Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2021-2022 - Laurea "Triennale" – Secondo semestre - Secondo anno

Università degli Studi di Napoli "Parthenope"

Stefano Perna

Color legend

New formulas, important considerations, important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Outline

- Radiation problem for extended antennas
- Field regions

Extended antennas

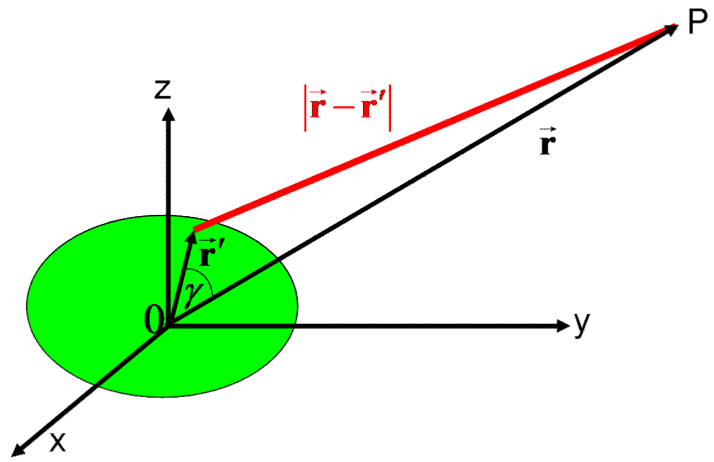
$J(r)$

$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$ Vector Helmholtz equation

$$\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi}\int \mathbf{J}(\mathbf{r}')\frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}d\mathbf{r}'$$

$$\boldsymbol{H} = \frac{1}{\mu}\nabla \times \boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega \boldsymbol{A} + \frac{\nabla\nabla \cdot \boldsymbol{A}}{j\omega\varepsilon\mu}$$

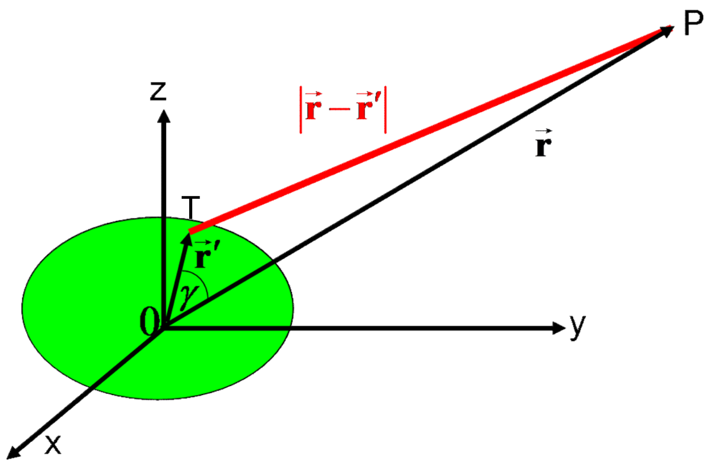
$E(r)$
$H(r)$



Is it possible to simplify the expressions of the fields, possibly via proper approximation of the vector potential A?

Extended antennas

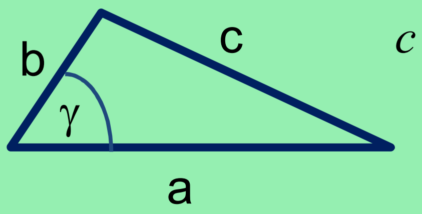
$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}') \frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} d\,\vec{\mathbf{r}}'$$



$$|\vec{\mathbf{r}}| = r \qquad |\vec{\mathbf{r}}'| = r'$$

$$|\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = \sqrt{r^2 + (r')^2 - 2rr'\cos\gamma} = \sqrt{r^2\left[1 + \left(\frac{r'}{r}\right)^2 - 2\frac{r'}{r}\cos\gamma\right]}$$

$$= r\sqrt{1 + \left(\frac{r'}{r}\right)^2 - 2\frac{r'}{r}\cos\gamma}$$



$$c = \sqrt{a^2 + b^2 - 2ab\cos\gamma}$$

Extended antennas

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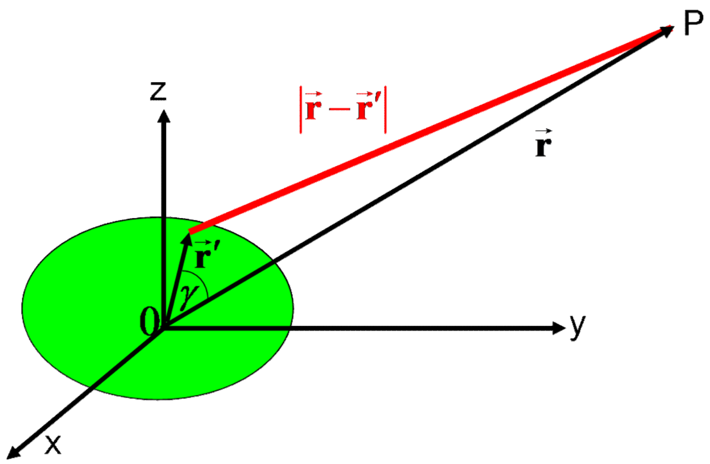

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$$= r\sqrt{1+\left(\frac{r'}{r}\right)^2-2\frac{r'}{r}\cos\gamma}$$

$$\sqrt{1+x} = 1+\frac{1}{2}x-\frac{1}{8}x^2+\cdots$$

$$\sqrt{1+\left(\frac{r'}{r}\right)^2-2\frac{r'}{r}\cos\gamma} = 1+\frac{1}{2}\left[\left(\frac{r'}{r}\right)^2-2\frac{r'}{r}\cos\gamma\right]-\frac{1}{8}\left[\left(\frac{r'}{r}\right)^2-2\frac{r'}{r}\cos\gamma\right]^2+\cdots = 1+\frac{1}{2}\left(\frac{r'}{r}\right)^2-\frac{r'}{r}\cos\gamma-\frac{1}{8}\left[\cancel{\left(\frac{r'}{r}\right)^4}+4\left(\frac{r'}{r}\right)^2\cos^2\gamma\cancel{-4\left(\frac{r'}{r}\right)^3\cos\gamma}\right]+\cdots$$

$$= 1+\frac{1}{2}\left(\frac{r'}{r}\right)^2-\frac{r'}{r}\cos\gamma-\frac{1}{2}\left(\frac{r'}{r}\right)^2\cos^2\gamma+\cdots = 1-\frac{r'}{r}\cos\gamma+\frac{1}{2}\left(\frac{r'}{r}\right)^2\left(1-\cos^2\gamma\right)+\cdots = 1-\frac{r'}{r}\cos\gamma+\frac{1}{2}\left(\frac{r'}{r}\right)^2\sin^2\gamma+\cdots$$

Extended antennas

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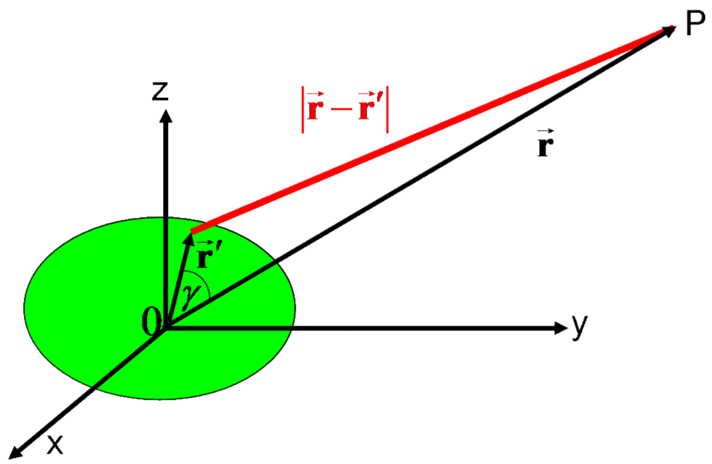

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$$= r\sqrt{1 + \left(\frac{r'}{r}\right)^2 - 2\frac{r'}{r}\cos\gamma} = r - r'\cos\gamma + \frac{(r')^2}{2r}\sin^2\gamma + \cdots$$

$$\sqrt{1 + \left(\frac{r'}{r}\right)^2 - 2\frac{r'}{r}\cos\gamma}$$

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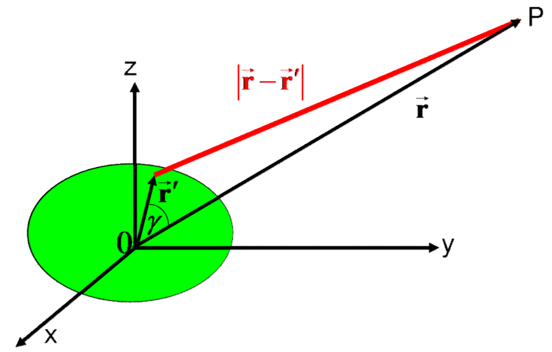

$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} =$$

$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} =$$

Extended antennas

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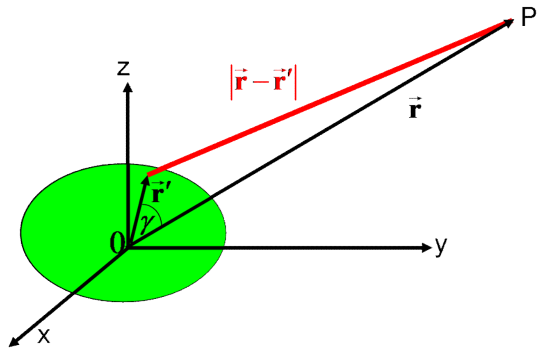

$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - r'\cos\gamma + \frac{(r')^2}{2r}\sin^2\gamma + \cdots} \approx \frac{1}{r}$$

$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = e^{-j\beta r}e^{j\beta r'\cos\gamma}e^{-j\beta\frac{(r')^2}{2r}\sin^2\gamma}\cdots$$

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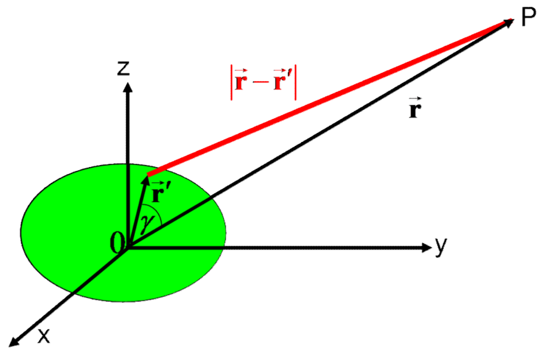

$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - r'\cos\gamma + \frac{(r')^2}{2r}\sin^2\gamma + \cdots} \approx \frac{1}{r} \quad \text{if} \quad r' << r$$

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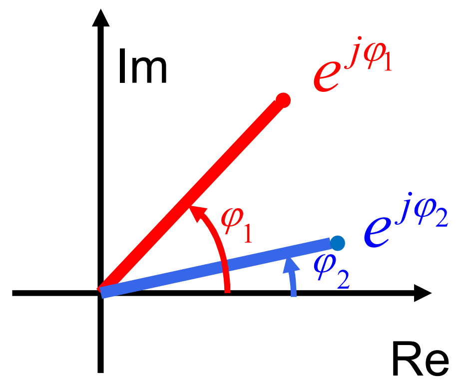

$$e^{j\beta r'\cos\gamma} \approx 1 \quad \Rightarrow \quad \frac{2\pi}{\lambda}r' << 2\pi \quad \Rightarrow \quad r' << \lambda$$

Extended antennas

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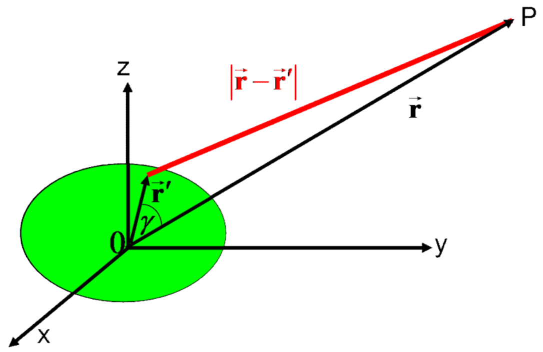

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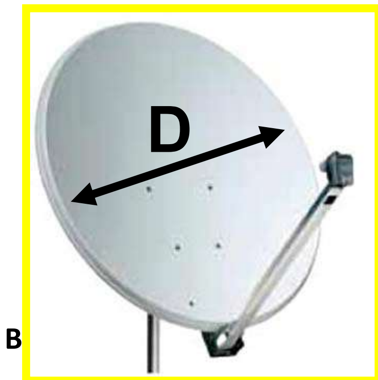

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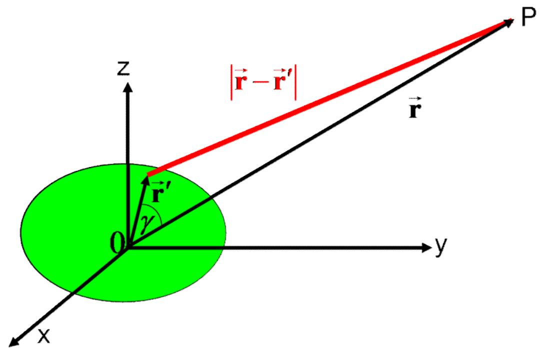

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if $D << r$

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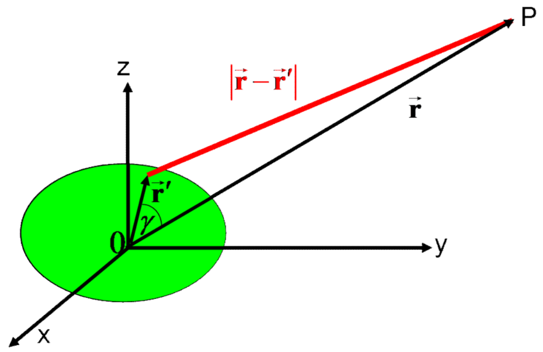

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When the antennas are **small** with respect to the wavelength **and** to the distance from the observation point

$$\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx \frac{e^{-j\beta r}}{r} \quad \Longrightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi}\frac{e^{-j\beta r}}{r}\int \mathbf{J}(\vec{\mathbf{r}}')d\,\vec{\mathbf{r}}'$$

Extended antennas

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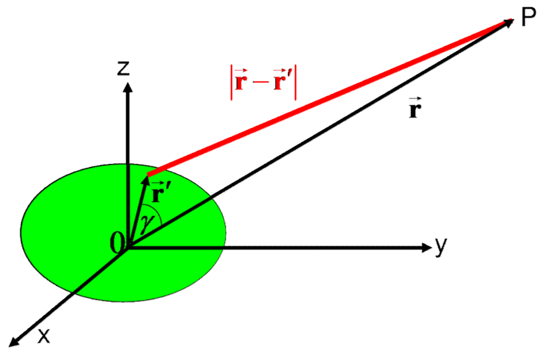

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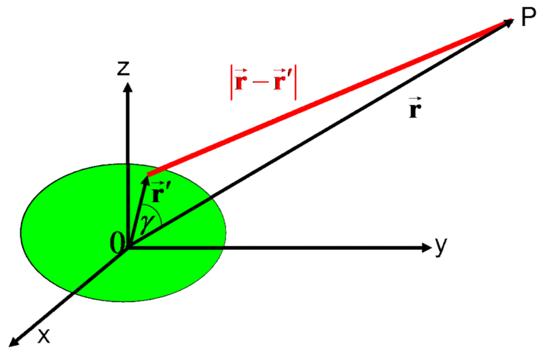

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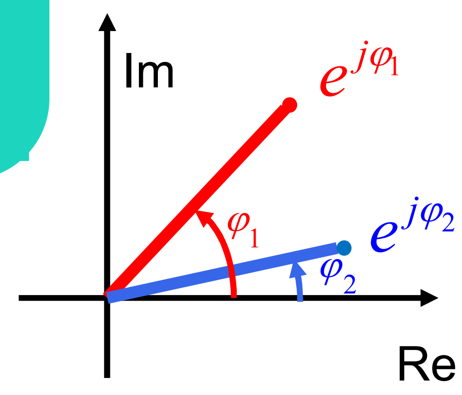

$$e^{-j\beta\frac{(r')^2}{2r}\sin^2\gamma} \approx 1 \;\Rightarrow\; \beta\frac{(r')^2}{2r}\sin^2\gamma << 2\pi \;\Rightarrow\; \frac{2\pi}{\lambda}\left(\frac{D}{2}\right)^2\frac{1}{2r} < \frac{\pi}{8} \;\Rightarrow\; r > \frac{2D^2}{\lambda}$$

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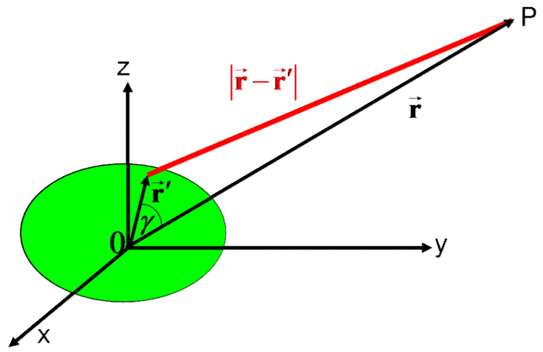

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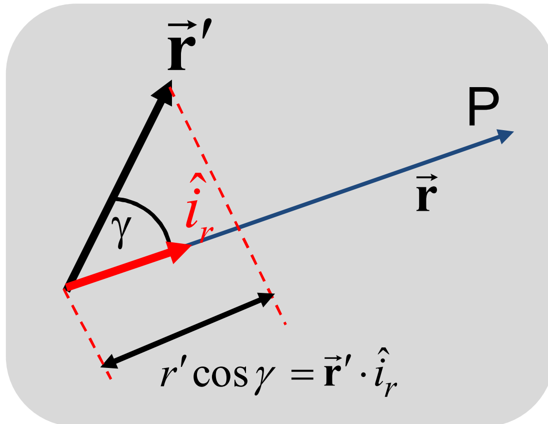

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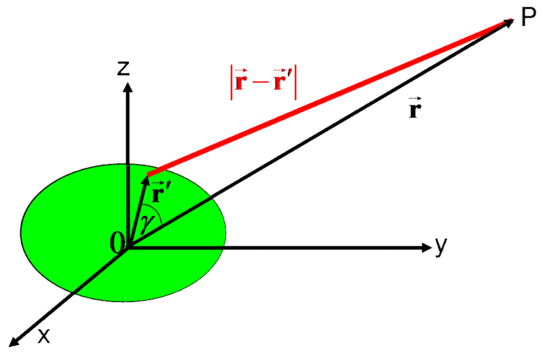

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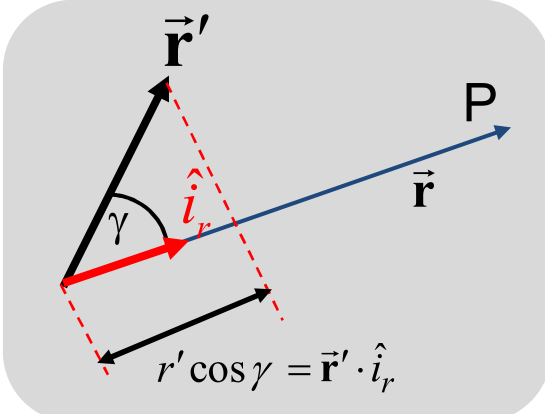

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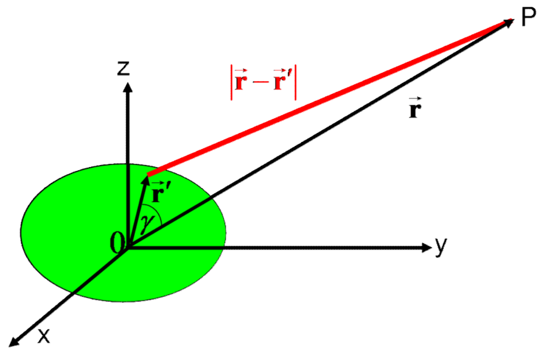

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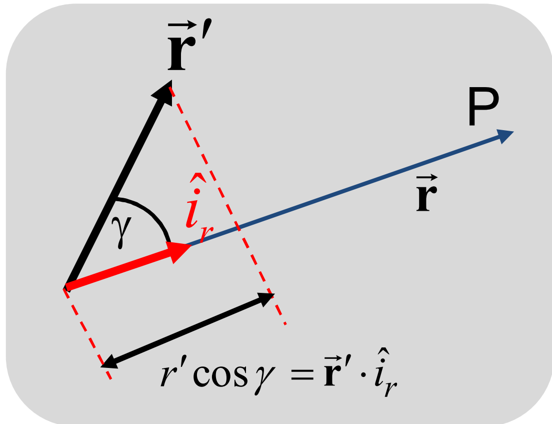

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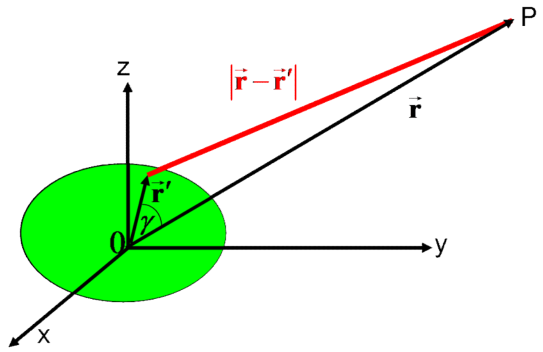

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$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = e^{-j\beta r}e^{j\beta r'\cos\gamma}e^{-j\beta\frac{(r')^2}{2r}\sin^2\gamma}\cdots \approx e^{-j\beta r}e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r} \quad if \quad r > \frac{2D^2}{\lambda}$$

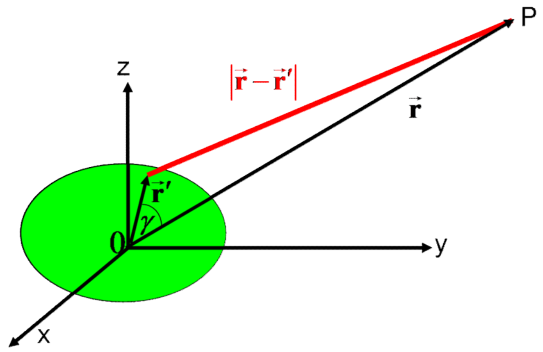
For **<u>all</u>** the antennas, if the distance from the observation point **is sufficiently large**

$$\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx \frac{e^{-j\beta r}e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}}{r} \quad \Rightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi}\frac{e^{-j\beta r}}{r}\int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}\,d\,\vec{\mathbf{r}}'$$

Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\,\vec{\mathbf{r}}'$$

$$|\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = r - \vec{\mathbf{r}}'\cdot\hat{i}_r + \frac{(r')^2}{2r}\sin^2\gamma + \cdots$$



$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - r'\cos\gamma + \frac{(r')^2}{2r}\sin^2\gamma + \cdots} \approx \frac{1}{r} \quad if \quad D << r$$

$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = e^{-j\beta r}e^{j\beta r'\cos\gamma}e^{-j\beta\frac{(r')^2}{2r}\sin^2\gamma}\cdots \approx e^{-j\beta r}e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r} \quad if \quad r > \frac{2D^2}{\lambda}$$

Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\,\vec{\mathbf{r}}'$$

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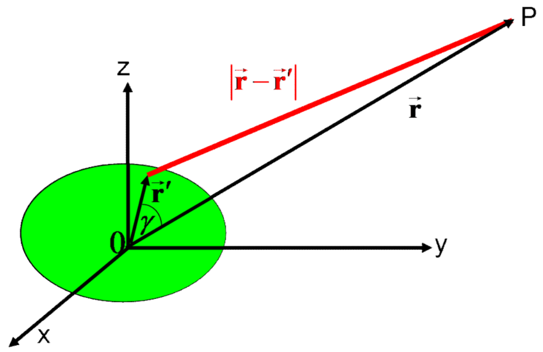

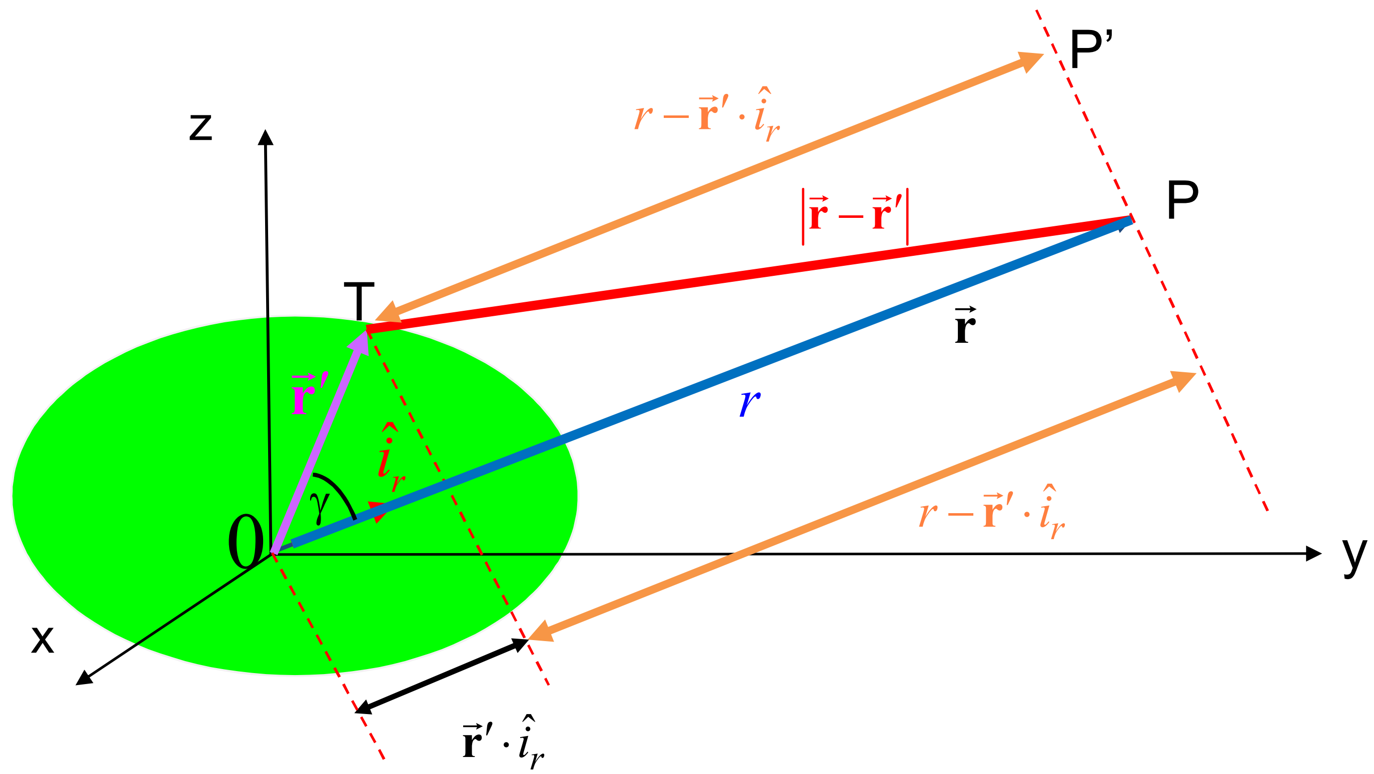

Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\,\vec{\mathbf{r}}'$$

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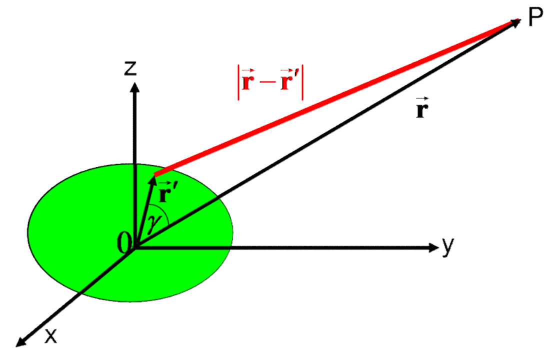

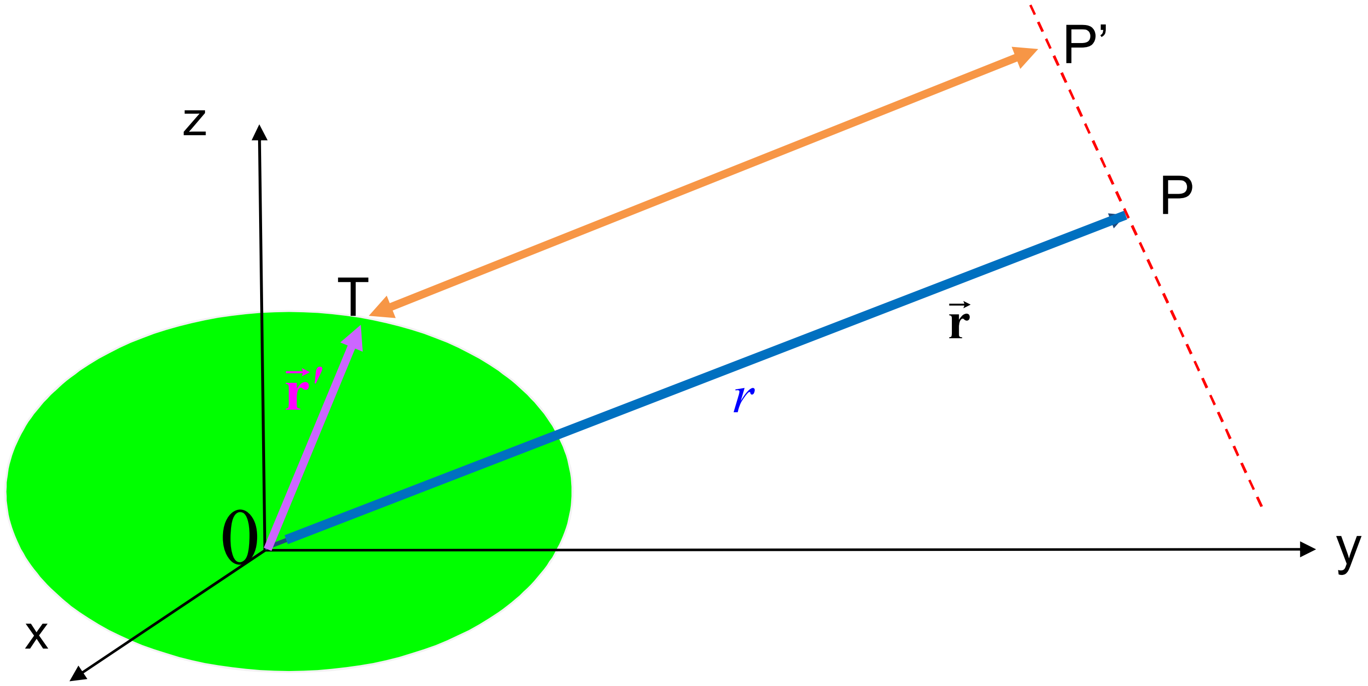

Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\vec{\mathbf{r}}'$$

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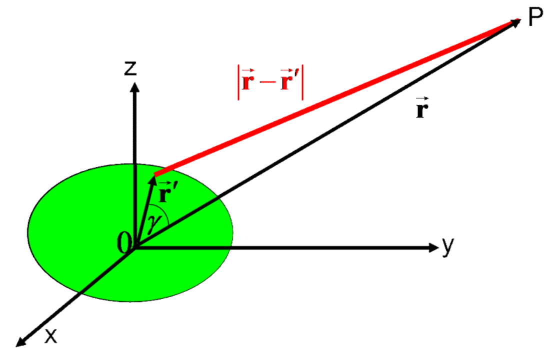
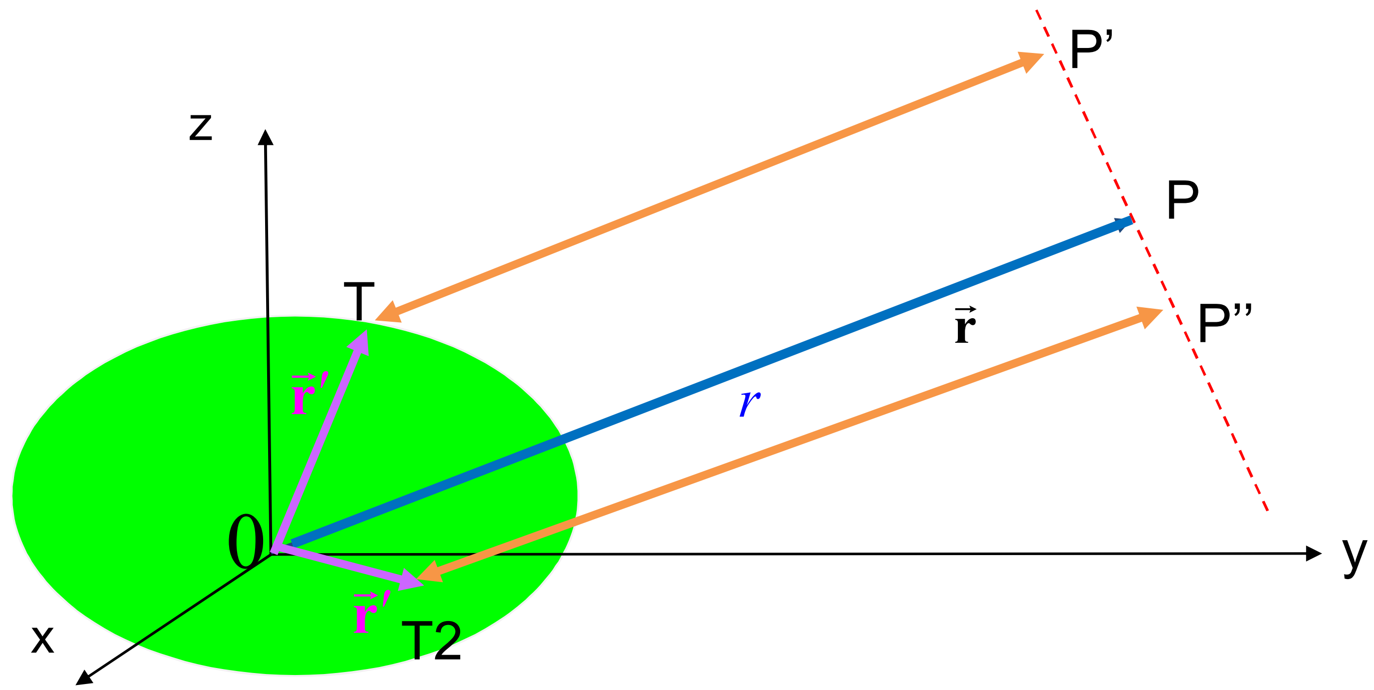
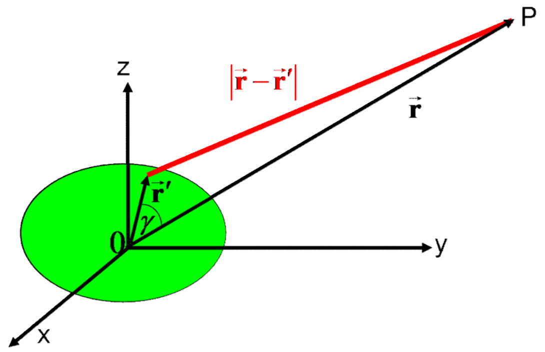




Extended antennas

$$\mathbf{A}(\vec{\mathbf{r}}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}d\,\vec{\mathbf{r}}'$$

$$|\vec{\mathbf{r}}-\vec{\mathbf{r}}'| = r - \vec{\mathbf{r}}'\cdot\hat{i}_r + \cancel{\frac{(r')^2}{2r}\sin^2\gamma} + \cdots$$



$$\frac{1}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = \frac{1}{r - \cancel{r'\cos\gamma} + \cancel{\frac{(r')^2}{2r}\sin^2\gamma} + \cdots} \approx \frac{1}{r} \quad \textit{if} \quad D << r$$

$$e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} = e^{-j\beta r}e^{j\beta r'\cos\gamma}e^{\cancel{-j\beta\frac{(r')^2}{2r}\sin^2\gamma}}\cdots \approx e^{-j\beta r}e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r} \quad \textit{if} \quad r > \frac{2D^2}{\lambda}$$

For <u>**all**</u> the antennas, if the distance from the observation point **is sufficiently large**

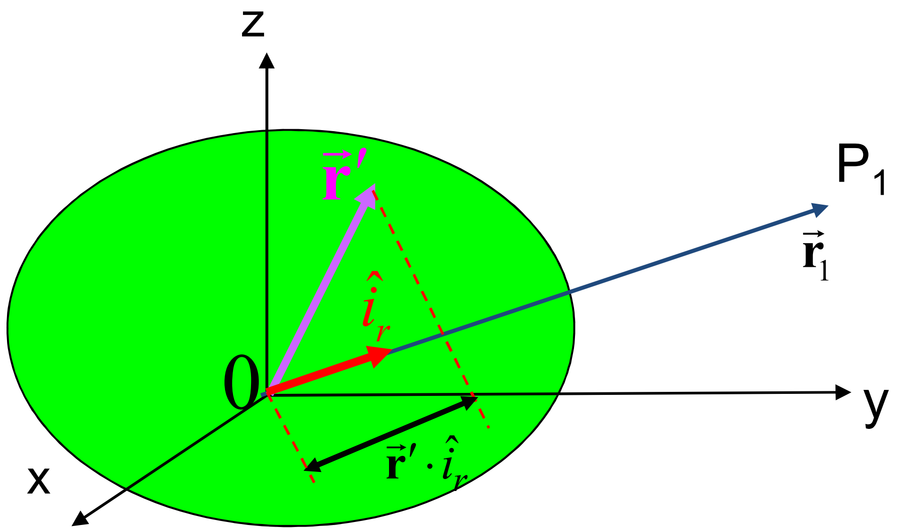
$$\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx \frac{e^{-j\beta r}e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}}{r} \quad \Rightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi}\frac{e^{-j\beta r}}{r}\int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}d\,\vec{\mathbf{r}}'$$

Extended antennas

$$r >> D$$

$$r > \frac{2D^2}{\lambda}$$

$$\int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\,\vec{\mathbf{r}}' =$$



For **all** the antennas, if the distance from the observation point **is sufficiently large**

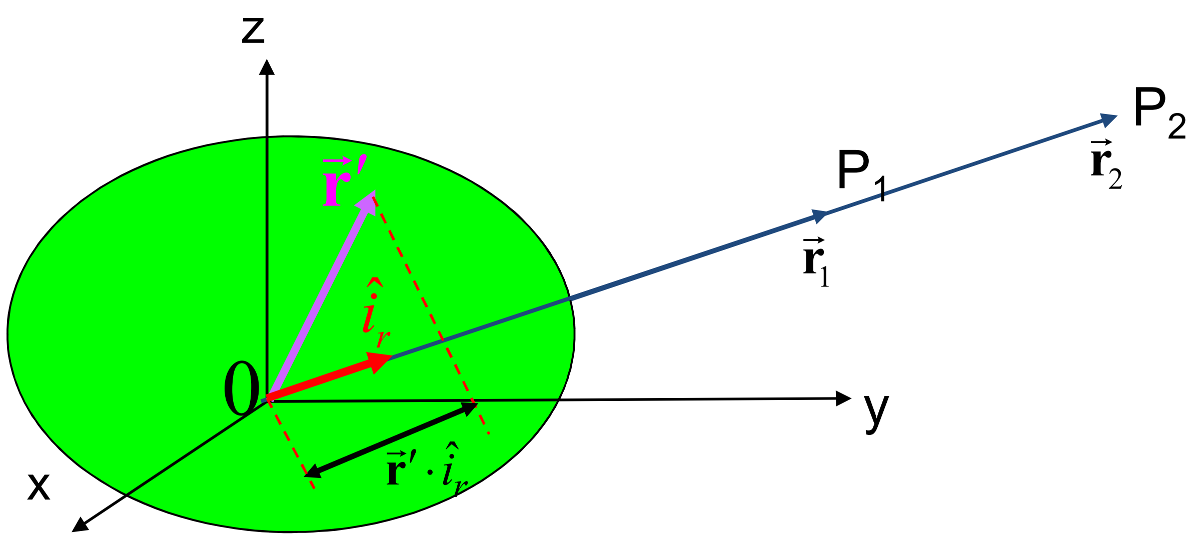
$$\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx \frac{e^{-j\beta r} e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r}}{r} \quad \Rightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\,\vec{\mathbf{r}}'$$

Extended antennas

$r >> D$

$r > \frac{2D^2}{\lambda}$

$\int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\,\vec{\mathbf{r}}' =$



For **all** the antennas, if the distance from the observation point **is sufficiently large**

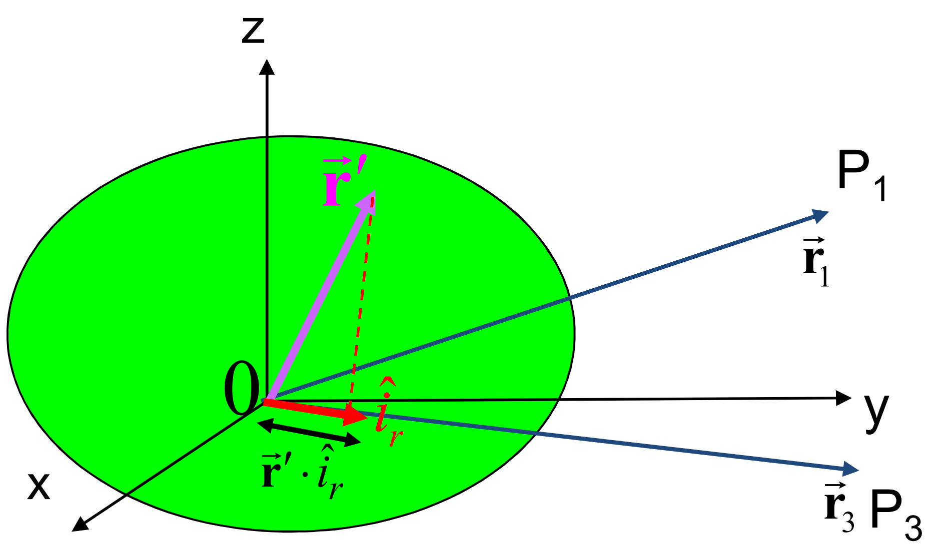
$$\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx \frac{e^{-j\beta r} e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r}}{r} \quad \Rightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\,\vec{\mathbf{r}}'$$

Extended antennas

$$r >> D$$

$$r > \frac{2D^2}{\lambda}$$

$$\int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\,\vec{\mathbf{r}}' = \mathbf{M}(\vartheta, \varphi) \quad \Rightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \mathbf{M}(\vartheta, \varphi)$$



For **all** the antennas, if the distance from the observation point **is sufficiently large**

$$\frac{e^{-j\beta|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|}}{|\vec{\mathbf{r}}-\vec{\mathbf{r}}'|} \approx \frac{e^{-j\beta r} e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r}}{r} \quad \Rightarrow \quad \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\,\vec{\mathbf{r}}'$$

Extended antennas

$$r >> D$$

$$r > \frac{2D^2}{\lambda}$$

$$\int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\,\vec{\mathbf{r}}' = \mathbf{M}(\vartheta, \varphi) \Longrightarrow \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \mathbf{M}(\vartheta, \varphi)$$

Fraunhofer region

$r >> D$

$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

$$\int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\vec{\mathbf{r}}' = \mathbf{M}(\vartheta, \varphi) \Rightarrow \mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \mathbf{M}(\vartheta, \varphi)$$

$$\begin{cases} \nabla\nabla \cdot \mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}}) \hat{i}_r \\ \nabla \times \mathbf{A}(\vec{\mathbf{r}}) \approx -j\beta \hat{i}_r \times \mathbf{A}(\vec{\mathbf{r}}) \end{cases}$$

Fraunhofer region

Radiation problem for extended antennas

$\downarrow$ $\boldsymbol{J}(\boldsymbol{r})$

$$\nabla^2 \boldsymbol{A} + k^2 \boldsymbol{A} = -\mu \boldsymbol{J}$$ **Vector Helmholtz equation**

$$\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi}\int \mathbf{J}(\vec{\mathbf{r}}')\frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}d\vec{\mathbf{r}}' \approx \frac{\mu}{4\pi}\frac{e^{-j\beta r}}{r}\int \mathbf{J}(\mathbf{r}')e^{j\beta \vec{\mathbf{r}}'\cdot\hat{i}_r}d\mathbf{r}'$$

$$\boldsymbol{H} = \frac{1}{\mu}\nabla \times \boldsymbol{A} \quad \text{and} \quad \boldsymbol{E} = -j\omega \boldsymbol{A} + \frac{\nabla\nabla\cdot \boldsymbol{A}}{j\omega\varepsilon\mu}$$

$\nabla \times \boldsymbol{A} \rightarrow -j\beta\,\hat{i}_r \times \mathbf{A}$

$\nabla\nabla\cdot \boldsymbol{A} \rightarrow -\beta^2 A_r \hat{i}_r$

$\downarrow$ $\boldsymbol{E}(\boldsymbol{r})$, $\boldsymbol{H}(\boldsymbol{r})$

Fraunhofer region

$r >> D$

$r > \frac{2D^2}{\lambda}$

$r >> \lambda$

$$\int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\vec{\mathbf{r}}' = \mathbf{M}(\vartheta, \varphi)$$

$$\mathbf{A}(\vec{\mathbf{r}}) \approx \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \mathbf{M}(\vartheta, \varphi)$$

$$\begin{cases} \nabla\nabla \cdot \mathbf{A}(\vec{\mathbf{r}}) \approx -\beta^2 A_r(\vec{\mathbf{r}}) \hat{i}_r \\ \nabla \times \mathbf{A}(\vec{\mathbf{r}}) \approx -j\beta \hat{i}_r \times \mathbf{A}(\vec{\mathbf{r}}) \end{cases}$$

$$\mathbf{A}(\vec{\mathbf{r}}) = \cancel{A_r(\vec{\mathbf{r}})} \hat{i}_r + A_\vartheta(\vec{\mathbf{r}}) \hat{i}_\vartheta + A_\varphi(\vec{\mathbf{r}}) \hat{i}_\varphi$$

Fraunhofer region

$$\mathbf{E}(\vec{\mathbf{r}}) = -j\omega \left[\mathbf{A}(\vec{\mathbf{r}}) - A_r(\vec{\mathbf{r}}) \hat{i}_r \right] = -j\omega \left[\frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} \mathbf{M}(\vartheta, \varphi) - \frac{\mu}{4\pi} \frac{e^{-j\beta r}}{r} M_r(\vartheta, \varphi) \hat{i}_r \right] = -\frac{j\omega\mu}{4\pi} \frac{e^{-j\beta r}}{r} \left[\mathbf{M}(\vartheta, \varphi) - M_r(\vartheta, \varphi) \hat{i}_r \right]$$

$$\zeta \mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

Fraunhofer region

$$r >> D$$

$$r > \frac{2D^2}{\lambda}$$

$$r >> \lambda$$

$$\int \mathbf{J}(\vec{\mathbf{r}}') e^{j\beta \vec{\mathbf{r}}' \cdot \hat{i}_r} d\,\vec{\mathbf{r}}' = \mathbf{M}(\vartheta, \varphi)$$

Fraunhofer region

$$\mathbf{E}(\vec{\mathbf{r}}) = \quad = -\frac{j\omega\mu}{4\pi} \frac{e^{-j\beta r}}{r} \left[\mathbf{M}(\vartheta,\varphi) - M_r(\vartheta,\varphi)\hat{i}_r \right]$$

$$\zeta \mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

Fraunhofer region

$$r >> D$$

$$r > \frac{2D^2}{\lambda}$$

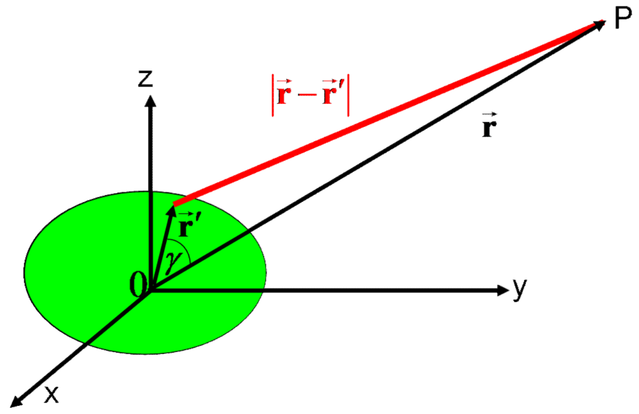
$$r >> \lambda$$

Fraunhofer region

$$\mathbf{E}(\vec{\mathbf{r}}) = \mathbf{E}(r,\vartheta,\varphi) = -\frac{j\omega\mu}{4\pi}\frac{e^{-j\beta r}}{r}\left[\mathbf{M}(\vartheta,\varphi) - M_r(\vartheta,\varphi)\hat{i}_r\right]$$

$$\zeta\mathbf{H}(\vec{\mathbf{r}}) = \hat{i}_r \times \mathbf{E}(\vec{\mathbf{r}})$$

$$\mathbf{M}(\vartheta,\varphi) = \int \mathbf{J}(\vec{\mathbf{r}}')e^{j\beta\vec{\mathbf{r}}'\cdot\hat{i}_r}\,d\,\vec{\mathbf{r}}'$$



- the e.m. field propagates along $\hat{i}_r$
- the e.m. field lies on the plane orthogonal to the propagation direction
- |E| and |H| exhibit the decaying factor 1/r
- |E| and |H| are proportional through ζ