P_{rad}=\frac{1}{2}R_{rad}|I|^2$$

$$P_{in}=\frac{1}{2}R_{in}|I|^2=\frac{1}{2}(R_{rad}+R_{\Omega})|I|^2$$

Radiation Efficiency: $\eta=\dfrac{P_{rad}}{P_{in}}=\dfrac{R_{rad}}{R_{rad}+R_{\Omega}}=\dfrac{G}{D}$

Parameters of the Tx Antenna

- Effective length
  - Radiation pattern
  - Radiation pattern lobes
  - Beamwidth
- Directivity
- Gain
- Radiation Resistance
- Equivalent circuit of the tx antenna
- Input Impedance and Input Resistance