Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2021-2022 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Outline

- Radiation problem for extended antennas
- Field regions

Extended antennas

$J(r)$

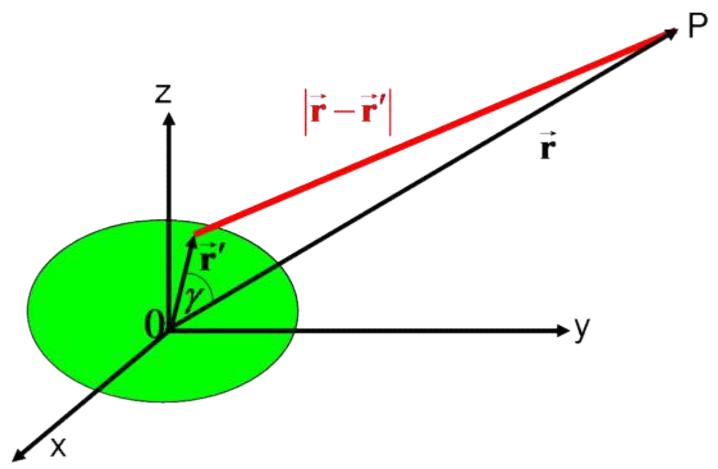
$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$ Vector Helmholtz equation

$$\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi}\int \mathbf{J}(\mathbf{r}')\frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'$$

$$\boldsymbol{H} = \frac{1}{\mu}\nabla \times \boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega \boldsymbol{A} + \frac{\nabla\nabla\cdot \boldsymbol{A}}{j\omega\varepsilon\mu}$$

$E(r)$

$H(r)$

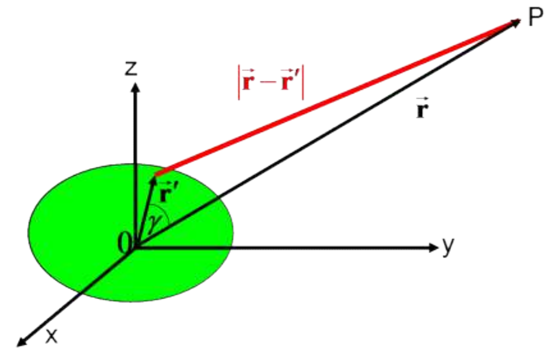


Is it possible to simplify the expressions of the fields, possibly via proper approximation of the vector potential A?

Extended antennas

$$\boxed{\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\vec{\mathbf{r}}'}$$

$$|\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = r - r'\cos\gamma + \frac{(r')^2}{2r}\sin^2\gamma + \cdots$$



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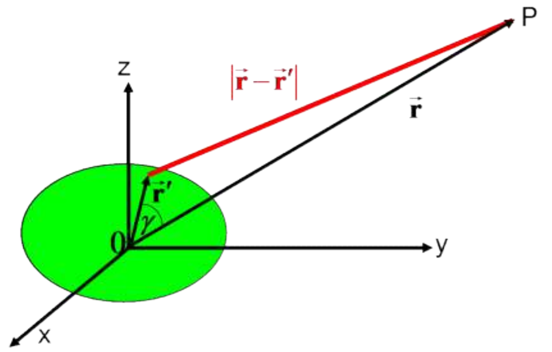

$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - \cancel{r'\cos\gamma} + \cancel{\frac{(r')^2}{2r}\sin^2\gamma} + \cdots} \approx \frac{1}{r}$$

$if \quad D << r$

$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = e^{-j\beta r}\cancel{e^{j\beta r'\cos\gamma}}\cancel{e^{-j\beta\frac{(r')^2}{2r}\sin^2\gamma}}\cdots \approx e^{-j\beta r}$$

$if \quad D << \lambda$

When the antennas are **small** with respect to the wavelength **and** to the distance from the observation point

$$\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx \frac{e^{-j\beta r}}{r} \quad \Longrightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi}\frac{e^{-j\beta r}}{r}\int \mathbf{J}(\vec{\mathbf{r}}')\,d\,\vec{\mathbf{r}}'$$

Extended antennas

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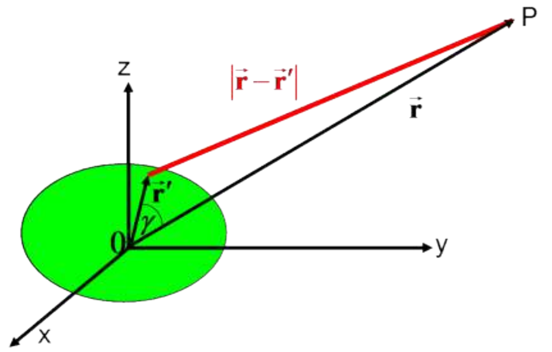

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For <u>**all**</u> the antennas, if the distance from the observation point **is sufficiently large**

$$\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx \frac{e^{-j\beta r}e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}}{r} \Rightarrow \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi}\frac{e^{-j\beta r}}{r}\int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta r'\cos\gamma}\,d\vec{\mathbf{r}}'$$

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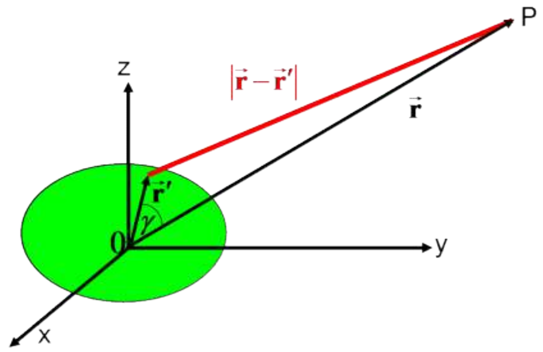

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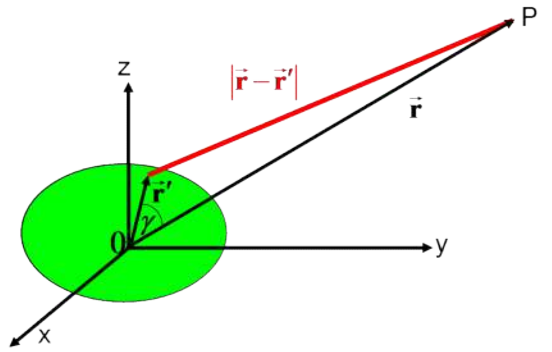

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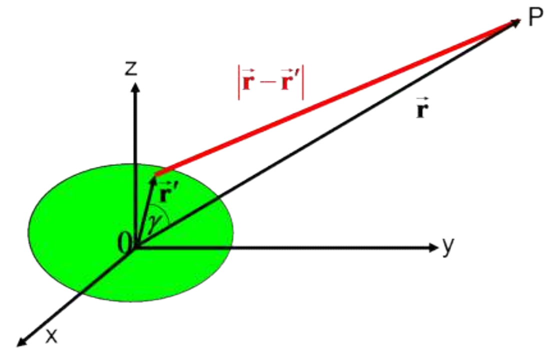

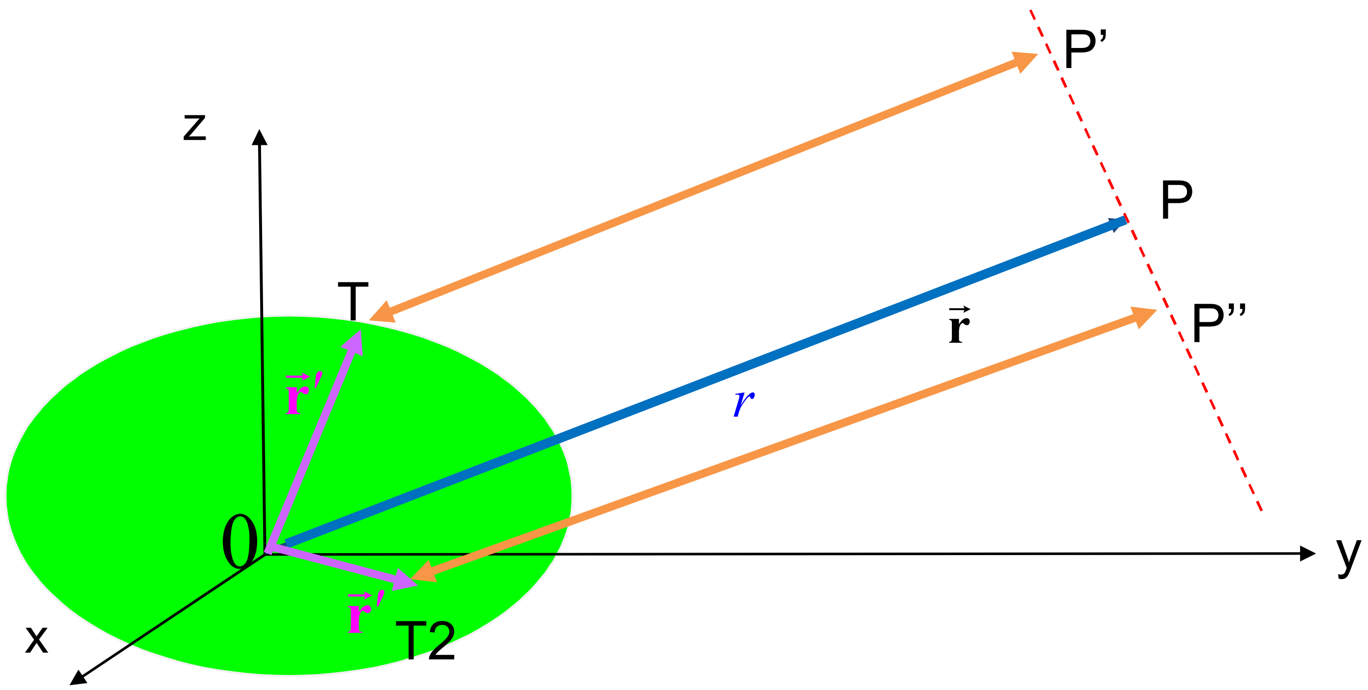

Extended antennas

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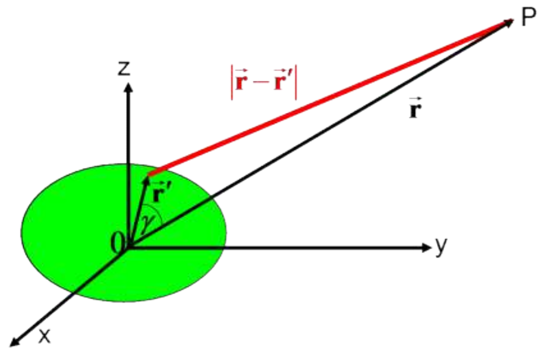

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For **all** the antennas, if the distance from the observation point **is sufficiently large**

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Fraunhofer region

$$r >> D$$

$$r > \frac{2D^2}{\lambda}$$

$$\int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\vec{\mathbf{r}}' = \mathbf{M}(\vartheta, \varphi) \quad \Rightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \mathbf{M}(\vartheta, \varphi)$$

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Fraunhofer region

$$\mathbf{E}(\vec{\mathbf{r}}) = -\frac{j\omega\mu}{4\pi} \frac{e^{-j\beta r}}{r} \left[\mathbf{M}(\vartheta, \varphi) - M_r(\vartheta, \varphi) \hat{i}_r \right]$$

$$\zeta \mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

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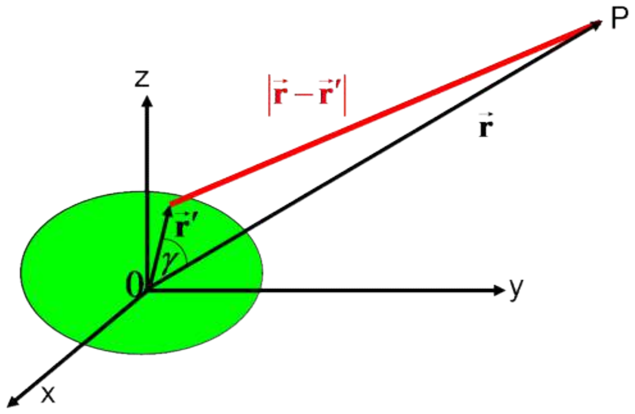

- the e.m. field propagates along $\hat{i}_r$
- the e.m. field lies on the plane orthogonal to the propagation direction
- |E| and |H| exhibit the decaying factor 1/r
- |E| and |H| are proportional through ζ

Field regions

Far-field (Fraunhofer) region is defined as “that region of the field of an antenna where the angular field distribution is essentially independent of the distance from the antenna. If the antenna has a maximum overall dimension D ($D>\lambda$), the far-field region is commonly taken to exist at distances greater than $2D^2/\lambda$ from the antenna, λ being the wavelength”.

In this region, the field components are essentially transverse

Fraunhofer region

$r >> D$

$r > \dfrac{2D^2}{\lambda}$

$r >> \lambda$

Fraunhofer region

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$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}d\,\vec{\mathbf{r}}'$$

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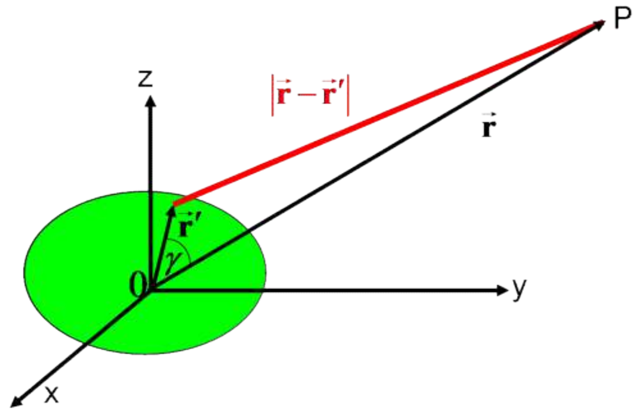

The radiation condition

$$r >> D$$

$$r > \frac{2D^2}{\lambda}$$

$$r >> \lambda$$

Fraunhofer region

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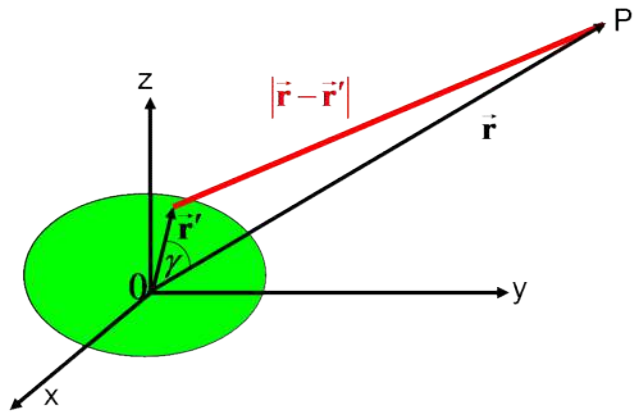
$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}d\,\vec{\mathbf{r}}'$$

$$\mathbf{E} \sim O\left(\frac{1}{r}\right)$$

$$\mathbf{H} \sim O\left(\frac{1}{r}\right) \qquad as \quad r \to \infty$$

$$\zeta\mathbf{H} - \hat{i}_r \times \mathbf{E} \sim o\left(\frac{1}{r}\right)$$



Fraunhofer region

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$r > \dfrac{2D^2}{\lambda}$

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Fraunhofer region

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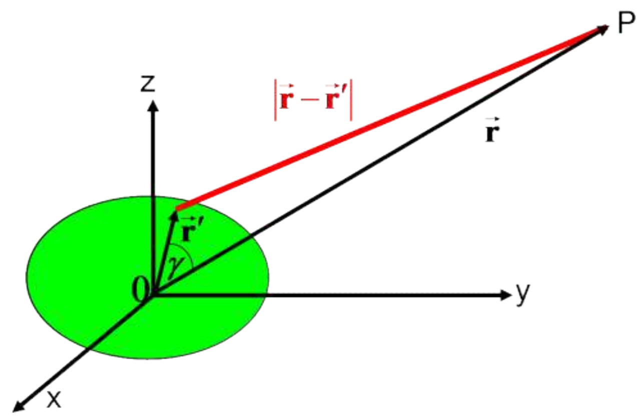

Fraunhofer region

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$r > \frac{2D^2}{\lambda}$

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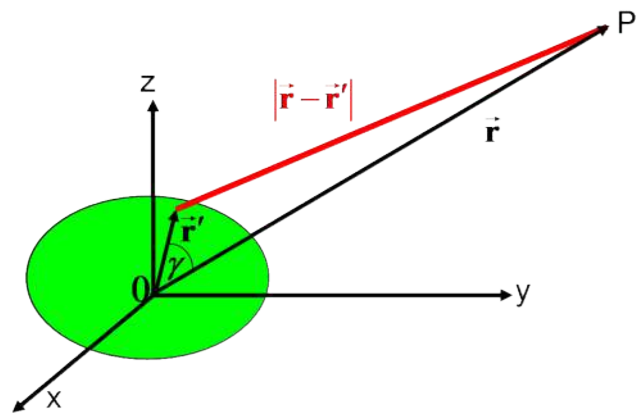

Fraunhofer region

$$\vec{\mathbf{S}} = \frac{1}{2}\vec{\mathbf{E}}\times\vec{\mathbf{H}}^* = \frac{1}{2\zeta}\vec{\mathbf{E}}\times\left(\hat{i}_r\times\vec{\mathbf{E}}\right)^* = \frac{1}{2\zeta}\vec{\mathbf{E}}\times\left(\hat{i}_r\times\vec{\mathbf{E}}^*\right) = \frac{1}{2\zeta}\left[\left|\vec{\mathbf{E}}\right|^2\hat{i}_r - \cancel{\left(\hat{i}_r\cdot\vec{\mathbf{E}}\right)}\vec{\mathbf{E}}^*\right] = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{i}_r$$

$$\vec{\mathbf{S}} = \frac{1}{2}\vec{\mathbf{E}}\times\vec{\mathbf{H}}^* = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{i}_r$$

$$\vec{\mathbf{A}}\times\left(\vec{\mathbf{B}}\times\vec{\mathbf{C}}\right) = \vec{\mathbf{B}}\left(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}}\right) - \vec{\mathbf{C}}\left(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}\right)$$

Fraunhofer region

$r >> D$

$r > \dfrac{2D^2}{\lambda}$

$r >> \lambda$

Fraunhofer region

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = -\frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r}\left[\mathbf{M}(\vartheta,\varphi) - M_r(\vartheta,\varphi)\hat{i}_r\right] = -\frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r}\left[M_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + M_\varphi(\vartheta,\varphi)\hat{i}_\varphi\right]$$

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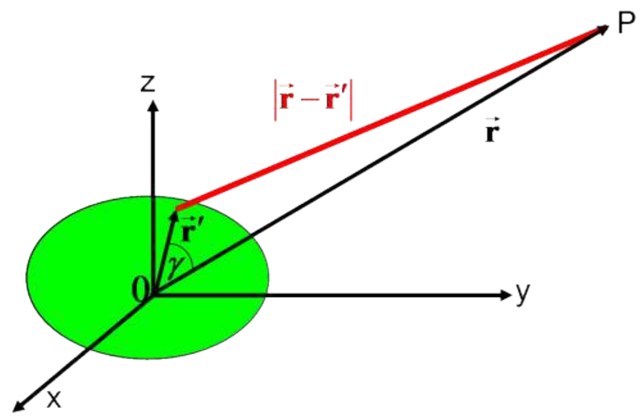
$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{i}_r$$



Fraunhofer region

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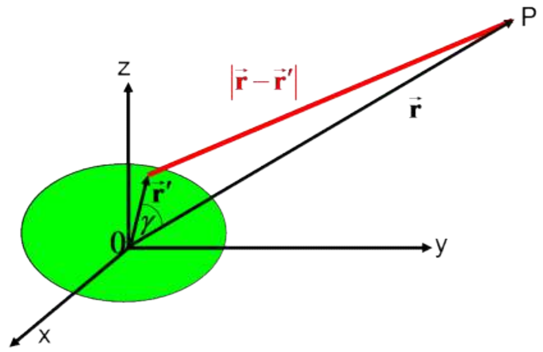

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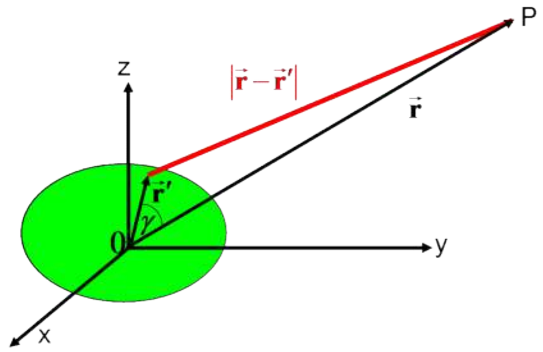
Fresnel region

$$\boxed{\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}\,d\vec{\mathbf{r}}'}$$

$$|\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = r - \vec{\mathbf{r}}'\cdot\hat{i}_r + \frac{(r')^2}{2r}\sin^2\gamma + \cancel{\cdots}$$



$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - r'\cancel{\cos}\gamma + \cancel{\frac{(r')^2}{2r}\sin^2}\gamma + \cdots} \approx \frac{1}{r} \quad \text{if} \quad D \ll r$$

$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx e^{-j\beta r}\, e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}\, e^{-j\beta\frac{(r')^2}{2r}\sin^2\gamma}\, \cancel{\cdots} \qquad \text{if} \quad r > 0.62\sqrt{\frac{D^3}{\lambda}}$$

Fresnel region

Fraunhofer region

$r >> D$

$r > \dfrac{2D^2}{\lambda}$

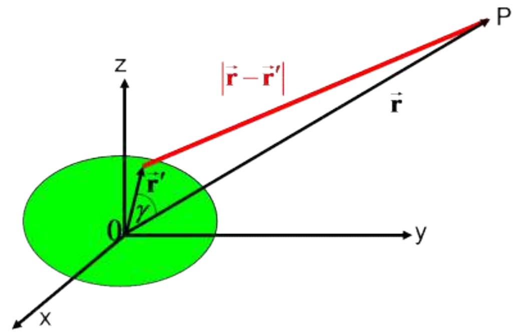
$r >> \lambda$

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = -\frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r}\left[\mathbf{M}(\vartheta,\varphi) - M_r(\vartheta,\varphi)\hat{i}_r\right] = -\boxed{\frac{j\omega\mu}{4\pi}}\frac{e^{-j\beta r}}{r}\left[M_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + M_\varphi(\vartheta,\varphi)\hat{i}_\varphi\right]$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2 \hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2 \hat{i}_r$$

$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}d\,\vec{\mathbf{r}}'$$



Fraunhofer region

$$\frac{j\omega\mu}{4\pi} = \frac{\boxed{j\omega\sqrt{\mu}}\boxed{\sqrt{\mu}}}{4\pi}\frac{\boxed{\sqrt{\varepsilon}}}{\boxed{\sqrt{\varepsilon}}} = \frac{j\zeta}{4\pi}\beta = \frac{j\zeta}{4\pi}\frac{2\pi}{\lambda} = \boxed{\frac{j\zeta}{2\lambda}}$$

$$\boxed{\omega\sqrt{\mu\varepsilon} = \beta} = \frac{2\pi}{\lambda}$$

$$\boxed{\zeta = \sqrt{\frac{\mu}{\varepsilon}}}$$

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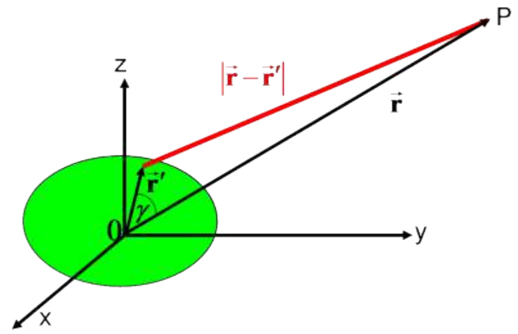

Fraunhofer region

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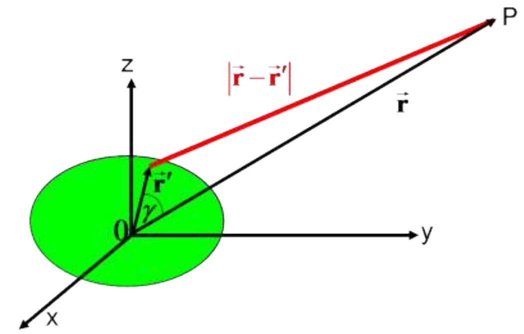

Fraunhofer region

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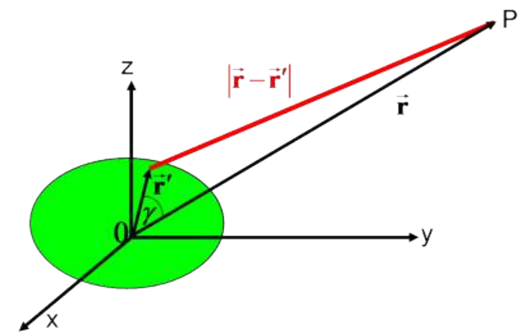

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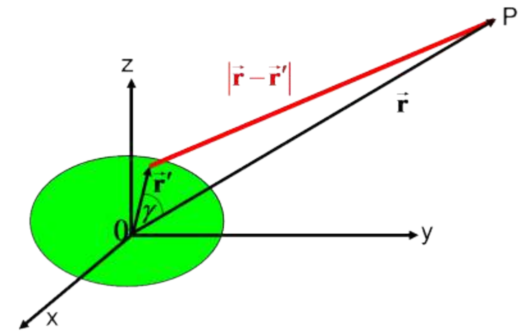
$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

I is the phasor associated to the input current at the antenna input terminals

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2 \hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2 \hat{i}_r$$

$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\,\vec{\mathbf{r}}'$$



Fraunhofer region

Fraunhofer region

$r >> D$

$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

Fraunhofer region

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = = \frac{j\zeta}{2\lambda}\frac{e^{-j\beta r}}{r} I \left[\frac{-M_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta - M_\varphi(\vartheta,\varphi)\hat{i}_\varphi}{I}\right]$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

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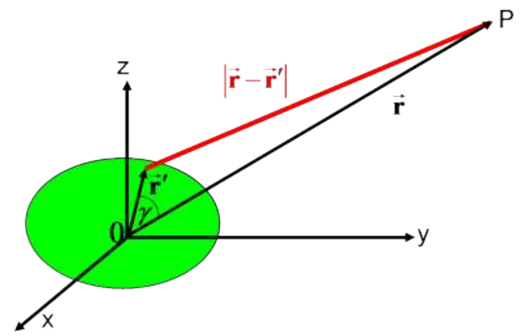

Fraunhofer region

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Fraunhofer region

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\left[\boxed{\frac{-M_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta - M_\varphi(\vartheta,\varphi)\hat{i}_\varphi}{I}}\right] = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\mathbf{l}(\vartheta,\varphi)$$

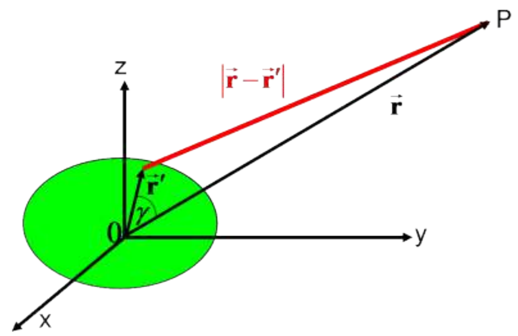
$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

I is the phasor associated to the input current at the antenna input terminals

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{i}_r$$

$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}\,d\,\vec{\mathbf{r}}'$$

$$[\mathbf{M}] = \frac{Ampere}{m^2}\times m^3 = Ampere \times m$$



$$\boxed{\frac{-M_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta - M_\varphi(\vartheta,\varphi)\hat{i}_\varphi}{I} = \mathbf{l}(\vartheta,\varphi) = l_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + l_\varphi(\vartheta,\varphi)\hat{i}_\varphi}$$

is said **effective length** of the antenna

Fraunhofer region

$$r >> D$$

$$r > \frac{2D^2}{\lambda}$$

$$r >> \lambda$$

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r, \vartheta, \varphi) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\mathbf{l}(\vartheta, \varphi)$$

$$\zeta \mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

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$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2 \hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2 \hat{i}_r$$

Antenna Parameters

Introduction

- To describe the performance of an antenna, definitions of various parameters are necessary.
- Some of the parameters are interrelated and not all of them need be specified for complete description of the antenna performance.

Antenna Parameters

Parameters of the Tx Antenna

Parameters of the Rx Antenna

Parameters of the Tx Antenna

- Effective length
  - Radiation pattern
  - Radiation pattern lobes
  - Beamwidth
- Directivity
- Gain
- Radiation Resistance
- Equivalent circuit of the tx antenna
- Input Impedance and Input Resistance

Effective Length

$r >> D$

$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\mathbf{l}(\vartheta,\varphi)$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{l}(\vartheta,\varphi) = l_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + l_\varphi(\vartheta,\varphi)\hat{i}_\varphi$$

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2\hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2\hat{i}_r$$

$\mathbf{l}(\vartheta,\varphi) = l_\vartheta(\vartheta,\varphi)\hat{i}_\vartheta + l_\varphi(\vartheta,\varphi)\hat{i}_\varphi$ **effective length** of the antenna

Effective Length

$$r >> D$$

$$r > \frac{2D^2}{\lambda}$$

$$r >> \lambda$$

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\mathbf{l}(\vartheta,\varphi)$$

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$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2 \hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2 \hat{i}_r$$

Elementary electrical dipole

$$\mathbf{E}(\vec{\mathbf{r}}) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}\Delta z \sin\vartheta\,\hat{i}_\vartheta$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta\,\hat{i}_\vartheta$$

Small loop antenna

$$\mathbf{E}(\vec{\mathbf{r}}) = \frac{j\zeta I}{2\lambda}\frac{e^{-j\beta r}}{r}(-j\beta\Delta S)\sin\vartheta\,\hat{i}_\varphi$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{l}(\vartheta,\varphi) = -j\beta\Delta S \sin\vartheta\,\hat{i}_\varphi$$

Parameters of the Tx Antenna

- Effective length
  - Radiation pattern
  - Radiation pattern lobes
  - Beamwidth
- Directivity
- Gain
- Radiation Resistance
- Equivalent circuit of the tx antenna
- Input Impedance and Input Resistance

Radiation pattern

$$r >> D$$

$$r > \frac{2D^2}{\lambda}$$

$$r >> \lambda$$

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r, \vartheta, \varphi) = \frac{j\zeta I}{2\lambda} \frac{e^{-j\beta r}}{r} \mathbf{l}(\vartheta, \varphi)$$

$$\zeta \mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{l}(\vartheta, \varphi) = l_\vartheta(\vartheta, \varphi)\hat{i}_\vartheta + l_\varphi(\vartheta, \varphi)\hat{i}_\varphi$$

$$\vec{\mathbf{S}} = \frac{1}{2\zeta}\left|\vec{\mathbf{E}}\right|^2 \hat{i}_r = \frac{\zeta}{2}\left|\vec{\mathbf{H}}\right|^2 \hat{i}_r$$

An antenna *radiation pattern* or *antenna pattern* is defined as “a mathematical function or a graphical representation of the radiation properties of the antenna as a function of space coordinates”.

In most cases, the radiation pattern is determined in the *far- field region* and is represented as a function of the directional coordinates.

We can describe the angular behavior of the field radiated by the antenna by representing its effective length.

Radiation pattern

a. *field* pattern (*in linear scale*) typically represents a plot of the magnitude of the electric or magnetic field as a function of the angular space.

b. *power* pattern (*in linear scale*) typically represents a plot of the square of the magnitude of the electric or magnetic field as a function of the angular space.

c. *power* pattern (*in dB*) represents the magnitude of the electric or magnetic field, in decibels, as a function of the angular space.

Often the *field* and *power* patterns are properly normalized, yielding *normalized field* and *power patterns*

Radiation pattern

an example: the electrical elementary dipole

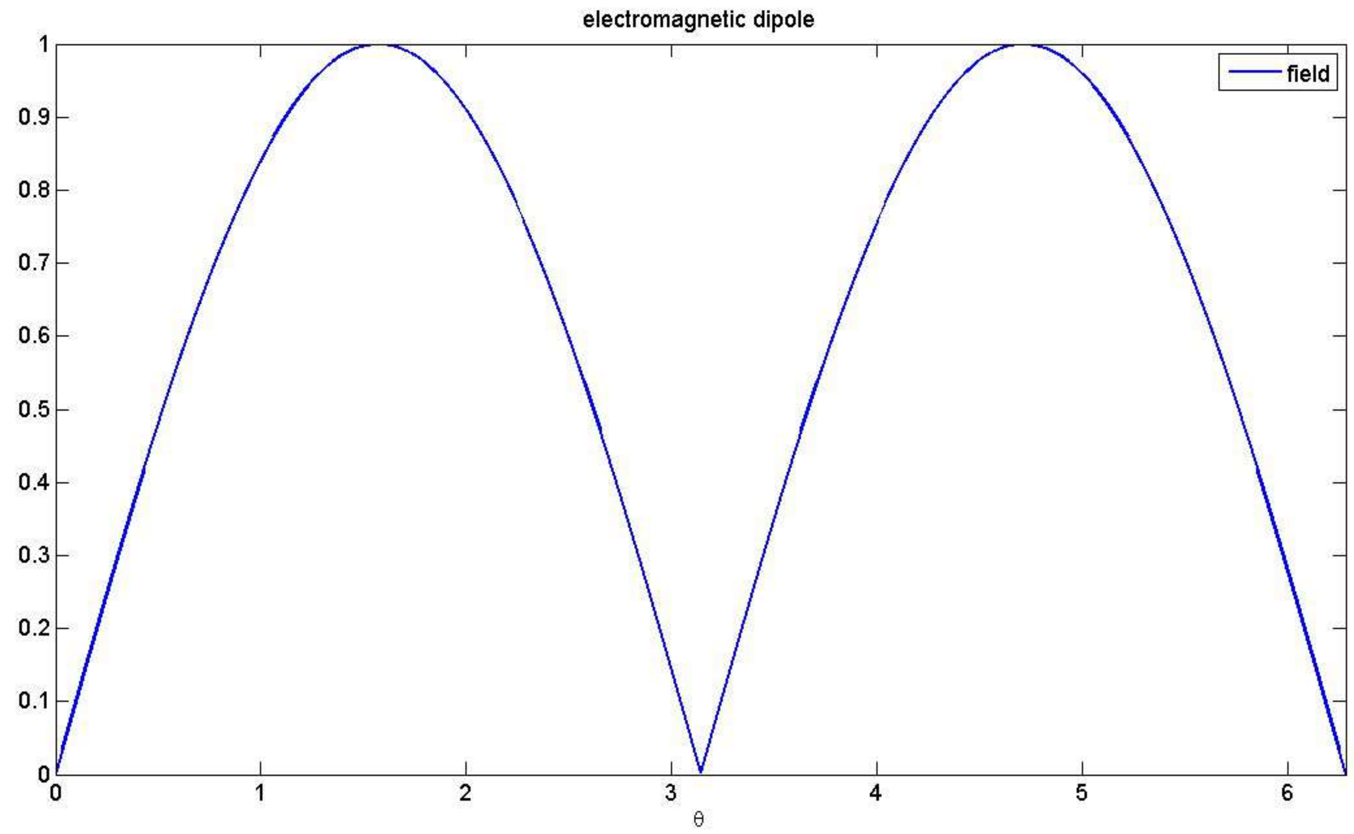
$$\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta\, \hat{i}_\vartheta$$

Radiation pattern

an example: the electrical elementary dipole

$$\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta\, \hat{i}_\vartheta$$

Vertical plane (φ=0)

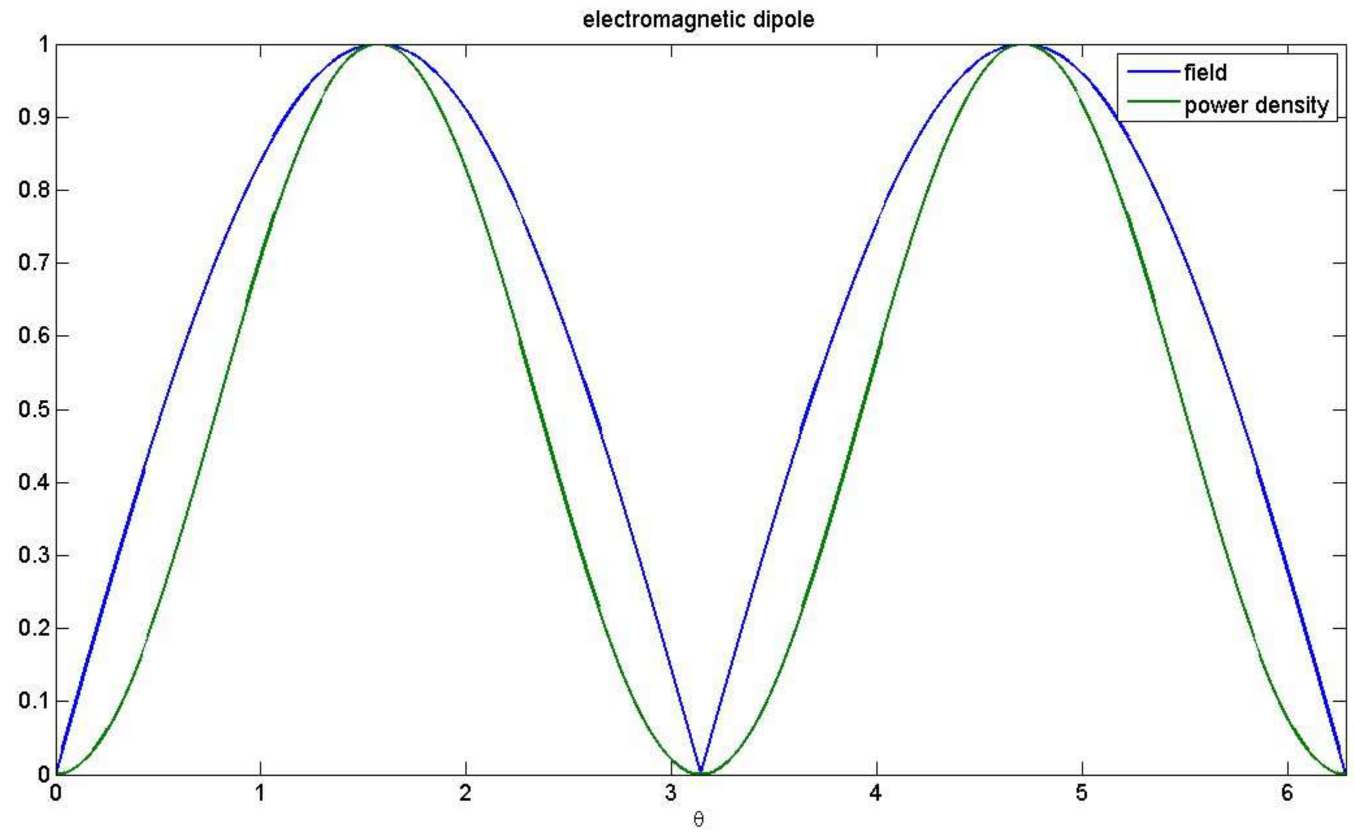


Radiation pattern

an example: the electrical elementary dipole

$$\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta\, \hat{i}_\vartheta$$

Vertical plane (φ=0)



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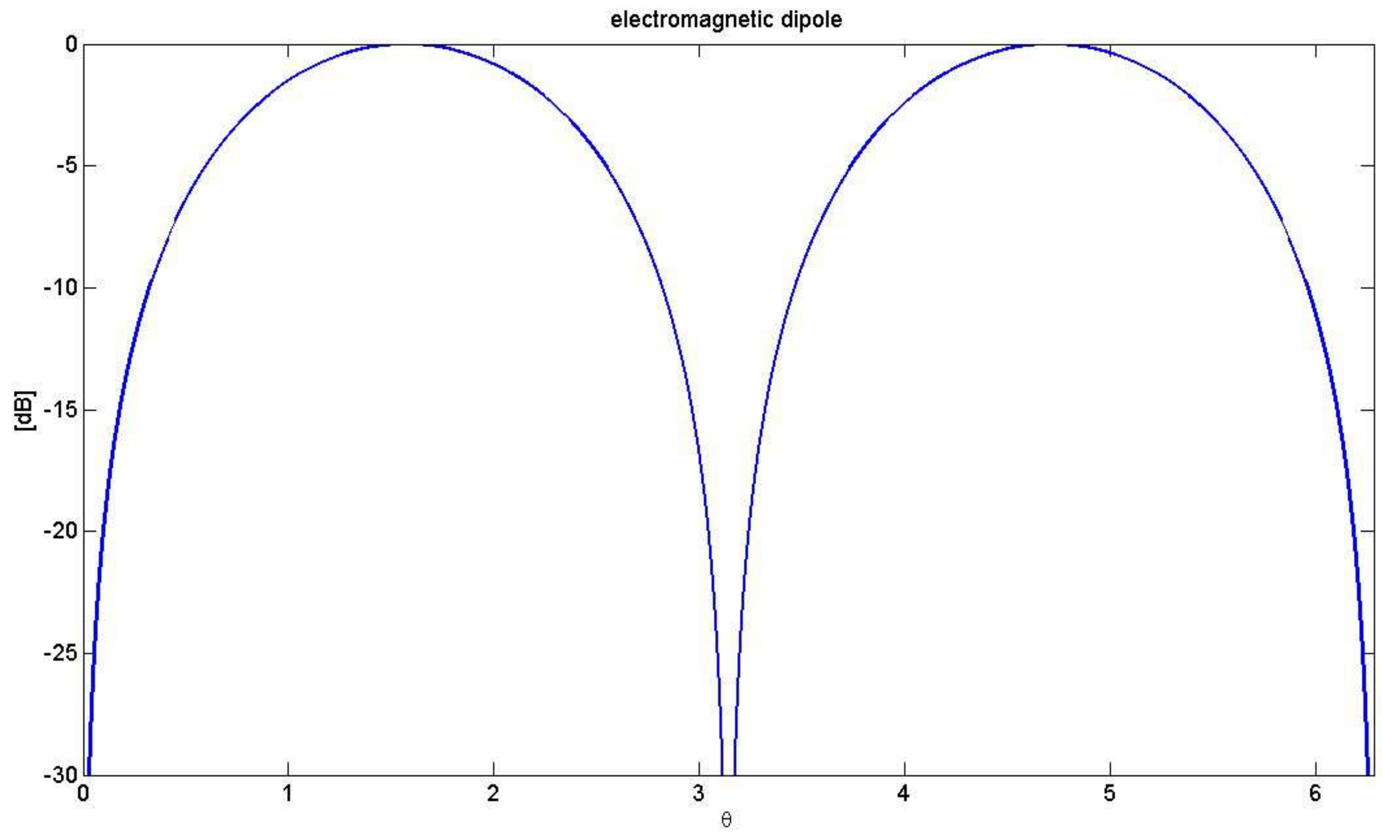

Radiation pattern

an example: the electrical elementary dipole

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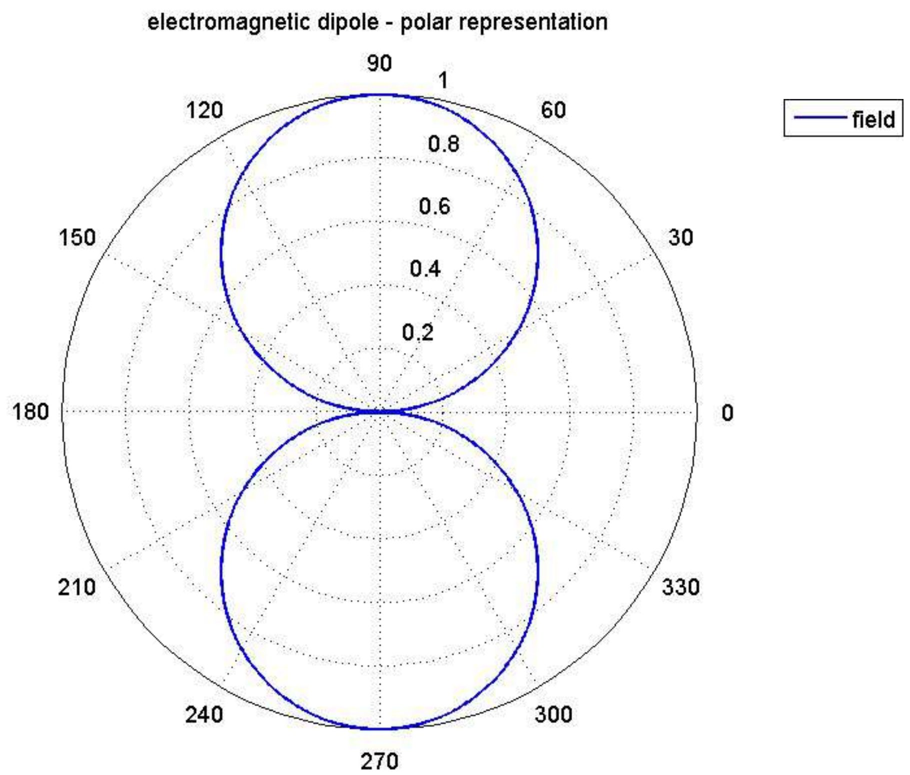
Vertical plane (φ=0)



Radiation pattern

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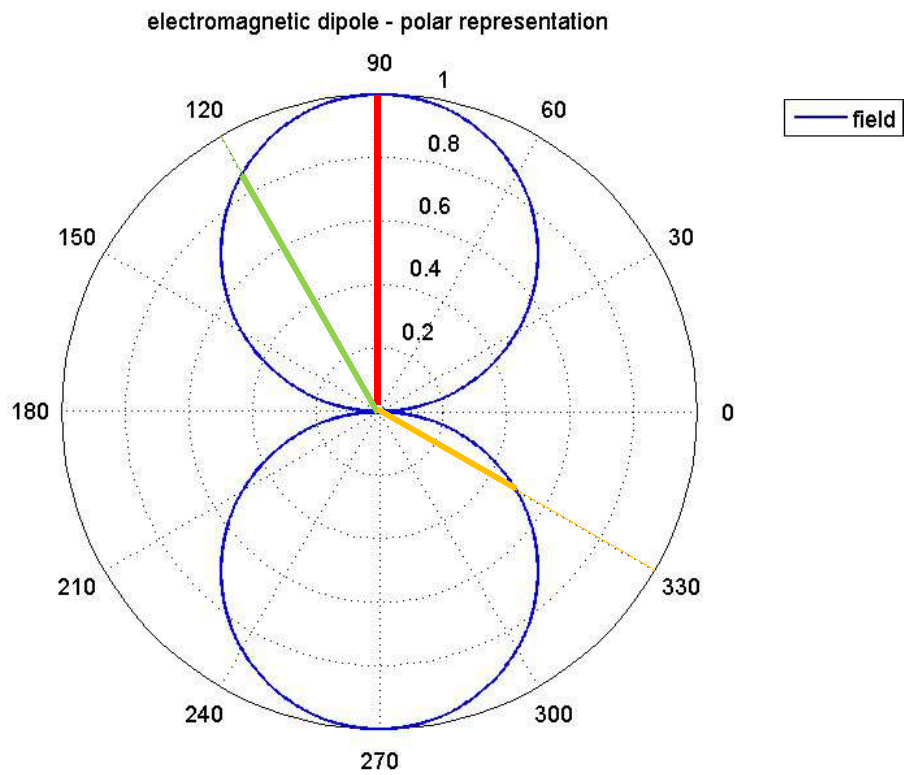
Vertical plane (φ=0)



Radiation pattern

an example: the electrical elementary dipole

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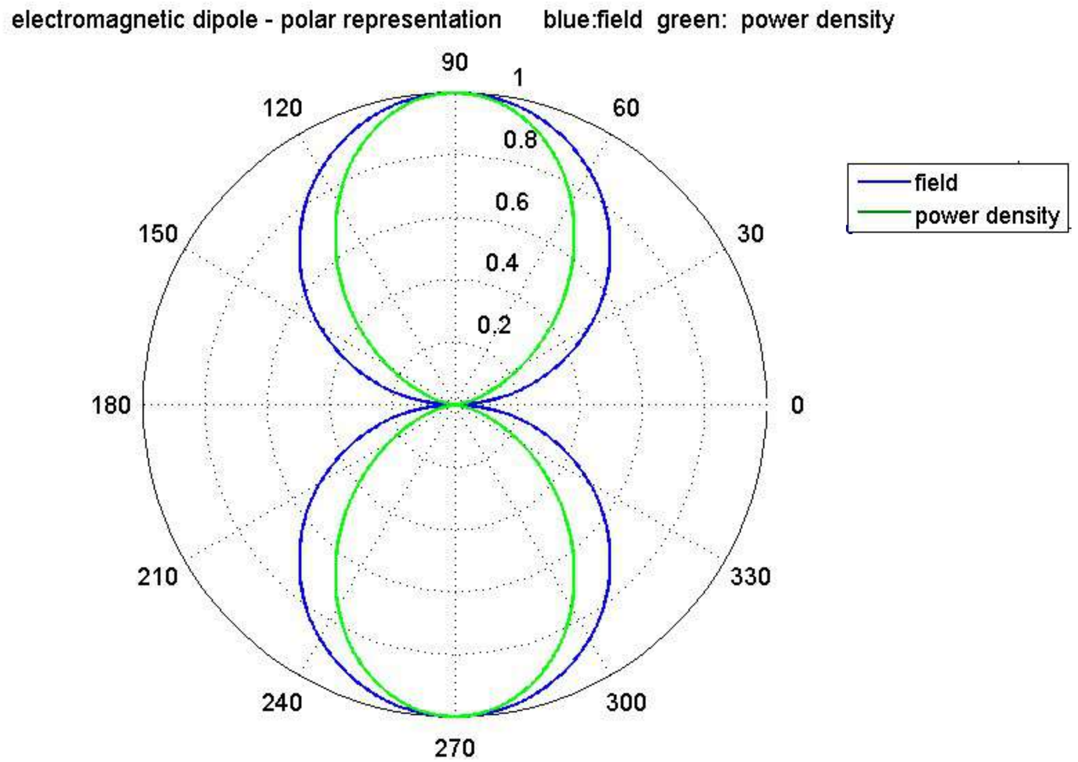
Vertical plane (φ=0)

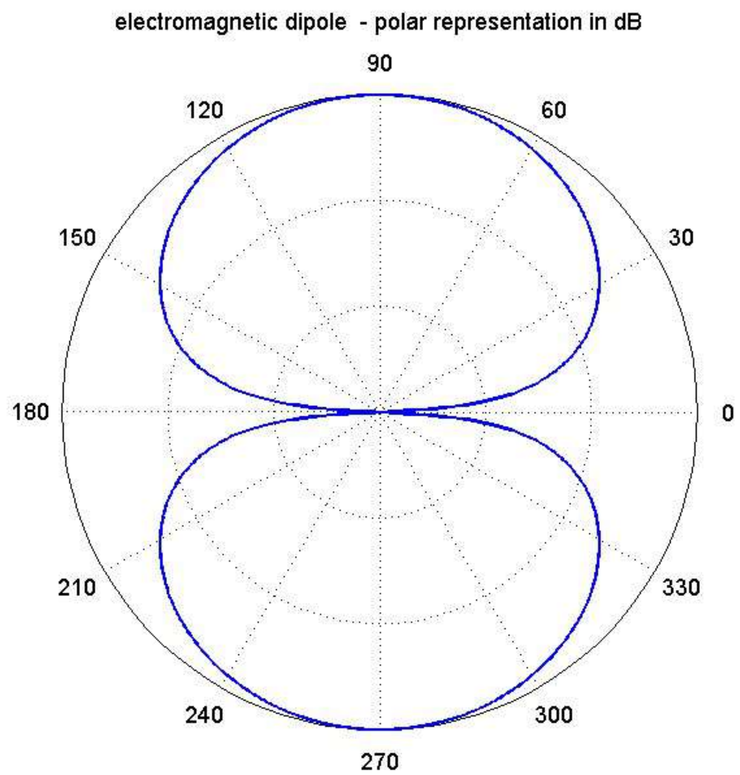


Radiation pattern

an example: the electrical elementary dipole

$$\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta \,\hat{i}_\vartheta$$

Vertical plane (φ=0)

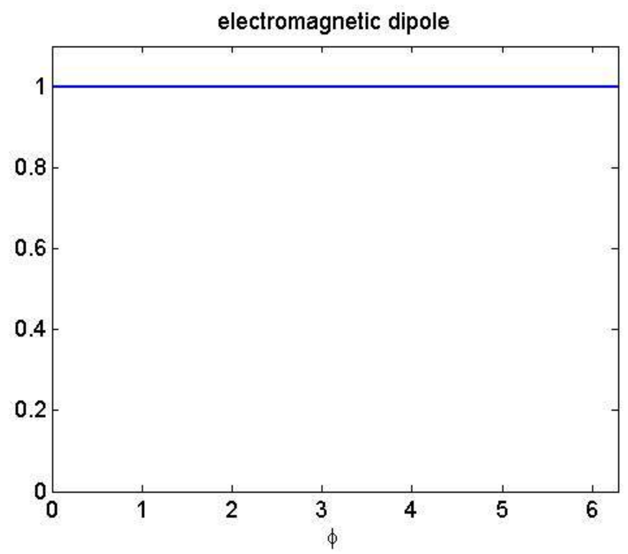


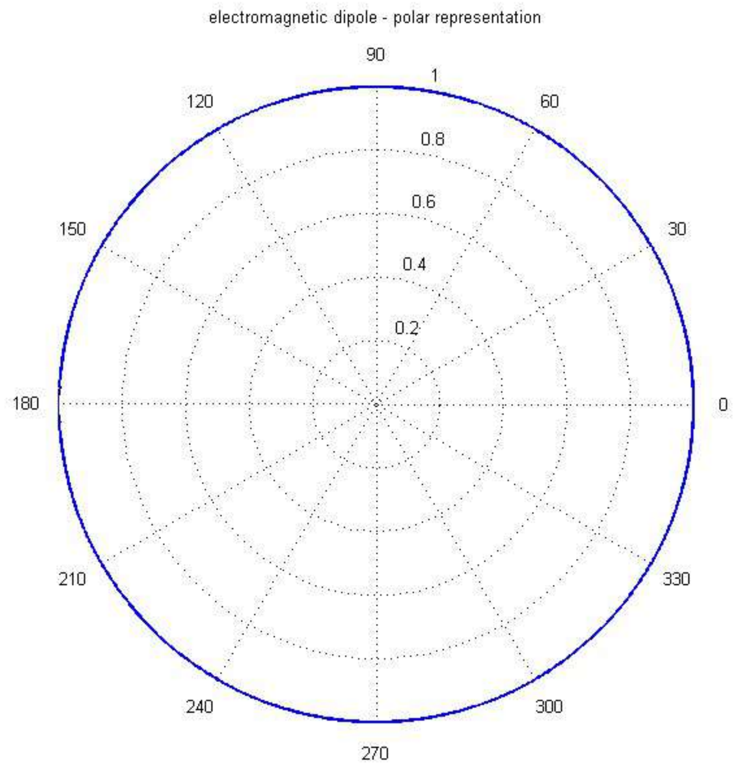
Radiation pattern

an example: the electrical elementary dipole

$$\mathbf{l}(\vartheta,\varphi) = \Delta z \sin\vartheta\, \hat{i}_{\vartheta}$$

Horizontal plane (θ=π/2)





Parameters of the Tx Antenna

- Effective length
  - Radiation pattern
  - Radiation pattern lobes
  - Beamwidth
- Directivity
- Gain
- Radiation Resistance
- Equivalent circuit of the tx antenna
- Input Impedance and Input Resistance

Radiation pattern lobes

- In some very specific directions there are zeros, or *nulls*, in the pattern indicating no radiation.
- The protuberances between the nulls are referred to as *lobes*, and the main, or major, lobe is in the direction of maximum radiation.

- There are also *side lobes* and *back lobes*.
  - A *back lobe* is "a radiation lobe whose axis makes an angle of approximately 180◦ with respect to the beam of an antenna." Usually it refers to a minor lobe that occupies the hemisphere in a direction opposite to that of the major (main) lobe.
  - *Side lobes* and *back lobes* divert power away from the main beam and are desired as small as possible.