Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2021-2022 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations, important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence (PD)

Plane Waves

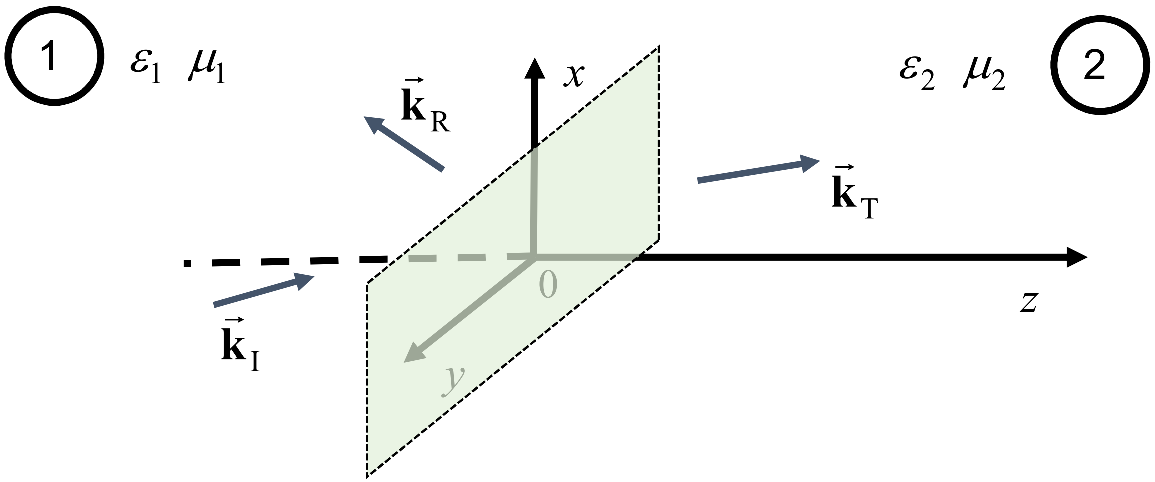
Incidence on a dielectric half-space

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x} e^{-jk_{\mathrm{I}y}y} e^{-jk_{\mathrm{I}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}\hat{i}_x + k_{\mathrm{I}y}\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}^2 + k_{\mathrm{I}y}^2 + k_{\mathrm{I}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x} e^{-jk_{\mathrm{R}y}y} e^{-jk_{\mathrm{R}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}\hat{i}_x + k_{\mathrm{R}y}\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}^2 + k_{\mathrm{R}y}^2 + k_{\mathrm{R}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x} e^{-jk_{\mathrm{T}y}y} e^{-jk_{\mathrm{T}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}\hat{i}_x + k_{\mathrm{T}y}\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}^2 + k_{\mathrm{T}y}^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$



$$\vec{\mathbf{E}}_1\left(\vec{\mathbf{r}}\right) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

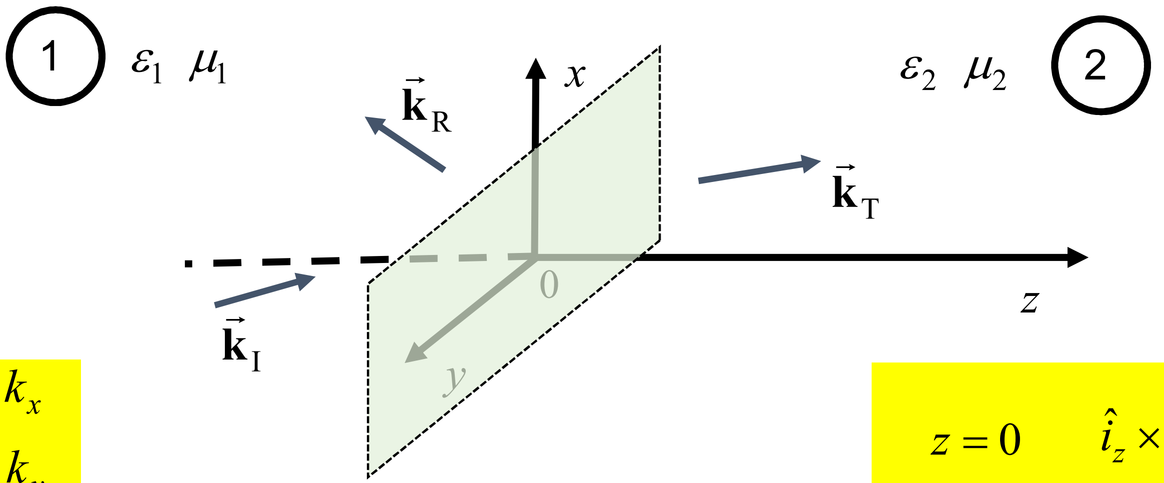
$$\vec{\mathbf{E}}_2\left(\vec{\mathbf{r}}\right) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x} e^{-jk_{\mathrm{I}y}y} e^{-jk_{\mathrm{I}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}\hat{i}_x + k_{\mathrm{I}y}\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}^2 + k_{\mathrm{I}y}^2 + k_{\mathrm{I}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x} e^{-jk_{\mathrm{R}y}y} e^{-jk_{\mathrm{R}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}\hat{i}_x + k_{\mathrm{R}y}\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}^2 + k_{\mathrm{R}y}^2 + k_{\mathrm{R}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x} e^{-jk_{\mathrm{T}y}y} e^{-jk_{\mathrm{T}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}\hat{i}_x + k_{\mathrm{T}y}\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}^2 + k_{\mathrm{T}y}^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$



$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

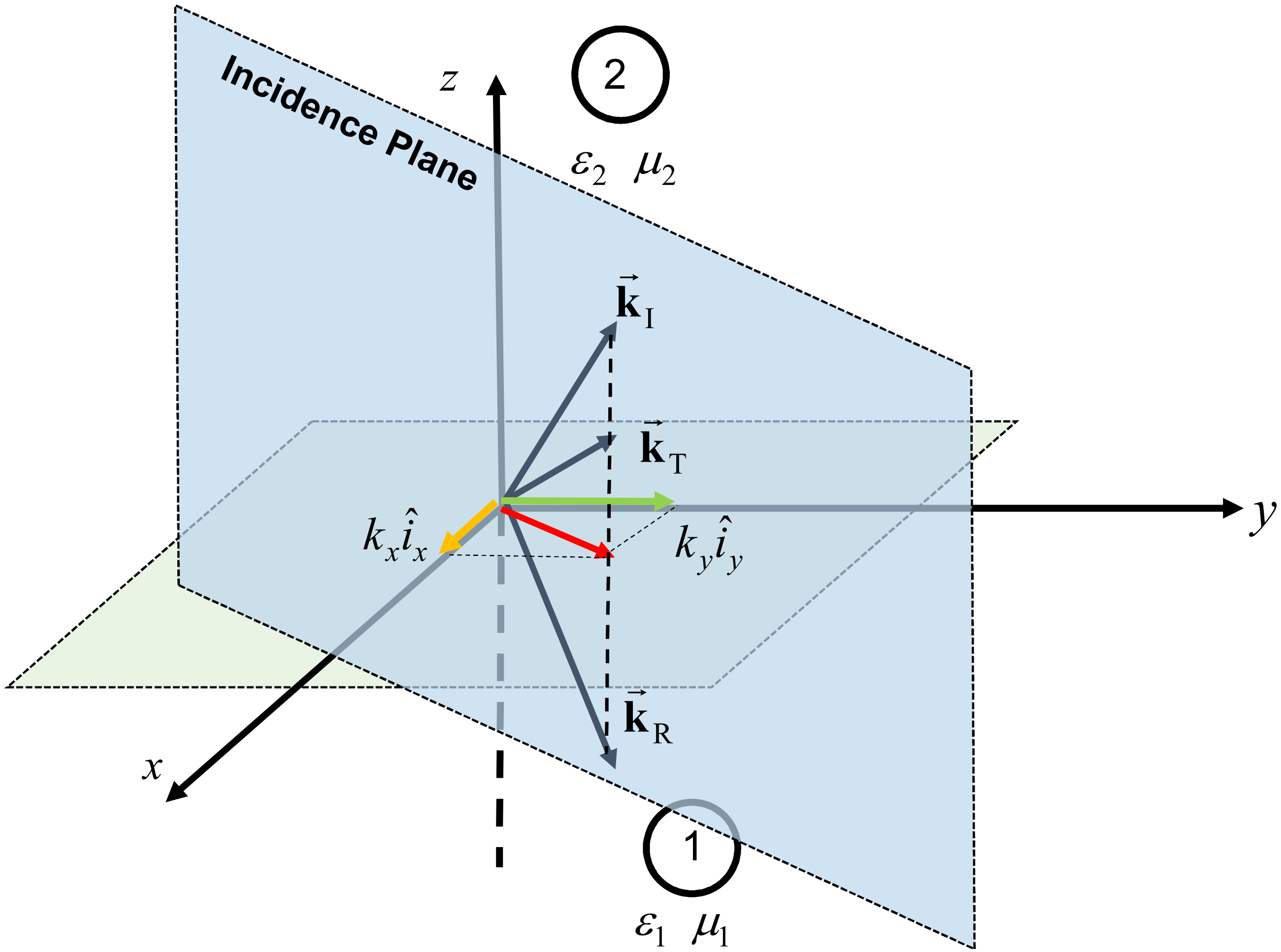
$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$k_{\mathrm{I}x} = k_{Rx} = k_{\mathrm{T}x} = k_x$$

$$k_{\mathrm{I}y} = k_{Ry} = k_{\mathrm{T}y} = k_y$$

$$z = 0 \qquad \hat{i}_z \times \left(\vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1\right) = \mathbf{0} \qquad \hat{i}_z \times \vec{\mathbf{E}}_1 = \hat{i}_z \times \vec{\mathbf{E}}_2$$

Incidence on a dielectric half-space

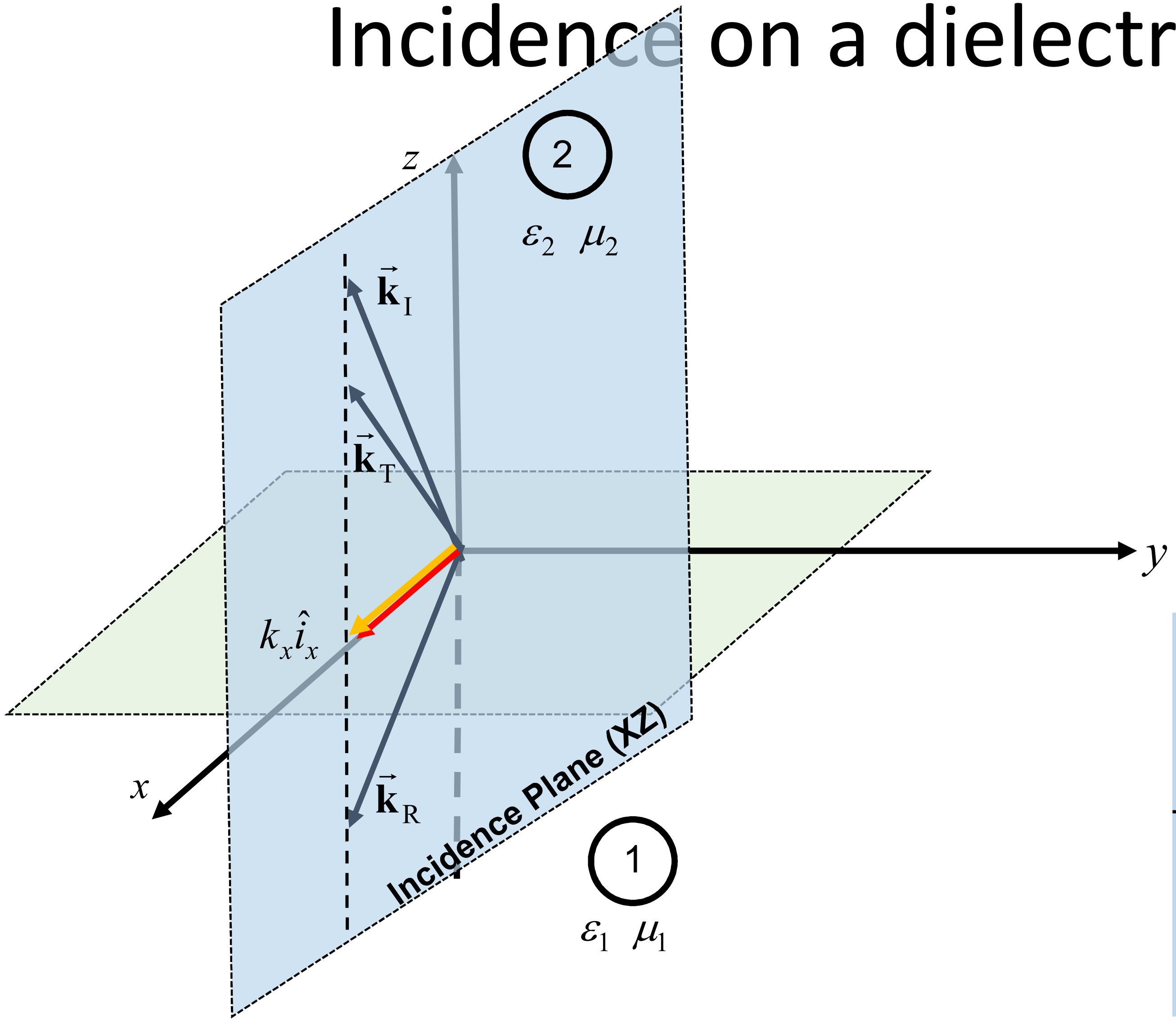


$$\vec{\mathbf{k}}_\mathrm{I} = k_x \hat{i}_x + k_y \hat{i}_y + k_{\mathrm{I}z} \hat{i}_z$$

$$\vec{\mathbf{k}}_\mathrm{R} = k_x \hat{i}_x + k_y \hat{i}_y + k_{\mathrm{R}z} \hat{i}_z$$

$$\vec{\mathbf{k}}_\mathrm{T} = k_x \hat{i}_x + k_y \hat{i}_y + k_{\mathrm{T}z} \hat{i}_z$$

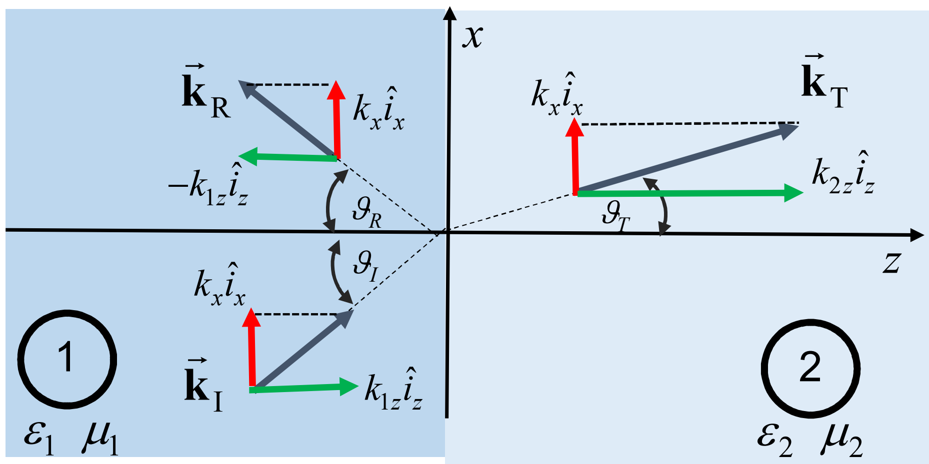
Incidence on a dielectric half-space



$$\vec{\mathbf{k}}_\mathrm{I} = k_x\hat{i}_x + k_{\mathrm{I}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_\mathrm{R} = k_x\hat{i}_x + k_{\mathrm{R}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_\mathrm{T} = k_x\hat{i}_x + k_{\mathrm{T}z}\hat{i}_z$$



Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}}$$

$$k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

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$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

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$$k_x = k_1 \sin\vartheta_{\mathrm{I}}$$

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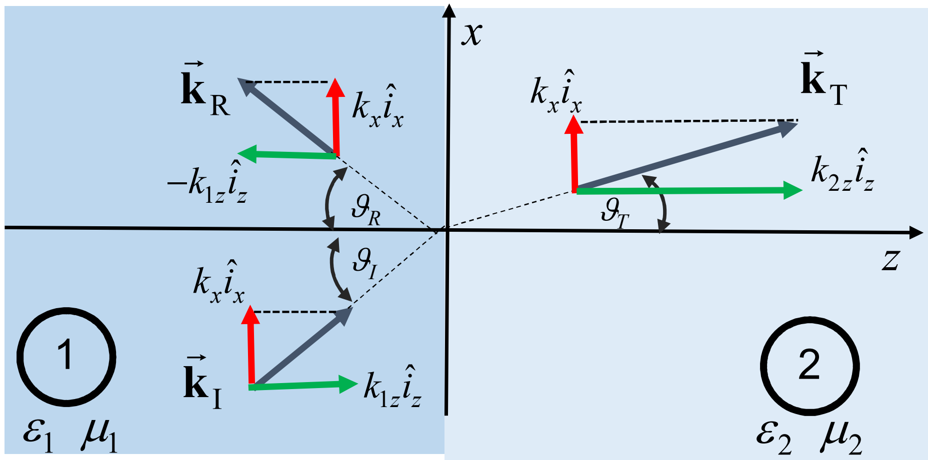
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

Perpendicular Polarization ⊥

$$\left[H_x, E_y, H_z\right]$$

Parallel Polarization ||

$$\left[E_x, H_y, E_z\right]$$



Incidence on a dielectric half-space: ‖ polarization

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}} \qquad k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** ‖

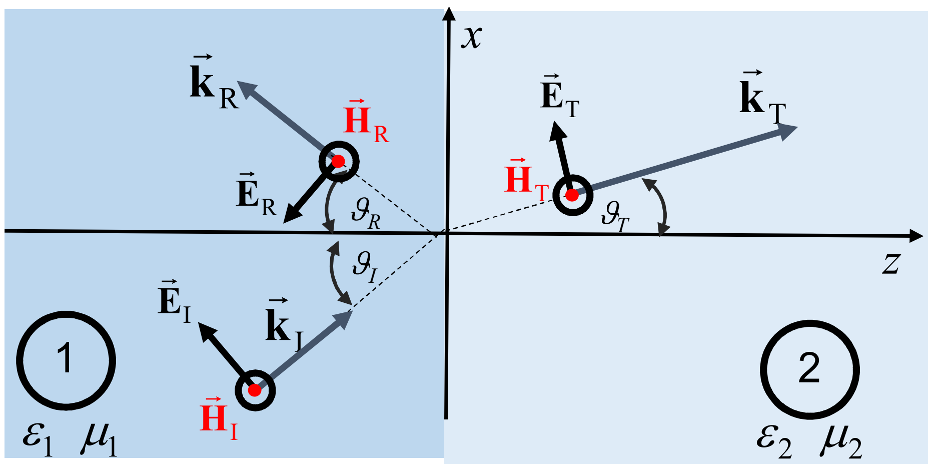
$$H_{1y}(z,x) = \left[H_{\mathrm{I}} e^{-jk_{1z}z} + H_{\mathrm{R}} e^{jk_{1z}z}\right] e^{-jk_x x} \qquad 1$$

$$H_{2y}(z,x) = H_{\mathrm{T}} e^{-jk_{2z}z} e^{-jk_x x} \qquad 2$$

$$\Gamma_{\parallel} \triangleq -\frac{H_{\mathrm{R}}}{H_{\mathrm{I}}} \qquad Z_1 = \frac{k_{1z}}{\omega\varepsilon_1}$$

$$\mathrm{T}_{\parallel} \triangleq \frac{Z_2 H_{\mathrm{T}}}{Z_1 H_{\mathrm{I}}} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_{\parallel} = -\frac{\cos\vartheta_{\mathrm{I}} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$



Incidence on a dielectric half-space: ⊥ polarization

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2 \varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I} \qquad k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

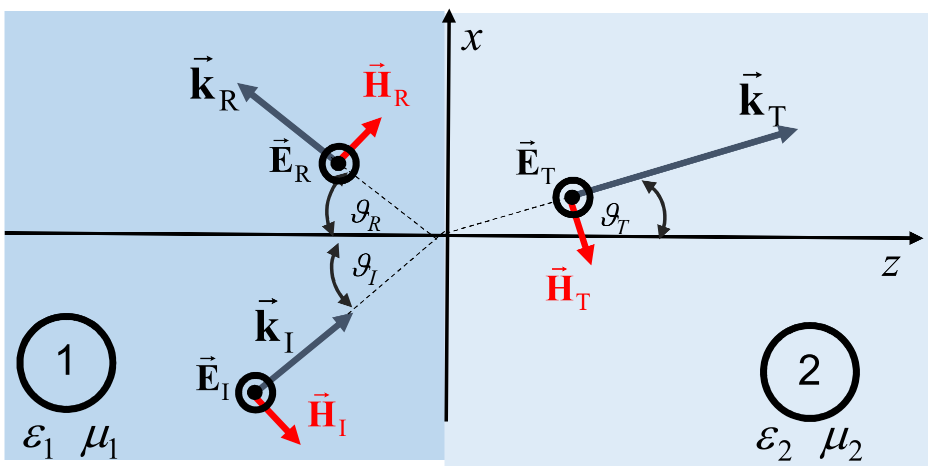
$$E_{1y}(z,x) = \left[E_\mathrm{I} e^{-jk_{1z}z} + E_\mathrm{R} e^{jk_{1z}z}\right] e^{-jk_x x} \qquad 1$$

$$E_{2y}(z,x) = E_\mathrm{T} e^{-jk_{2z}z} e^{-jk_x x} \qquad 2$$

$$\Gamma_\perp \triangleq \frac{E_\mathrm{R}}{E_\mathrm{I}} \qquad Z_1 = \frac{\omega\mu_1}{k_{1z}}$$

$$\mathrm{T}_\perp \triangleq \frac{E_\mathrm{T}}{E_\mathrm{I}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_\mathrm{I} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$



Incidence on a dielectric half-space

$$\begin{cases} 1-\Gamma = \frac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$\mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I}$$

$$k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$E_{1y}(z,x) = \left[E_\mathrm{I} e^{-jk_{1z}z} + E_\mathrm{R} e^{jk_{1z}z}\right] e^{-jk_x x} \qquad 1$$

$$E_{2y}(z,x) = E_\mathrm{T} e^{-jk_{2z}z} e^{-jk_x x} \qquad 2$$

$$\Gamma_\perp \triangleq \frac{E_\mathrm{R}}{E_\mathrm{I}} \qquad Z_1 = \frac{\omega\mu_1}{k_{1z}}$$

$$\mathrm{T}_\perp \triangleq \frac{E_\mathrm{T}}{E_\mathrm{I}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_\mathrm{I} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** $\parallel$

$$H_{1y}(z,x) = \left[H_\mathrm{I} e^{-jk_{1z}z} + H_\mathrm{R} e^{jk_{1z}z}\right] e^{-jk_x x} \qquad 1$$

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$$\Gamma_\parallel \triangleq -\frac{H_\mathrm{R}}{H_\mathrm{I}} \qquad Z_1 = \frac{k_{1z}}{\omega\varepsilon_1}$$

$$\mathrm{T}_\parallel \triangleq \frac{Z_2 H_\mathrm{T}}{Z_1 H_\mathrm{I}} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\parallel = -\frac{\cos\vartheta_\mathrm{I} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$

Incidence on a dielectric half-space

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

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$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

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$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}}$$

$$Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_{\perp} = \frac{\cos\vartheta_{\mathrm{I}} - \left(\mu_1/\mu_2\right)\sqrt{\left(k_2/k_1\right)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + \left(\mu_1/\mu_2\right)\sqrt{\left(k_2/k_1\right)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** $\|$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1}$$

$$Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_{\|} = -\frac{\cos\vartheta_{\mathrm{I}} - \left(\varepsilon_1/\varepsilon_2\right)\sqrt{\left(k_2/k_1\right)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + \left(\varepsilon_1/\varepsilon_2\right)\sqrt{\left(k_2/k_1\right)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

Incidence on a dielectric half-space

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

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$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_{\mathrm{I}}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_{\mathrm{I}} - \left(\mu_1/\mu_2\right)\sqrt{\left(k_2/k_1\right)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + \left(\mu_1/\mu_2\right)\sqrt{\left(k_2/k_1\right)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** $\parallel$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\parallel = -\frac{\cos\vartheta_{\mathrm{I}} - \left(\varepsilon_1/\varepsilon_2\right)\sqrt{\left(k_2/k_1\right)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + \left(\varepsilon_1/\varepsilon_2\right)\sqrt{\left(k_2/k_1\right)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

Normal Incidence

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$\mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}}$$

$$\boxed{k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$

$$\boxed{k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

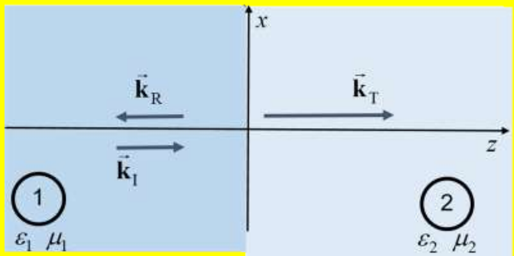
$$\Gamma_\perp = \frac{\cos\vartheta_{\mathrm{I}} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

$[E_x, H_y, E_z]$ **Parallel Polarization** $\parallel$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\parallel = -\frac{\cos\vartheta_{\mathrm{I}} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

Normal incidence



$$\vartheta_I = 0° \Rightarrow \quad k_{1z} = k_1 \qquad k_{2z} = k_2$$

Normal Incidence

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

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$$k_x = k_1 \sin\vartheta_\mathrm{I} \qquad k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** $\perp$

$$\boxed{Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}}$$

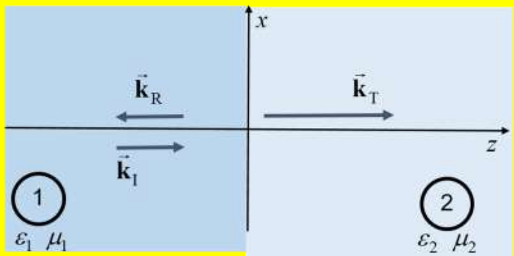
$$\Gamma_\perp = \frac{\cos\vartheta_\mathrm{I} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$

$[E_x, H_y, E_z]$ **Parallel Polarization** $\|$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\| = -\frac{\cos\vartheta_\mathrm{I} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$

Normal incidence



$$\vartheta_I = 0° \Rightarrow \boxed{k_{1z} = k_1} \qquad k_{2z} = k_2$$

$\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} = \frac{\omega\mu_1}{k_1} = \frac{\omega\mu_1}{\omega\sqrt{\mu_1\varepsilon_1}} = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \zeta_1$$

$$Z_2 = \frac{\omega\mu_2}{k_{2z}} = \zeta_2$$

Normal Incidence

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\boxed{\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I \qquad k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

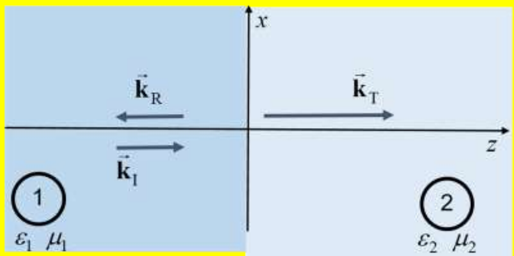
$$\Gamma_\perp = \frac{\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

$[E_x, H_y, E_z]$ **Parallel Polarization** $\|$

$$\boxed{Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}}$$

$$\Gamma_\| = -\frac{\cos\vartheta_I - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

Normal incidence



$$\vartheta_I = 0° \Rightarrow \boxed{k_{1z} = k_1} \qquad k_{2z} = k_2$$

$\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} = \frac{\omega\mu_1}{k_1} = \frac{\omega\mu_1}{\omega\sqrt{\mu_1\varepsilon_1}} = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \zeta_1$$

$$Z_2 = \frac{\omega\mu_2}{k_{2z}} = \zeta_2$$

$\|$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} = \frac{k_1}{\omega\varepsilon_1} = \frac{\omega\sqrt{\mu_1\varepsilon_1}}{\omega\varepsilon_1} = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \zeta_1$$

$$Z_2 = \frac{k_{2z}}{\omega\varepsilon_2} = \zeta_2$$

Normal Incidence

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

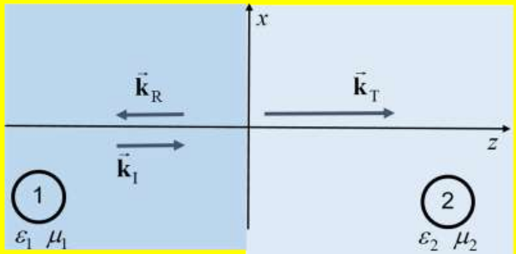
$$\boxed{\begin{aligned} \Gamma &= \frac{Z_2 - Z_1}{Z_1 + Z_2} \\ \mathrm{T} &= \frac{2Z_2}{Z_1 + Z_2} \end{aligned}}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$
$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$
$$k_x = k_1 \sin\vartheta_I$$
$$k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$
$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

In the case of normal incidence, perpendicular and parallel polarizations behave the same

Normal incidence



$$\vartheta_I = 0° \Rightarrow \begin{aligned} k_{1z} &= k_1 \\ k_{2z} &= k_2 \end{aligned}$$

$\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} = \frac{\omega\mu_1}{k_1} = \frac{\omega\mu_1}{\omega\sqrt{\mu_1\varepsilon_1}} = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \zeta_1$$

$$Z_2 = \frac{\omega\mu_2}{k_{2z}} = \zeta_2$$

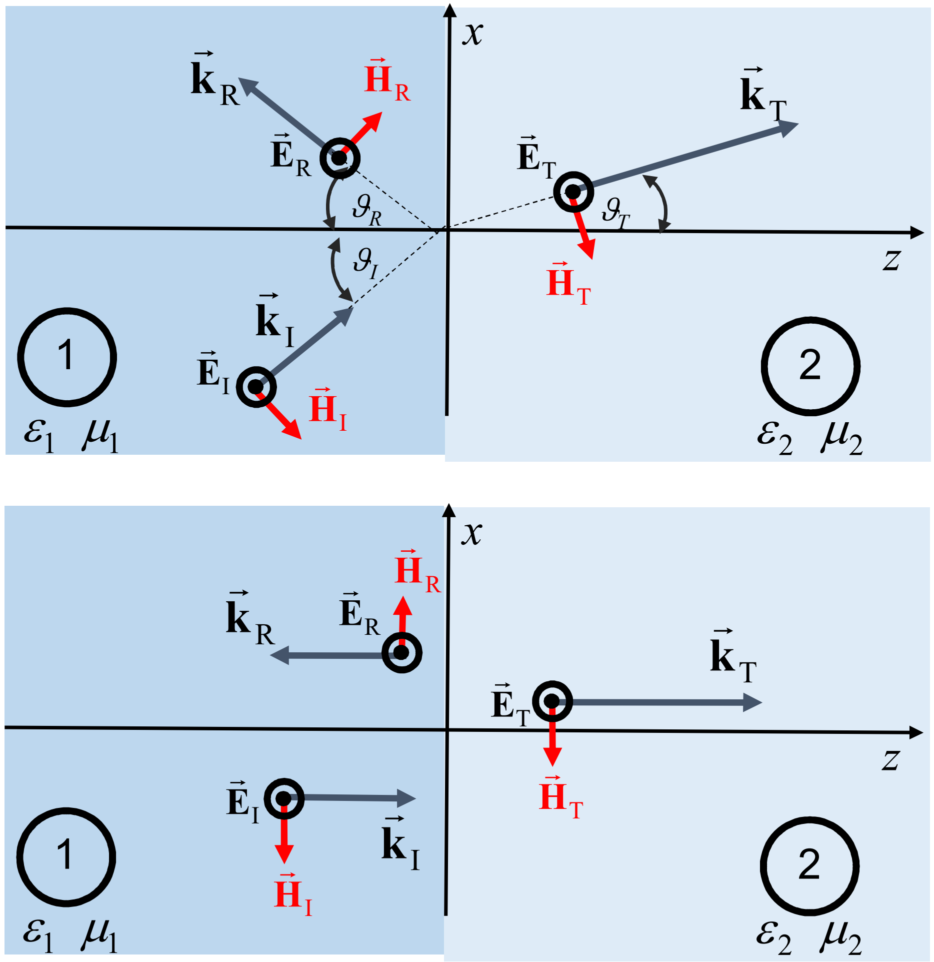
$\parallel$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} = \frac{k_1}{\omega\varepsilon_1} = \frac{\omega\sqrt{\mu_1\varepsilon_1}}{\omega\varepsilon_1} = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \zeta_1$$

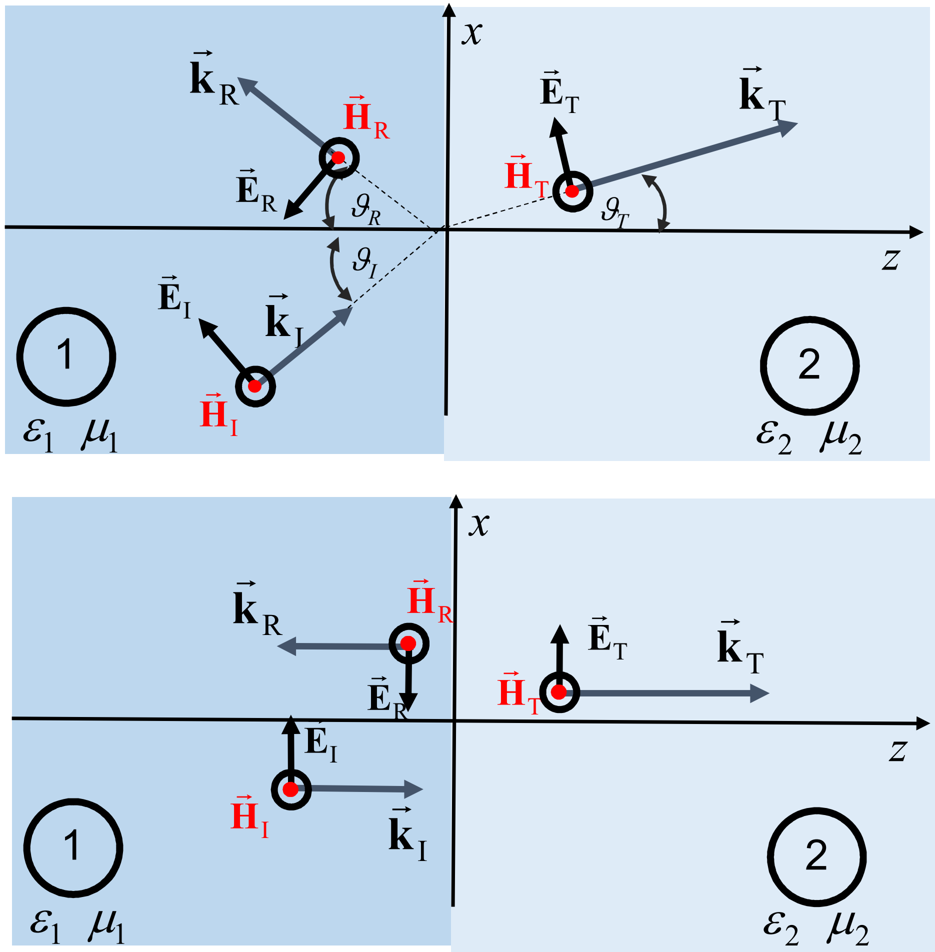
$$Z_2 = \frac{k_{2z}}{\omega\varepsilon_2} = \zeta_2$$

Normal Incidence

Perpendicular Polarization ⊥



Parallel Polarization ∥

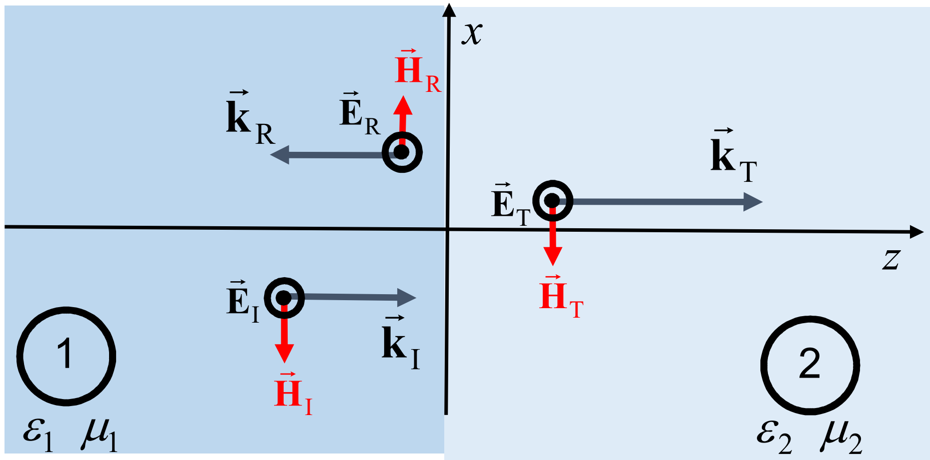


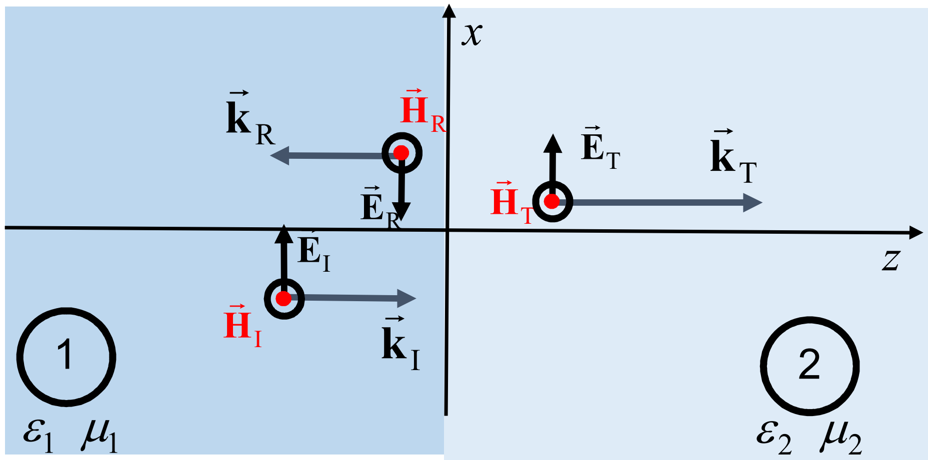
Normal Incidence

Perpendicular Polarization $\perp$

Parallel Polarization $\parallel$

In the case of normal incidence, perpendicular and parallel polarizations behave the same





$$\vartheta_{\mathrm{I}} = \pi/2$$

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}} \qquad k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_{\mathrm{I}} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** $\|$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\| = -\frac{\cos\vartheta_{\mathrm{I}} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

$$\Gamma_\perp = \frac{-(\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - 1}}{(\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - 1}} \Rightarrow \left|\Gamma_\perp\right| = 1$$

$$\Gamma_\| = -\frac{-(\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - 1}}{(\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - 1}} \Rightarrow \left|\Gamma_\|\right| = \left|\Gamma_\perp\right| = 1$$

Limit angle

$$\vec{\mathbf{k}}_T = k_x \hat{i}_x + k_{2z} \hat{i}_z$$
$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}}$$

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2 \varepsilon_2}$$
$$k_x = k_1 \sin\vartheta_I \qquad k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$
$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$\left[H_x, E_y, H_z\right]$ Perpendicular Polarization $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

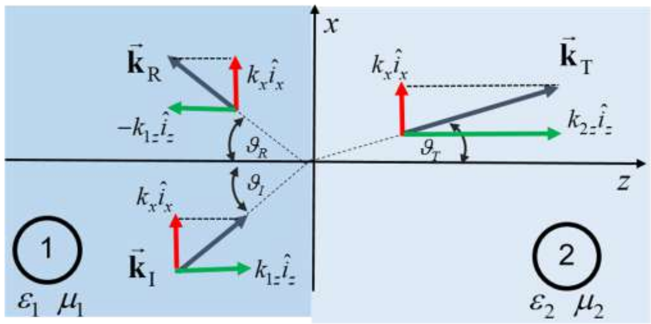
$$\Gamma_\perp = \frac{\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

$\left[E_x, H_y, E_z\right]$ Parallel Polarization $\parallel$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\parallel = -\frac{\cos\vartheta_I - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

$$\vec{\mathbf{k}}_I = k_x \hat{i}_x + k_{1z} \hat{i}_z$$
$$\vec{\mathbf{k}}_R = k_x \hat{i}_x - k_{1z} \hat{i}_z$$
$$\vec{\mathbf{k}}_T = k_x \hat{i}_x + k_{2z} \hat{i}_z$$



Limit angle

$$\vec{\mathbf{k}}_T = k_x \hat{i}_x + k_{2z}\hat{i}_z$$
$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T\cdot\vec{\mathbf{r}}}$$

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma = \frac{Z_2-Z_1}{Z_1+Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1+Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I \qquad k_{1z} = k_1\cos\vartheta_I$$

$$\vartheta_I = \vartheta_R \qquad k_1\sin\vartheta_I = k_2\sin\vartheta_T$$

$$\boxed{k_{2z} = \sqrt{k_2^2 - k_1^2\sin^2\vartheta_I}}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{\cancel{(k_2/k_1)^2 - \sin^2\vartheta_I}}}{\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{\cancel{(k_2/k_1)^2 - \sin^2\vartheta_I}}}$$

$[E_x, H_y, E_z]$ **Parallel Polarization** $\|$

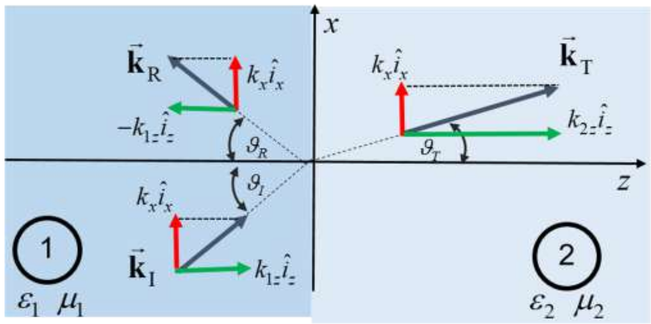
$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\| = -\frac{\cos\vartheta_I - (\varepsilon_1/\varepsilon_2)\sqrt{\cancel{(k_2/k_1)^2 - \sin^2\vartheta_I}}}{\cos\vartheta_I + (\varepsilon_1/\varepsilon_2)\sqrt{\cancel{(k_2/k_1)^2 - \sin^2\vartheta_I}}}$$

$$if\ \frac{k_2}{k_1} < 1,\ \text{an angle } \bar{\vartheta}_I \text{ exists such that } \sin\bar{\vartheta}_I = \frac{k_2}{k_1}$$

$$\vartheta = \bar{\vartheta}_I \ \Rightarrow\ \sin\vartheta_I = \frac{k_2}{k_1} \ \Rightarrow\ k_{2z} = \sqrt{k_2^2 - k_1^2\frac{k_2^2}{k_1^2}} = 0$$

$$\Rightarrow\ \vec{\mathbf{k}}_T = k_x\hat{i}_x \ \Rightarrow\ \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_T e^{-jk_x x} \qquad |\Gamma_\|| = |\Gamma_\perp| = 1$$



Limit angle

$$\vec{\mathbf{k}}_T = k_x \hat{i}_x + k_{2z} \hat{i}_z$$
$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}}$$

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2 \varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_I \qquad k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_I - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

$[E_x, H_y, E_z]$ **Parallel Polarization** $\parallel$

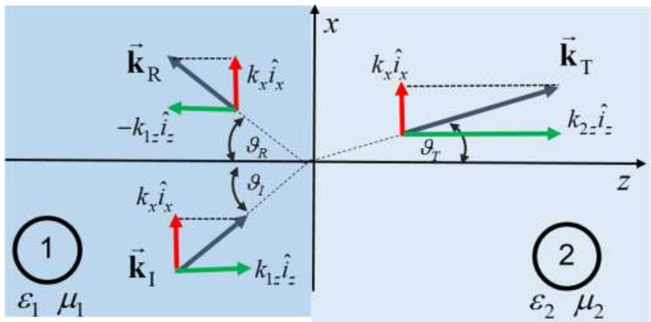
$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\parallel = -\frac{\cos\vartheta_I - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}{\cos\vartheta_I + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_I}}$$

if $\dfrac{k_2}{k_1} < 1$, an angle $\bar{\vartheta}_I$ exists such that $\sin\bar{\vartheta}_I = \dfrac{k_2}{k_1}$

$$\vartheta > \bar{\vartheta}_I \Rightarrow \sin\vartheta_I > \frac{k_2}{k_1} \Rightarrow k_2^2 - k_1^2\sin^2\vartheta_I < 0 \Rightarrow k_{2z} = -ja$$

$$\Rightarrow \vec{\mathbf{k}}_T = k_x\hat{i}_x - ja\hat{i}_z \Rightarrow \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_T e^{-jk_x x} e^{-az} \qquad |\Gamma_\parallel| = |\Gamma_\perp| = 1$$



Incidence: Limit angle

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}} \qquad k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

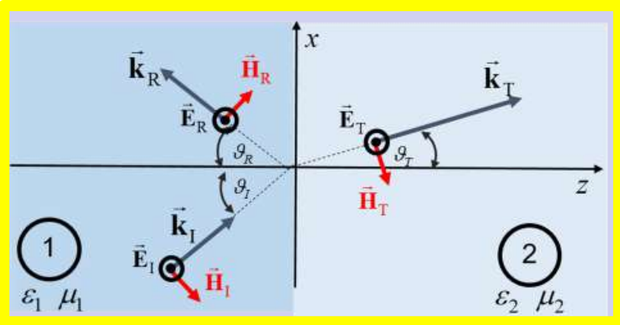
$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

$$\left[H_x, E_y, H_z\right]$$

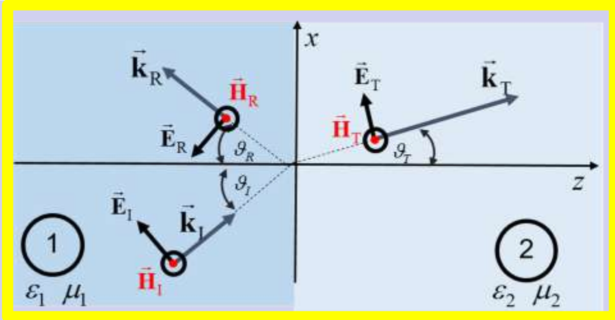
$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$



$\perp$

$$\left[E_x, H_y, E_z\right]$$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

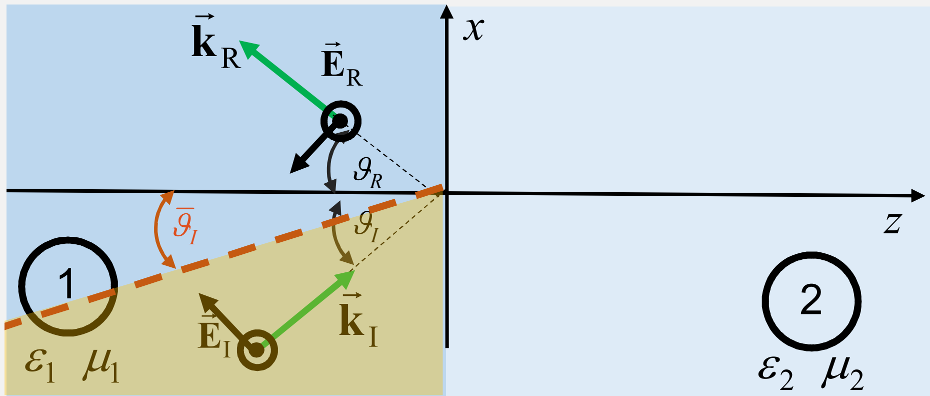


$\parallel$

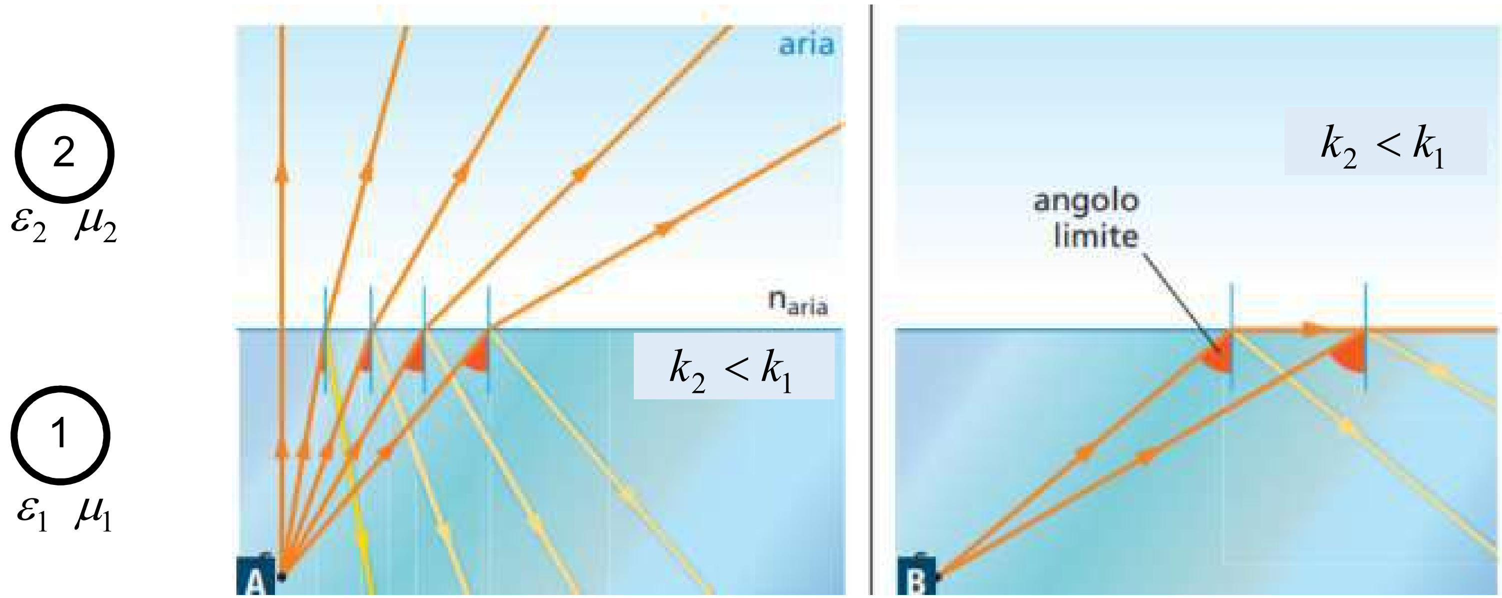
$$if \ \frac{k_2}{k_1} < 1$$

An angle $\bar{\vartheta}_{\mathrm{I}}$ exists, referred to as **limit angle**, such that for $\vartheta_{\mathrm{I}} \geq \bar{\vartheta}_I$ no propagation occurs in the second half-space

$$\sin\bar{\vartheta}_{\mathrm{I}} = \frac{k_2}{k_1}$$



Incidence: Limit angle



Brewster angle

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\boxed{\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}}$$

$$\mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$
$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}}$$
$$k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R$$
$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

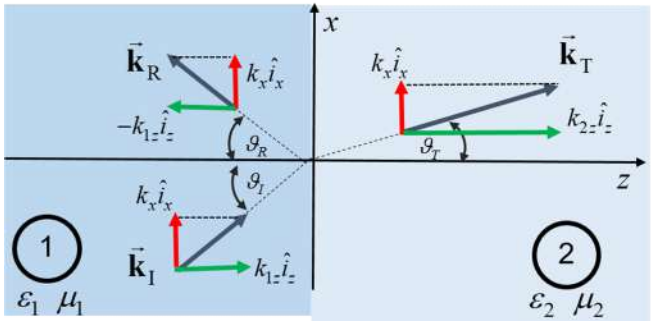
$$\Gamma_{\perp} = \frac{\cos\vartheta_{\mathrm{I}} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

$[E_x, H_y, E_z]$ **Parallel Polarization** $\|$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_{\|} = -\frac{\cos\vartheta_{\mathrm{I}} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

Let us explore the possible existence of an incidence angle such that there is no reflected wave.
Let us consider the simplest, yet important, nonmagnetic case (μ_1=μ_2). Of course, discontinuity implies in this case $\varepsilon_1 \neq \varepsilon_2$



Brewster angle

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\boxed{\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}}$$

$$\mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I}$$

$$\boxed{k_{1z} = k_1 \cos\vartheta_\mathrm{I}}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T$$

$$\boxed{k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$\boxed{Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_\mathrm{I} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** $\parallel$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

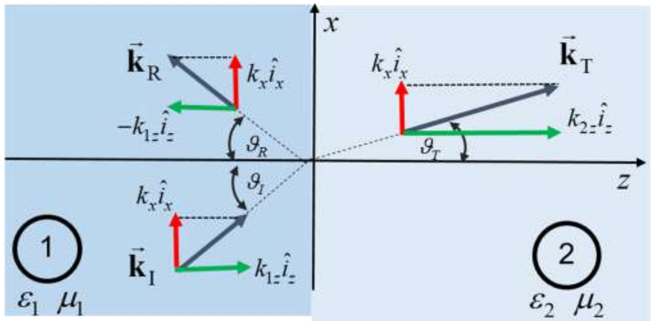
$$\Gamma_\parallel = -\frac{\cos\vartheta_\mathrm{I} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}{\cos\vartheta_\mathrm{I} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_\mathrm{I}}}$$

Let us explore the possible existence of an incidence angle such that there is no reflected wave.
Let us consider the simplest, yet important, nonmagnetic case (μ_1=μ_2). Of course, discontinuity implies in this case $\varepsilon_1 \neq \varepsilon_2$

Perpendicular Polarization $\perp$

$$Z_2 - Z_1 \quad \Rightarrow k_{1z} = k_{2z} \quad \Rightarrow k_1 = k_2$$

This condition cannot be enforced, since μ_1=μ_2 and $\varepsilon_1 \neq \varepsilon_2$



Brewster angle

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases}$$

$$\boxed{\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}}$$

$$\mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$
$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}}$$
$$k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R$$
$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos\vartheta_{\mathrm{I}} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** $\|$

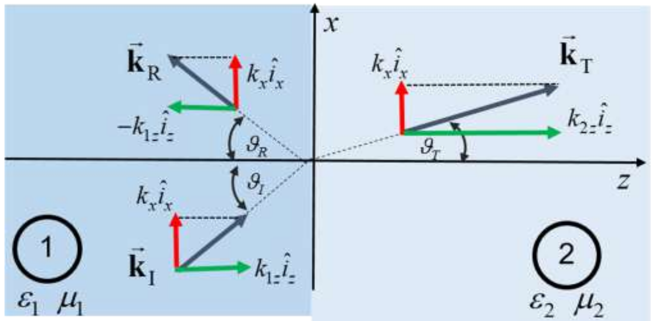
$$\boxed{Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}}$$

$$\Gamma_\| = -\frac{\cos\vartheta_{\mathrm{I}} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}{\cos\vartheta_{\mathrm{I}} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2\vartheta_{\mathrm{I}}}}$$

Let us explore the possible existence of an incidence angle such that there is no reflected wave.
Let us consider the simplest, yet important, nonmagnetic case ($\mu_1 = \mu_2$). Of course, discontinuity implies in this case $\varepsilon_1 \neq \varepsilon_2$

Parallel Polarization $\|$

$$Z_2 - Z_1 \quad \Rightarrow \frac{k_{1z}}{\varepsilon_1} = \frac{k_{2z}}{\varepsilon_2} \quad \Rightarrow \sin^2\vartheta = \frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$



Brewster angle

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1 \varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2 \varepsilon_2}$$

$$k_x = k_1 \sin \vartheta_{\mathrm{I}} \qquad k_{1z} = k_1 \cos \vartheta_{\mathrm{I}}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin \vartheta_{\mathrm{I}} = k_2 \sin \vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_{\mathrm{I}}}$$

$\left[H_x, E_y, H_z\right]$ **Perpendicular Polarization** $\perp$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$\Gamma_\perp = \frac{\cos \vartheta_{\mathrm{I}} - (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_{\mathrm{I}}}}{\cos \vartheta_{\mathrm{I}} + (\mu_1/\mu_2)\sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_{\mathrm{I}}}}$$

$\left[E_x, H_y, E_z\right]$ **Parallel Polarization** $\|$

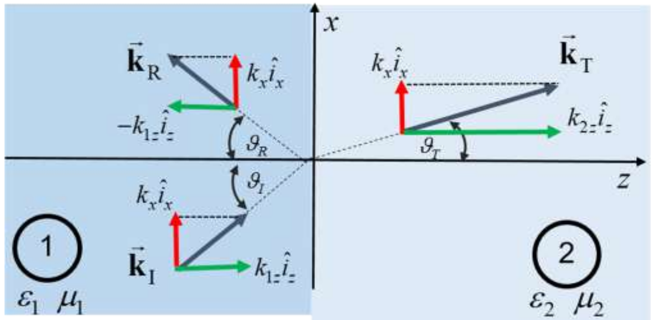
$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

$$\Gamma_\| = -\frac{\cos \vartheta_{\mathrm{I}} - (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_{\mathrm{I}}}}{\cos \vartheta_{\mathrm{I}} + (\varepsilon_1/\varepsilon_2)\sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_{\mathrm{I}}}}$$

Let us explore the possible existence of an incidence angle such that there is no reflected wave.
Let us consider the simplest, yet important, nonmagnetic case (μ_1=μ_2). Of course, discontinuity implies in this case $\varepsilon_1 \neq \varepsilon_2$

$$\sin^2 \vartheta_B = \frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2} \Rightarrow \begin{matrix} \Gamma_\| = 0 \\ \Gamma_\perp \neq 0 \end{matrix}$$

An unpolarized plane wave incident at angle ϑ_B is reflected with perpendicular polarization



Brewster angle

$$k_{1z} = k_1 \cos \vartheta_{\mathrm{I}} \qquad k_1^2 = \omega^2 \mu \varepsilon_1 \qquad \frac{k_1^2}{k_2^2} = \frac{\varepsilon_1}{\varepsilon_2}$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_{\mathrm{I}}} \qquad k_2^2 = \omega^2 \mu \varepsilon_2$$

Parallel Polarization $\parallel$

$$Z_1 = \frac{k_{1z}}{\omega \varepsilon_1}$$

$$Z_2 = \frac{k_{2z}}{\omega \varepsilon_2}$$

$$Z_1 = Z_2 \Rightarrow \frac{k_{1z}}{\varepsilon_1} = \frac{k_{2z}}{\varepsilon_2} \quad \Rightarrow \left(\frac{k_{1z}}{\varepsilon_1}\right)^2 = \left(\frac{k_{2z}}{\varepsilon_2}\right)^2$$

$$\Rightarrow {k_{1z}}^2 = \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 {k_{2z}}^2 \quad \Rightarrow {k_1}^2 \cos^2 \vartheta_I = \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 \left(k_2^2 - k_1^2 \sin^2 \vartheta_{\mathrm{I}}\right) \quad \Rightarrow {k_1}^2 \left(1 - \sin^2 \vartheta_I\right) = \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 k_2^2 \left(1 - \frac{k_1^2}{k_2^2} \sin^2 \vartheta_{\mathrm{I}}\right)$$

$$\Rightarrow \frac{{k_1}^2}{k_2^2}\left(1 - \sin^2 \vartheta_I\right) = \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 \left(1 - \frac{k_1^2}{k_2^2} \sin^2 \vartheta_{\mathrm{I}}\right) \quad \Rightarrow \frac{\varepsilon_1}{\varepsilon_2}\left(1 - \sin^2 \vartheta_I\right) = \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 \left(1 - \frac{\varepsilon_1}{\varepsilon_2} \sin^2 \vartheta_{\mathrm{I}}\right) \quad \Rightarrow 1 - \sin^2 \vartheta_I = \frac{\varepsilon_1}{\varepsilon_2} - \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 \sin^2 \vartheta_{\mathrm{I}}$$

$$\Rightarrow 1 - \frac{\varepsilon_1}{\varepsilon_2} = \left[1 - \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2\right] \sin^2 \vartheta_{\mathrm{I}} \quad \Rightarrow 1 - \frac{\varepsilon_1}{\varepsilon_2} = \left[1 - \frac{\varepsilon_1}{\varepsilon_2}\right]\left[1 + \frac{\varepsilon_1}{\varepsilon_2}\right] \sin^2 \vartheta_{\mathrm{I}} \quad \Rightarrow 1 = \left[1 + \frac{\varepsilon_1}{\varepsilon_2}\right] \sin^2 \vartheta_{\mathrm{I}} \quad \Rightarrow 1 = \left[\frac{\varepsilon_2 + \varepsilon_1}{\varepsilon_2}\right] \sin^2 \vartheta_{\mathrm{I}}$$

Incidence: Limit and Brewster angles

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

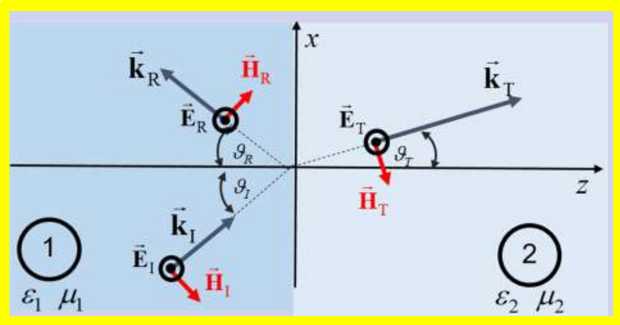
$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I} \qquad k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$$\left[H_x, E_y, H_z\right]$$

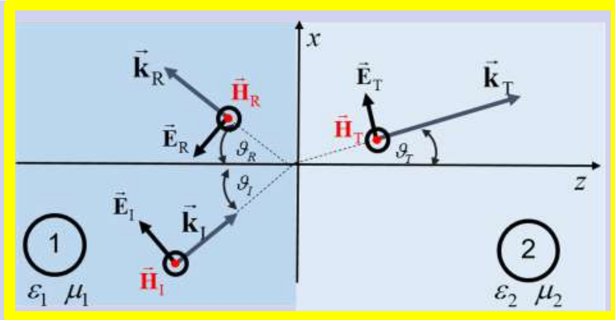
$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$



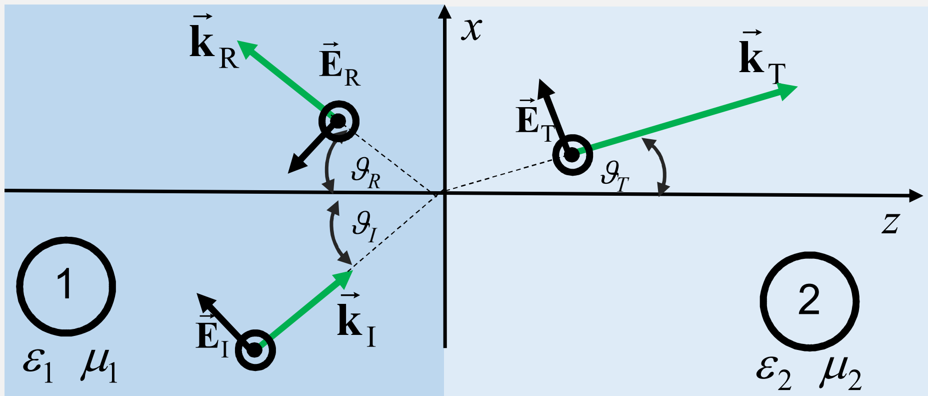
$\perp$

$$\left[E_x, H_y, E_z\right]$$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$



$\parallel$



Incidence: Brewster angle

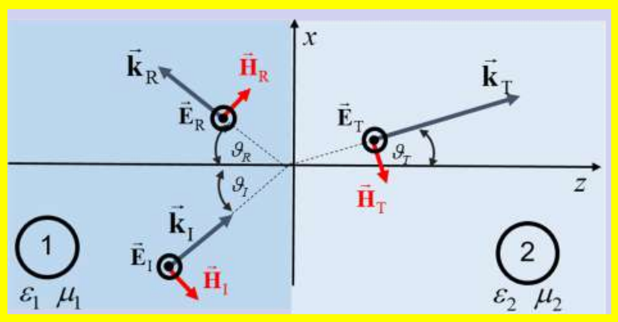
$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_{\mathrm{I}} \qquad k_{1z} = k_1 \cos\vartheta_{\mathrm{I}}$$

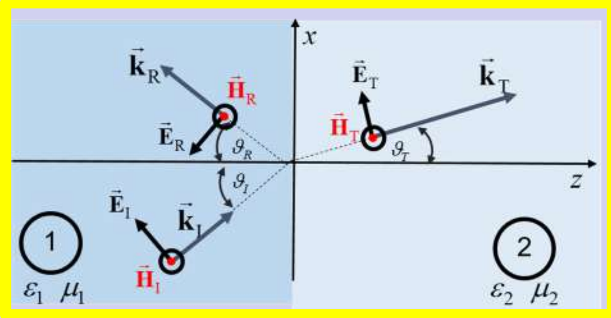
$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

$$\left[H_x, E_y, H_z\right] \qquad Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$



$\perp$

$$\left[E_x, H_y, E_z\right] \qquad Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

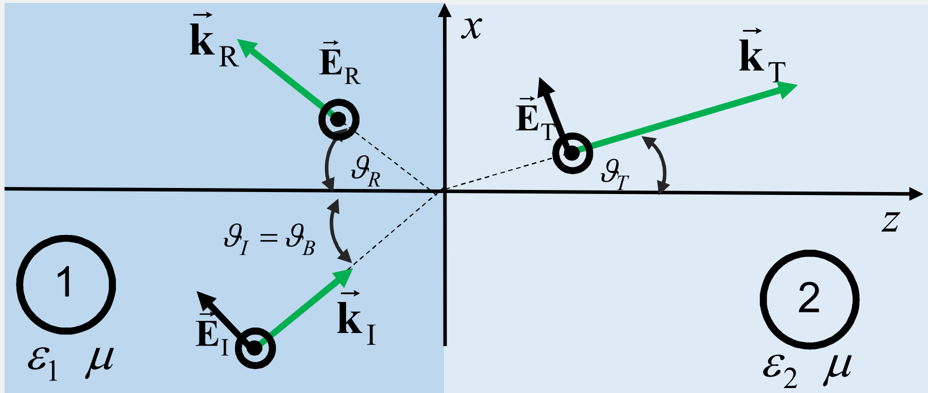


$\parallel$

if $\mu_1 = \mu_2$ *and* $\varepsilon_1 \neq \varepsilon_2$

An angle ϑ_{B} exists, referred to as **Brewster angle**, such that an unpolarized plane wave incident at angle $\vartheta_I = \vartheta_{\mathrm{B}}$ is reflected with perpendicular polarization

$$\sin^2\vartheta_B = \frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$



Incidence: Limit angle

$$\begin{cases} 1-\Gamma = \dfrac{Z_1}{Z_2}\mathrm{T} \\ 1+\Gamma = \mathrm{T} \end{cases} \qquad \Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \qquad \mathrm{T} = \frac{2Z_2}{Z_1 + Z_2}$$

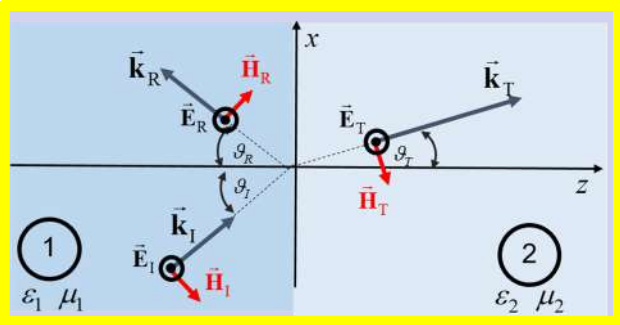
$$k_1 = \omega\sqrt{\mu_1\varepsilon_1} \qquad k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I} \qquad k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R \qquad k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T \qquad k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$$\left[H_x, E_y, H_z\right]$$

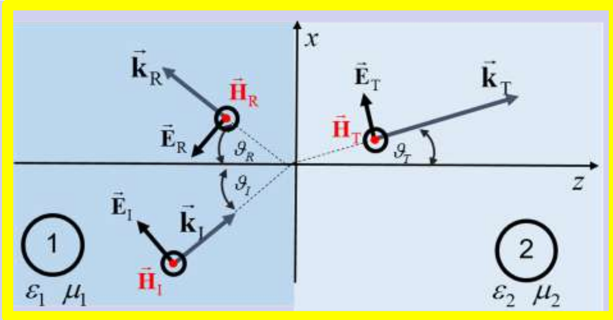
$$Z_1 = \frac{\omega\mu_1}{k_{1z}} \qquad Z_2 = \frac{\omega\mu_2}{k_{2z}}$$



$\perp$

$$\left[E_x, H_y, E_z\right]$$

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1} \qquad Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

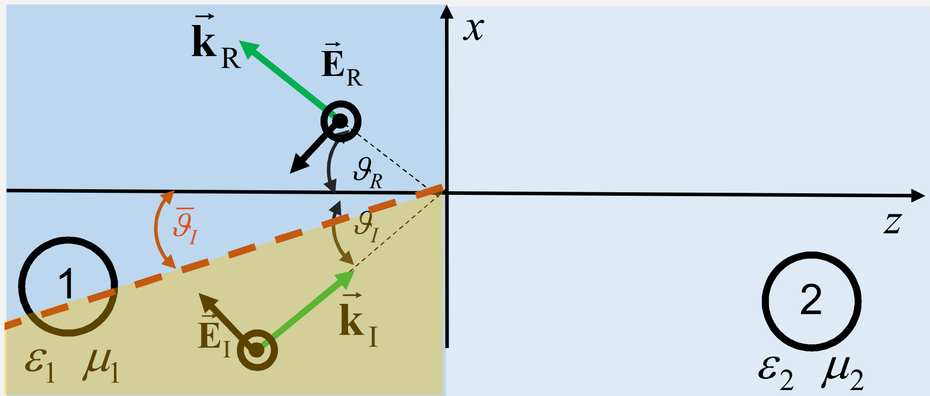


||

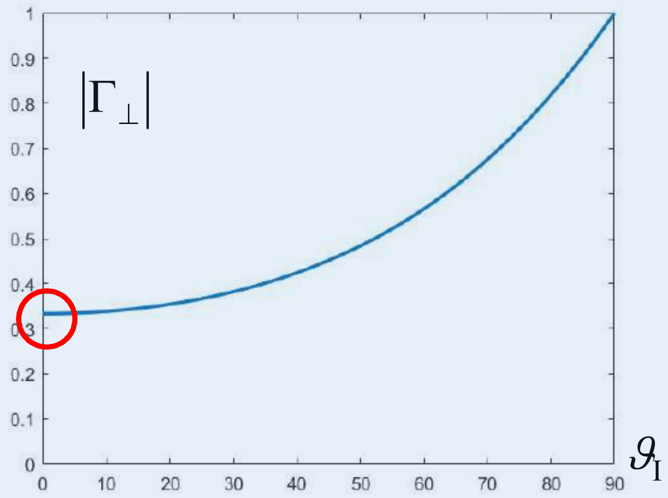
$$if \ \frac{k_2}{k_1} < 1$$

An angle $\bar{\vartheta}_\mathrm{I}$ exists, referred to as **limit angle**, such that for $\vartheta_\mathrm{I} \geq \bar{\vartheta}_I$ no propagation occurs in the second half-space

$$\sin\bar{\vartheta}_\mathrm{I} = \frac{k_2}{k_1}$$



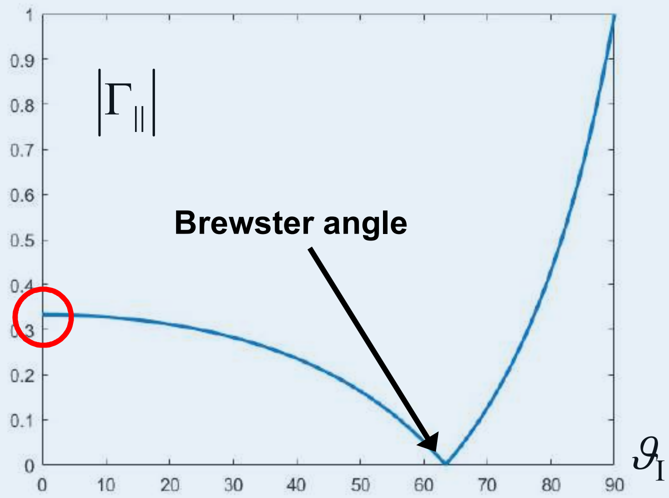
Fresnel coefficients



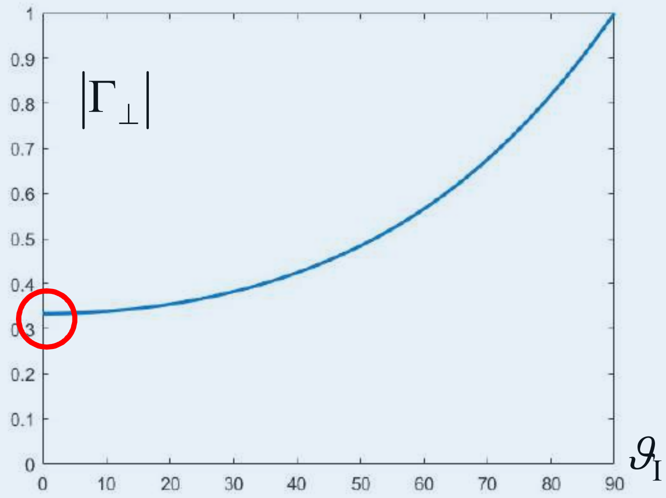
$$\mu_1 = \mu_2$$

$$\varepsilon_2 = 4\varepsilon_1$$

$$\frac{k_2}{k_1} > 1$$



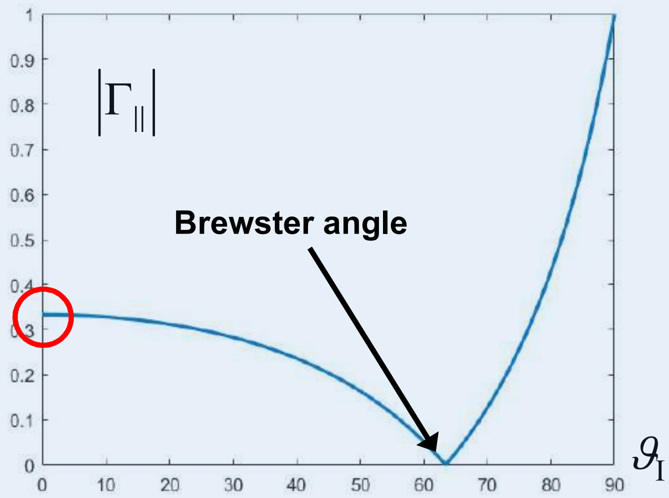
Fresnel coefficients

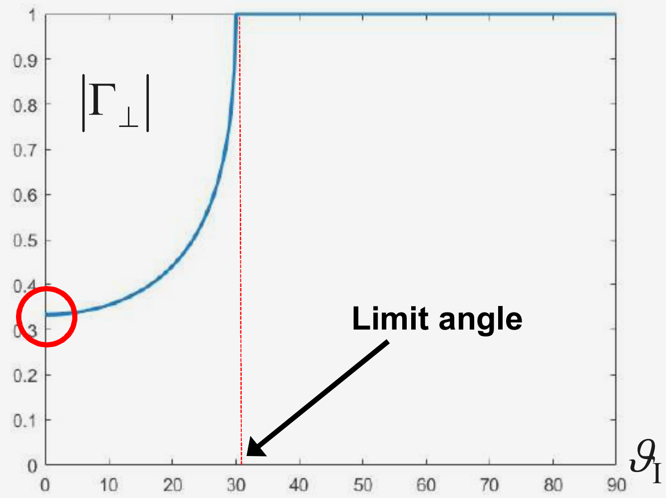


$$\mu_1 = \mu_2$$

$$\varepsilon_2 = 4\varepsilon_1$$

$$\frac{k_2}{k_1} > 1$$





$$\mu_1 = \mu_2$$

$$4\varepsilon_2 = \varepsilon_1$$

$$\frac{k_2}{k_1} < 1$$