

Campi Elettromagnetici

**Corso di Laurea in Ingegneria Informatica,
Biomedica e delle Telecomunicazioni**

a.a. 2021-2022 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

Phasor Domain (PD)

Fourier Domain (FD)

Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence (PD)

Plane Waves

Incidence on a dielectric half-space

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_{Ix}x} e^{-jk_{Iy}y} e^{-jk_{Iz}z}$$

$$\vec{\mathbf{k}}_I = k_{Ix}\hat{i}_x + k_{Iy}\hat{i}_y + k_{Iz}\hat{i}_z$$

$$\vec{\mathbf{k}}_I \cdot \vec{\mathbf{k}}_I = k_{Ix}^2 + k_{Iy}^2 + k_{Iz}^2 = \omega^2 \mu_1 \epsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_R e^{-jk_{Rx}x} e^{-jk_{Ry}y} e^{-jk_{Rz}z}$$

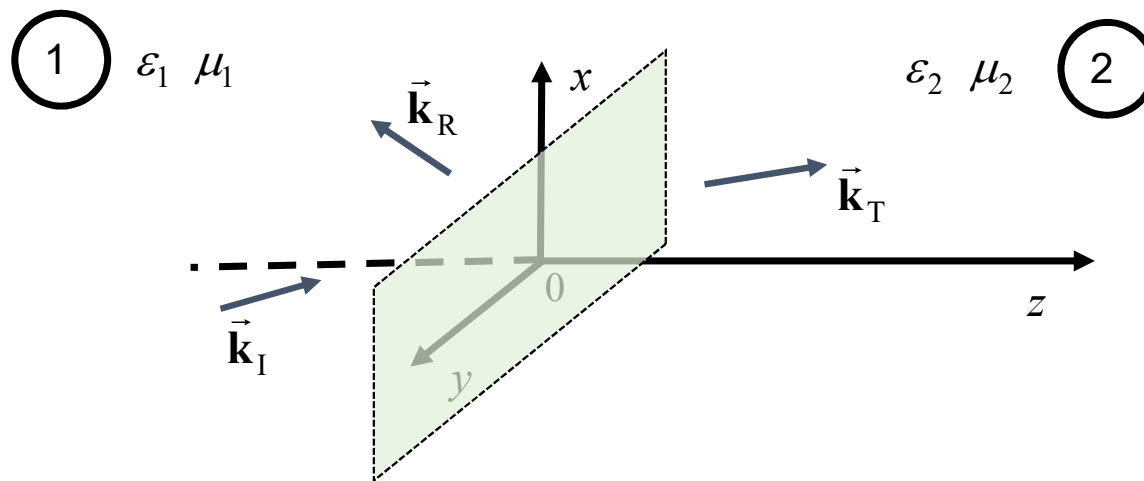
$$\vec{\mathbf{k}}_R = k_{Rx}\hat{i}_x + k_{Ry}\hat{i}_y + k_{Rz}\hat{i}_z$$

$$\vec{\mathbf{k}}_R \cdot \vec{\mathbf{k}}_R = k_{Rx}^2 + k_{Ry}^2 + k_{Rz}^2 = \omega^2 \mu_1 \epsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_{Tx}x} e^{-jk_{Ty}y} e^{-jk_{Tz}z}$$

$$\vec{\mathbf{k}}_T = k_{Tx}\hat{i}_x + k_{Ty}\hat{i}_y + k_{Tz}\hat{i}_z$$

$$\vec{\mathbf{k}}_T \cdot \vec{\mathbf{k}}_T = k_{Tx}^2 + k_{Ty}^2 + k_{Tz}^2 = \omega^2 \mu_2 \epsilon_2 = k_2^2$$



$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}}$$

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_{Ix}x} e^{-jk_{Iy}y} e^{-jk_{Iz}z}$$

$$\vec{\mathbf{k}}_I = k_{Ix}\hat{i}_x + k_{Iy}\hat{i}_y + k_{Iz}\hat{i}_z$$

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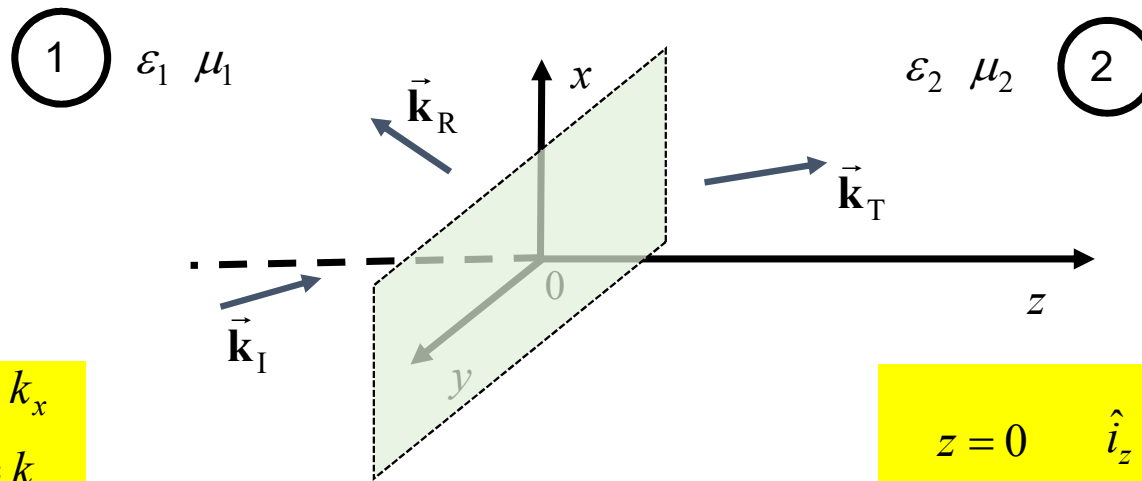
$$\vec{\mathbf{k}}_R = k_{Rx}\hat{i}_x + k_{Ry}\hat{i}_y + k_{Rz}\hat{i}_z$$

$$\vec{\mathbf{k}}_R \cdot \vec{\mathbf{k}}_R = k_{Rx}^2 + k_{Ry}^2 + k_{Rz}^2 = \omega^2 \mu_1 \epsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_{Tx}x} e^{-jk_{Ty}y} e^{-jk_{Tz}z}$$

$$\vec{\mathbf{k}}_T = k_{Tx}\hat{i}_x + k_{Ty}\hat{i}_y + k_{Tz}\hat{i}_z$$

$$\vec{\mathbf{k}}_T \cdot \vec{\mathbf{k}}_T = k_{Tx}^2 + k_{Ty}^2 + k_{Tz}^2 = \omega^2 \mu_2 \epsilon_2 = k_2^2$$



$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}}$$

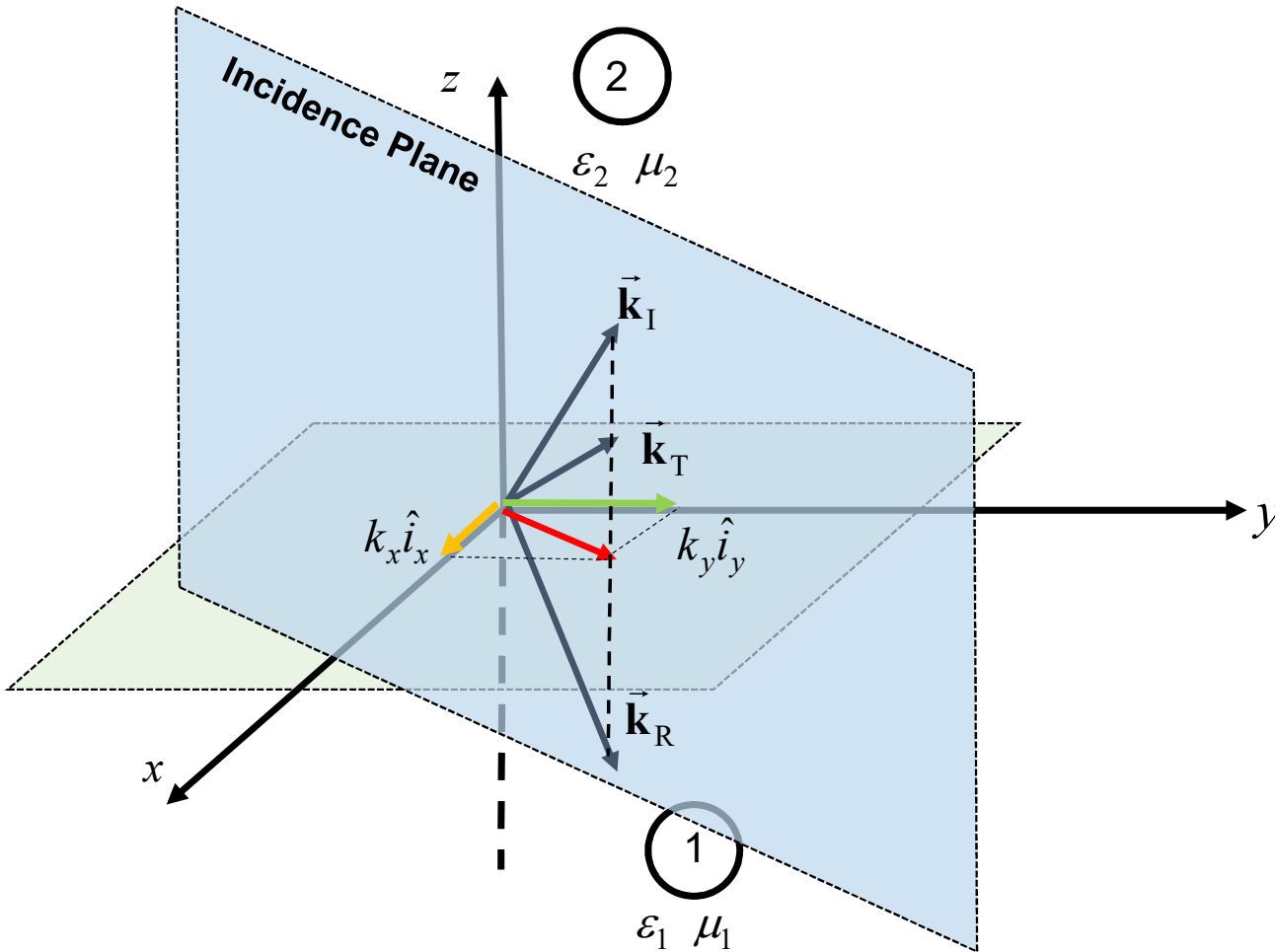
$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}}$$

$$k_{Ix} = k_{Rx} = k_{Tx} = k_x$$

$$k_{Iy} = k_{Ry} = k_{Ty} = k_y$$

$$z=0 \quad \hat{i}_z \times (\vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1) = \mathbf{0} \quad \hat{i}_z \times \vec{\mathbf{E}}_1 = \hat{i}_z \times \vec{\mathbf{E}}_2$$

Incidence on a dielectric half-space

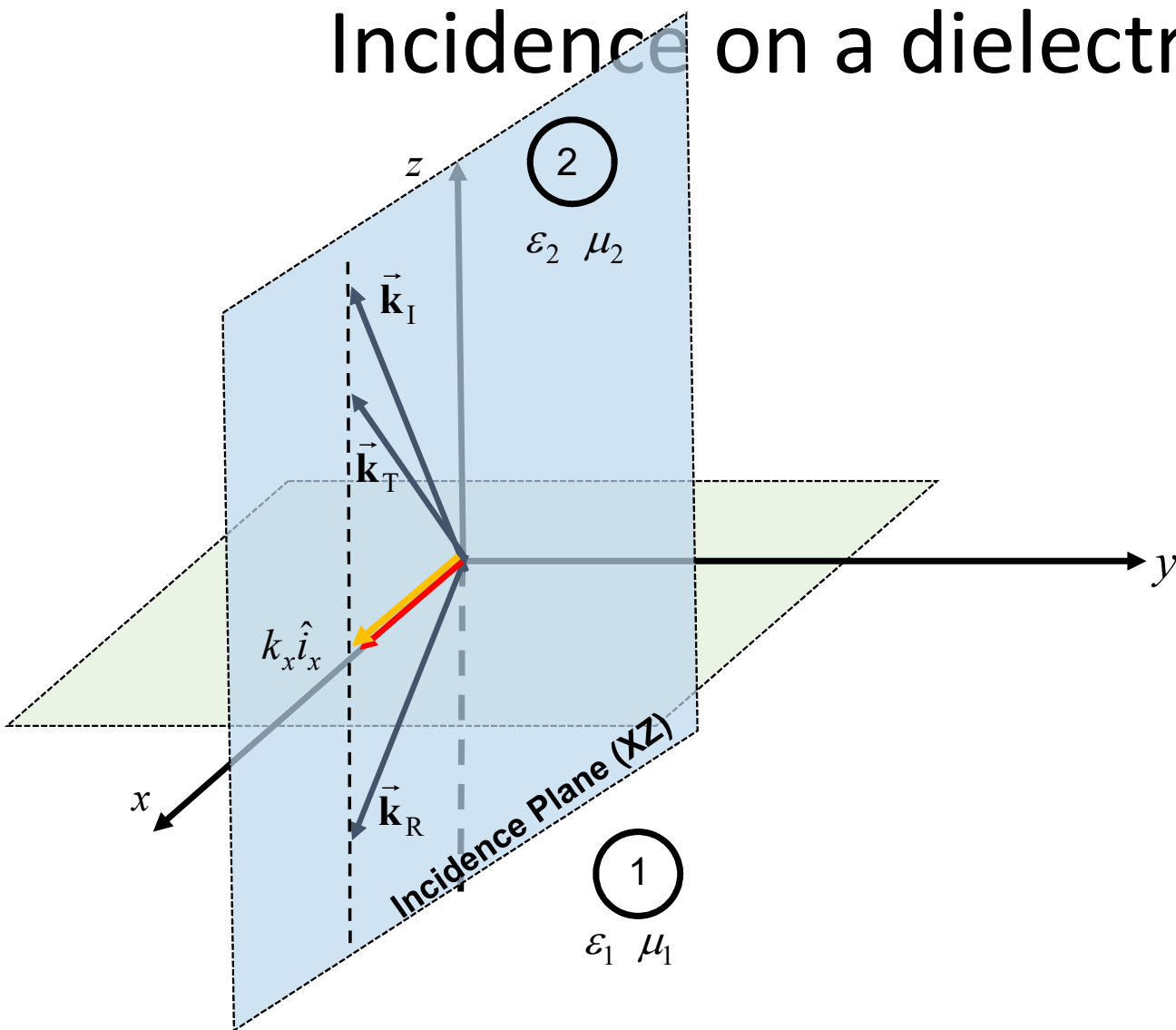


$$\vec{k}_I = k_x \hat{i}_x + k_y \hat{i}_y + k_{Iz} \hat{i}_z$$

$$\vec{k}_R = k_x \hat{i}_x + k_y \hat{i}_y + k_{Rz} \hat{i}_z$$

$$\vec{k}_T = k_x \hat{i}_x + k_y \hat{i}_y + k_{Tz} \hat{i}_z$$

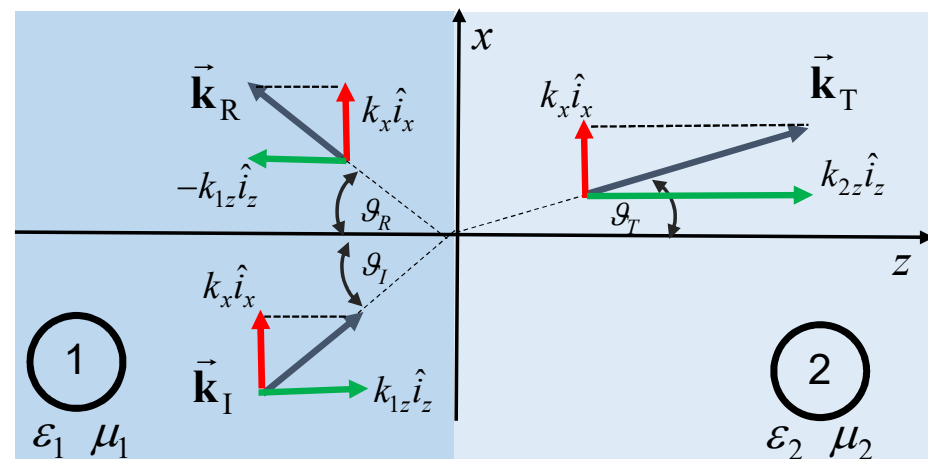
Incidence on a dielectric half-space



$$\vec{k}_I = k_x \hat{i}_x + k_{Iz} \hat{i}_z$$

$$\vec{k}_R = k_x \hat{i}_x + k_{Rz} \hat{i}_z$$

$$\vec{k}_T = k_x \hat{i}_x + k_{Tz} \hat{i}_z$$



Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega \sqrt{\mu_1 \epsilon_1}$$

$$k_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

$$k_x = k_1 \sin \vartheta_1$$

$$k_{1z} = k_1 \cos \vartheta_1$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_1 = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}$$

Incidence on a dielectric half-space

$$\begin{cases} \nabla \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \end{cases}$$

$$\begin{aligned} \vec{\mathbf{E}}_1 &= \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} \\ \vec{\mathbf{E}}_2 &= \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} \end{aligned}$$

$$\begin{aligned} \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z} \\ \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z} \\ \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z} \end{aligned}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

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$$k_x = k_1 \sin \vartheta_1$$

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$$k_1 \sin \vartheta_1 = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}$$

$[H_x, E_y, H_z]$ Perpendicular Polarization \perp

$$-\frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial H_x}{\partial z} + jk_x H_z = j\omega\varepsilon E_y$$

$$-jk_x E_y = -j\omega\mu H_z$$

$[E_x, H_y, E_z]$ Parallel Polarization \parallel

$$-\frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x$$

$$\frac{\partial E_x}{\partial z} + jk_x E_z = -j\omega\mu H_y$$

$$-jk_x H_y = j\omega\varepsilon E_z$$

Incidence on a dielectric half-space

$$\begin{aligned} \vec{\mathbf{E}}_1 &= \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z} \\ \vec{\mathbf{E}}_2 &= \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z} \\ & & \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z} \end{aligned}$$

$$\begin{aligned} k_1 &= \omega \sqrt{\mu_1 \epsilon_1} \\ k_2 &= \omega \sqrt{\mu_2 \epsilon_2} \end{aligned}$$

$$\begin{aligned} k_x &= k_1 \sin \vartheta_I \\ k_{1z} &= k_1 \cos \vartheta_I \end{aligned}$$

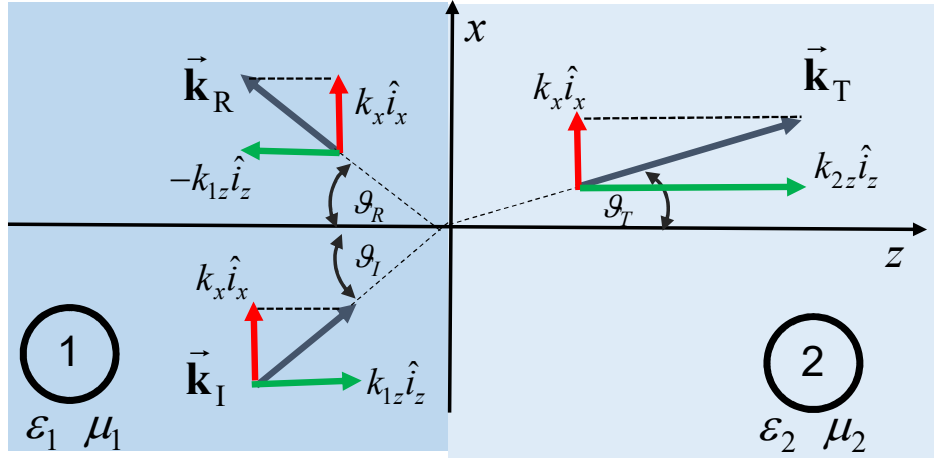
$$\begin{aligned} \vartheta_I &= \vartheta_R \\ k_1 \sin \vartheta_I &= k_2 \sin \vartheta_T \\ k_{2z} &= \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_I} \end{aligned}$$

Perpendicular Polarization \perp

$$\left[H_x, E_y, H_z \right]$$

Parallel Polarization \parallel

$$\left[E_x, H_y, E_z \right]$$



Incidence on a dielectric half-space

$$\begin{aligned}\vec{\mathbf{E}}_1 &= \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z} \\ \vec{\mathbf{E}}_2 &= \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z} \\ & & \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}\end{aligned}$$

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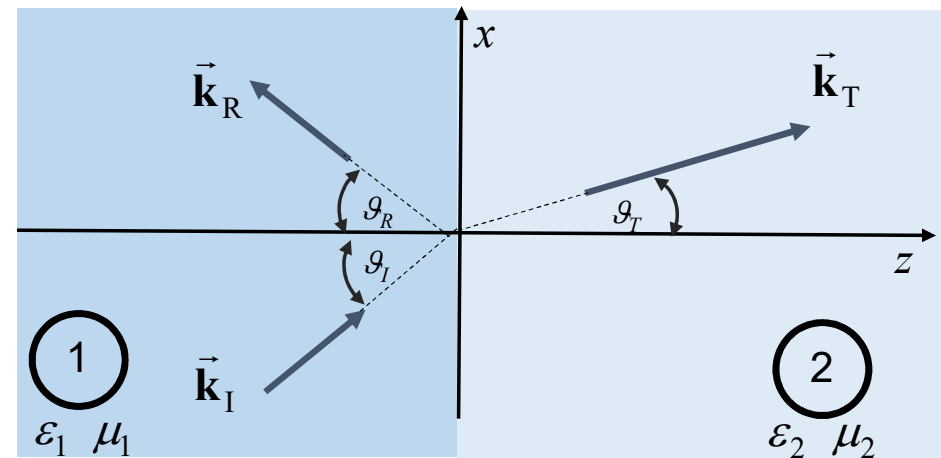
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_I}$$

Perpendicular Polarization \perp

$$[H_x, E_y, H_z]$$

Parallel Polarization \parallel

$$[E_x, H_y, E_z]$$



Incidence on a dielectric half-space: \perp polarization

$$\begin{cases} \vec{k} \times \vec{E} = \omega\mu\vec{H} \\ \vec{k} \times \vec{H} = -\omega\varepsilon\vec{E} \\ \vec{k} \cdot \vec{E} = 0 \\ \vec{k} \cdot \vec{H} = 0 \end{cases}$$

$$\begin{aligned} \vec{E}_1 &= \vec{E}_I e^{-j\vec{k}_I \cdot \vec{r}} + \vec{E}_R e^{-j\vec{k}_R \cdot \vec{r}} & \vec{E}_I e^{-j\vec{k}_I \cdot \vec{r}} &= \vec{E}_I e^{-jk_x x} e^{-jk_{1z} z} \\ \vec{E}_2 &= \vec{E}_T e^{-j\vec{k}_T \cdot \vec{r}} & \vec{E}_R e^{-j\vec{k}_R \cdot \vec{r}} &= \vec{E}_R e^{-jk_x x} e^{jk_{1z} z} \\ & & \vec{E}_T e^{-j\vec{k}_T \cdot \vec{r}} &= \vec{E}_T e^{-jk_x x} e^{-jk_{2z} z} \end{aligned}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

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$$k_1 \sin \vartheta_1 = k_2 \sin \vartheta_T$$

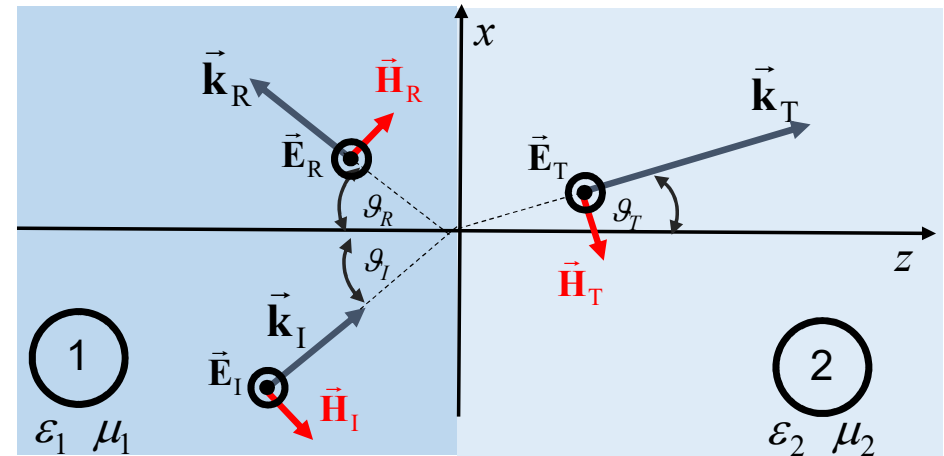
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}$$

$[H_x, E_y, H_z]$ Perpendicular Polarization \perp

$$-\frac{\partial E_y}{\partial z} = -j\omega\mu H_x \quad \longrightarrow \quad H_x = \frac{1}{j\omega\mu} \frac{\partial E_y}{\partial z}$$

$$\frac{\partial H_x}{\partial z} + jk_x H_z = j\omega\varepsilon E_y$$

$$-jk_x E_y = -j\omega\mu H_z \quad \longrightarrow \quad H_z = \frac{k_x}{\omega\mu} E_y$$



Incidence on a dielectric half-space: \perp polarization

$$\begin{aligned}\vec{\mathbf{E}}_1 &= \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z} \\ \vec{\mathbf{E}}_2 &= \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z} \\ & & \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}\end{aligned}$$

$$k_1 = \omega \sqrt{\mu_1 \epsilon_1}$$

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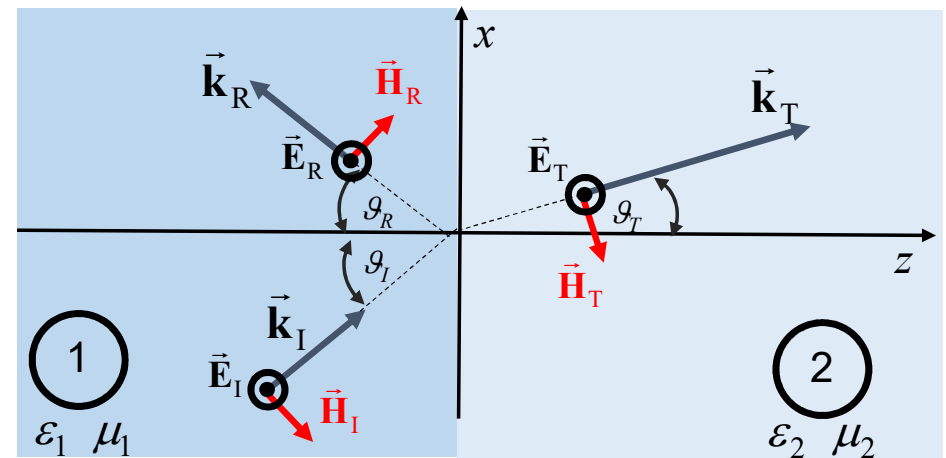
$[H_x, E_y, H_z]$ Perpendicular Polarization \perp

$$H_x = \frac{1}{j\omega\mu} \frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu} E_y$$

$$-\frac{\partial E_y}{\partial z} = -j\omega\mu H_x \quad \longrightarrow \quad H_x = \frac{1}{j\omega\mu} \frac{\partial E_y}{\partial z}$$

$$\frac{\partial H_x}{\partial z} + jk_x H_z = j\omega\epsilon E_y$$

$$-jk_x E_y = -j\omega\mu H_z \quad \longrightarrow \quad H_z = \frac{k_x}{\omega\mu} E_y$$



Incidence on a dielectric half-space: \perp polarization

$$\begin{aligned} \vec{\mathbf{E}}_1 &= \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z} \\ \vec{\mathbf{E}}_2 &= \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z} \\ & & \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z} \end{aligned}$$

$$k_1 = \omega \sqrt{\mu_1 \epsilon_1}$$

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$$k_x = k_1 \sin \vartheta_I$$

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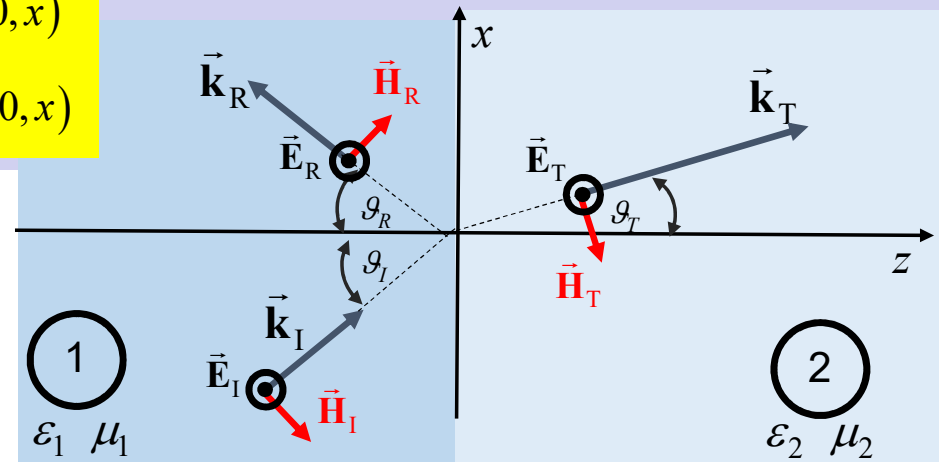
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_I}$$

$[H_x, E_y, H_z]$ Perpendicular Polarization \perp

$$H_x = \frac{1}{j\omega\mu} \frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu} E_y$$

$$E_{1y}(z=0, x) = E_{2y}(z=0, x)$$

$$H_{1x}(z=0, x) = H_{2x}(z=0, x)$$



MEMO

Fields at boundaries

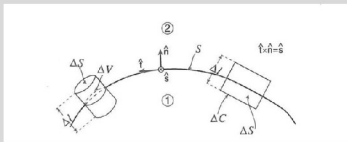
$$\hat{\mathbf{n}} \times (\hat{\mathbf{e}}_2 - \hat{\mathbf{e}}_1) = 0$$

$$\hat{\mathbf{n}} \times (\hat{\mathbf{h}}_2 - \hat{\mathbf{h}}_1) = \hat{\mathbf{j}}$$

$$(\hat{\mathbf{d}}_2 - \hat{\mathbf{d}}_1) \cdot \hat{\mathbf{n}} = \rho_s$$

$$(\hat{\mathbf{b}}_2 - \hat{\mathbf{b}}_1) \cdot \hat{\mathbf{n}} = 0$$

$$(\hat{\mathbf{i}}_2 - \hat{\mathbf{i}}_1) \cdot \hat{\mathbf{n}} = -\frac{\partial \rho_s}{\partial t}$$



Incidence on a dielectric half-space: \perp polarization

$$\begin{aligned}\vec{\mathbf{E}}_1 &= \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z} \\ \vec{\mathbf{E}}_2 &= \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z} \\ & & \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}\end{aligned}$$

$$k_1 = \omega \sqrt{\mu_1 \epsilon_1}$$

$$k_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

$$k_x = k_1 \sin \vartheta_I$$

$$k_{1z} = k_1 \cos \vartheta_I$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_I = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_I}$$

$[H_x, E_y, H_z]$ Perpendicular Polarization \perp

$$H_x = \frac{1}{j\omega\mu} \frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu} E_y$$

$$E_{1y}(z=0, x) = E_{2y}(z=0, x)$$

$$H_{1x}(z=0, x) = H_{2x}(z=0, x)$$

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} = E_I e^{-jk_x x} e^{-jk_{1z} z} \hat{\mathbf{i}}_y \quad \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} = E_R e^{-jk_x x} e^{jk_{1z} z} \hat{\mathbf{i}}_y \quad \textcircled{1}$$

$$\vec{\mathbf{E}}_1(z, x) = [E_I e^{-jk_{1z} z} + E_R e^{jk_{1z} z}] e^{-jk_x x} \hat{\mathbf{i}}_y$$

$$E_{1y}(z, x) = [E_I e^{-jk_{1z} z} + E_R e^{jk_{1z} z}] e^{-jk_x x}$$

$$H_{1x}(z, x) = \frac{1}{j\omega\mu_1} [-jk_{1z} E_I e^{-jk_{1z} z} + jk_{1z} E_R e^{jk_{1z} z}] e^{-jk_x x}$$

1

$$\vec{\mathbf{E}}_I = E_I \hat{\mathbf{i}}_y$$

$$\vec{\mathbf{E}}_R = E_R \hat{\mathbf{i}}_y$$

$$\vec{\mathbf{E}}_T = E_T \hat{\mathbf{i}}_y$$

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} = E_T e^{-jk_x x} e^{-jk_{2z} z} \hat{\mathbf{i}}_y \quad \textcircled{2}$$

$$\vec{\mathbf{E}}_2(z, x) = E_T e^{-jk_{2z} z} e^{-jk_x x} \hat{\mathbf{i}}_y$$

$$E_{2y}(z, x) = E_T e^{-jk_{2z} z} e^{-jk_x x}$$

$$H_{2x}(z, x) = \frac{1}{j\omega\mu_2} [-jk_{2z} E_T e^{-jk_{2z} z} e^{-jk_x x}]$$

2

Incidence on a dielectric half-space: \perp polarization

$$\begin{aligned}\vec{\mathbf{E}}_1 &= \vec{\mathbf{E}}_I e^{-j\vec{k}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{k}_R \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_1 e^{-j\vec{k}_1 \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z} \\ \vec{\mathbf{E}}_2 &= \vec{\mathbf{E}}_T e^{-j\vec{k}_T \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_R e^{-j\vec{k}_R \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z} \\ & & \vec{\mathbf{E}}_T e^{-j\vec{k}_T \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}\end{aligned}$$

$$k_1 = \omega \sqrt{\mu_1 \varepsilon_1}$$

$$k_2 = \omega \sqrt{\mu_2 \varepsilon_2}$$

$$k_x = k_1 \sin \vartheta_I$$

$$k_{1z} = k_1 \cos \vartheta_I$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_I = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_I}$$

$[H_x, E_y, H_z]$ Perpendicular Polarization \perp

$$H_x = \frac{1}{j\omega\mu} \frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu} E_y$$

$$E_{1y}(z=0, x) = E_{2y}(z=0, x)$$

$$H_{1x}(z=0, x) = H_{2x}(z=0, x)$$

$$E_{1y}(z=0, x) = [E_I + E_R] e^{-jk_x x}$$

$$H_{1x}(z=0, x) = -\frac{k_{1z}}{\omega\mu_1} [E_I - E_R] e^{-jk_x x}$$

$$E_{1y}(z, x) = [E_I e^{-jk_{1z} z} + E_R e^{jk_{1z} z}] e^{-jk_x x}$$

$$H_{1x}(z, x) = \frac{1}{j\omega\mu_1} [-jk_{1z} E_I e^{-jk_{1z} z} + jk_{1z} E_R e^{jk_{1z} z}] e^{-jk_x x}$$

1

$$Z_1 = \frac{\omega\mu_1}{k_{1z}}$$

$$E_{2y}(z=0, x) = E_T e^{-jk_x x}$$

$$H_{2x}(z=0, x) = -\frac{k_{2z}}{\omega\mu_2} E_T e^{-jk_x x}$$

$$E_{2y}(z, x) = E_T e^{-jk_{2z} z} e^{-jk_x x}$$

$$H_{2x}(z, x) = \frac{1}{j\omega\mu_2} [-jk_{2z} E_T e^{-jk_{2z} z} e^{-jk_x x}]$$

2

$$Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

Incidence on a dielectric half-space: \perp polarization

$$\begin{aligned}\vec{\mathbf{E}}_1 &= \vec{\mathbf{E}}_I e^{-j\vec{k}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{k}_R \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_1 e^{-j\vec{k}_1 \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z} \\ \vec{\mathbf{E}}_2 &= \vec{\mathbf{E}}_T e^{-j\vec{k}_T \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_R e^{-j\vec{k}_R \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z} \\ & & \vec{\mathbf{E}}_T e^{-j\vec{k}_T \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}\end{aligned}$$

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$$E_{2y}(z, x) = E_T e^{-jk_{2z} z} e^{-jk_x x}$$

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2

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Incidence on a dielectric half-space: \perp polarization

$$\begin{cases} 1 + \Gamma = T \\ 1 - \Gamma = \frac{Z_1}{Z_2} T \end{cases}$$

$$\Gamma \triangleq \frac{E_R}{E_I}$$

$$T \triangleq \frac{E_T}{E_I}$$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}}$$

$$Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$k_1 = \omega\sqrt{\mu_1\epsilon_1}$$

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$$\vartheta_I = \vartheta_R$$

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$$H_x = \frac{1}{j\omega\mu} \frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu} E_y$$

$$E_{1y}(z=0, x) = E_{2y}(z=0, x)$$

$$H_{1x}(z=0, x) = H_{2x}(z=0, x)$$

$$\begin{cases} E_I + E_R = E_T \\ E_I - E_R = \frac{Z_1}{Z_2} E_T \end{cases} \quad \begin{cases} 1 + \frac{E_R}{E_I} = \frac{E_T}{E_I} \\ 1 - \frac{E_R}{E_I} = \frac{Z_1}{Z_2} \frac{E_T}{E_I} \end{cases}$$

$$E_{1y}(z=0, x) = [E_I + E_R] e^{-jk_x x}$$

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2

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Incidence on a dielectric half-space: \perp polarization

| | | | | | |
|--|--|---|---|--|--|
| $\begin{cases} 1 + \Gamma = T \\ 1 - \Gamma = \frac{Z_1}{Z_2} T \end{cases}$ | $\Gamma \triangleq \frac{E_R}{E_I}$ $T \triangleq \frac{E_T}{E_I}$ | $\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$ $T = \frac{2Z_2}{Z_1 + Z_2}$ | $Z_1 = \frac{\omega\mu_1}{k_{1z}}$ $Z_2 = \frac{\omega\mu_2}{k_{2z}}$ | $k_1 = \omega\sqrt{\mu_1\varepsilon_1}$ $k_2 = \omega\sqrt{\mu_2\varepsilon_2}$ $k_x = k_1 \sin \vartheta_I$ $k_{1z} = k_1 \cos \vartheta_I$ | $\vartheta_I = \vartheta_R$ $k_1 \sin \vartheta_I = k_2 \sin \vartheta_T$ $k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_I}$ |
|--|--|---|---|--|--|

| | |
|---|--|
| $[H_x, E_y, H_z] \quad \text{Perpendicular Polarization} \quad \perp$ | $2 = \left[\frac{Z_1}{Z_2} + 1 \right] \Gamma = \left[\frac{Z_1 + Z_2}{Z_2} \right] \Gamma \Rightarrow T = \frac{2Z_2}{Z_1 + Z_2}$ |
| $H_x = \frac{1}{j\omega\mu} \frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu} E_y$ | $E_{1y}(z=0, x) = E_{2y}(z=0, x)$ $H_{1x}(z=0, x) = H_{2x}(z=0, x)$ |
| | $\Gamma = T - 1 = \frac{2Z_2}{Z_1 + Z_2} - 1 = \frac{Z_2 - Z_1}{Z_1 + Z_2}$ |

$$E_{1y}(z=0, x) = [E_I + E_R] e^{-jk_x x} \quad \textcircled{1}$$

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$$E_{2y}(z=0, x) = E_T e^{-jk_x x} \quad \textcircled{2}$$

$$H_{2x}(z=0, x) = -\frac{1}{Z_2} E_T e^{-jk_x x}$$

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Incidence on a dielectric half-space: \perp polarization

$$\begin{cases} 1 + \Gamma = T \\ 1 - \Gamma = \frac{Z_1}{Z_2} T \end{cases} \quad \begin{aligned} \Gamma &\triangleq \frac{E_R}{E_I} \\ T &\triangleq \frac{E_T}{E_I} \end{aligned}$$

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$$E_{2y}(z=0, x) = E_T e^{-jk_x x} \quad \textcircled{2}$$

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Incidence on a dielectric half-space: \perp polarization

$$\begin{cases} 1 + \Gamma = T \\ 1 - \Gamma = \frac{Z_1}{Z_2} T \end{cases} \quad \Gamma \triangleq \frac{E_R}{E_I} \quad T \triangleq \frac{E_T}{E_I}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

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$$Z_1 = \frac{\omega\mu_1}{k_{1z}}$$

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$$H_x = \frac{1}{j\omega\mu} \frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu} E_y$$

$$\Gamma = \frac{\cos \vartheta_1 - (\mu_1/\mu_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}{\cos \vartheta_1 + (\mu_1/\mu_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}$$

$$E_{1y}(z, x) = [E_I e^{-jk_{1z}z} + E_R e^{jk_{1z}z}] e^{-jk_x x}$$

1

$$E_{2y}(z, x) = E_T e^{-jk_{2z}z} e^{-jk_x x}$$

2

Incidence on a dielectric half-space: \perp polarization

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$Z_1 = \frac{\omega\mu_1}{k_{1z}}$$

$$Z_2 = \frac{\omega\mu_2}{k_{2z}}$$

$$k_{1z} = k_1 \cos \vartheta_1$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}$$

Fresnel reflection coefficient

$$\Gamma = \frac{\frac{\omega\mu_2}{k_{2z}} - \frac{\omega\mu_1}{k_{1z}}}{\frac{\omega\mu_2}{k_{2z}} + \frac{\omega\mu_1}{k_{1z}}} = \frac{k_{2z} k_{1z} \frac{\omega\mu_2}{k_{2z}} - k_{1z} k_{1z} \frac{\omega\mu_1}{k_{1z}}}{k_{2z} k_{1z} \frac{\omega\mu_2}{k_{2z}} + k_{1z} k_{1z} \frac{\omega\mu_1}{k_{1z}}} = \frac{k_{1z}\mu_2 - k_{2z}\mu_1}{k_{1z}\mu_2 + k_{2z}\mu_1} = \frac{k_{1z} - k_{2z}(\mu_1/\mu_2)}{k_{1z} + k_{2z}(\mu_1/\mu_2)} = \frac{k_1 \cos \vartheta_1 - (\mu_1/\mu_2) \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}}{k_1 \cos \vartheta_1 + (\mu_1/\mu_2) \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}}$$

$$= \frac{k_1 \cos \vartheta_1 - (\mu_1/\mu_2) k_1 \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}{k_1 \cos \vartheta_1 + (\mu_1/\mu_2) k_1 \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}} = \frac{\cos \vartheta_1 - (\mu_1/\mu_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}{\cos \vartheta_1 + (\mu_1/\mu_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}$$

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$$\begin{cases} 1 + \Gamma = T \\ 1 - \Gamma = \frac{Z_1}{Z_2} T \end{cases} \quad \Gamma \triangleq \frac{E_R}{E_I} \quad T \triangleq \frac{E_T}{E_I}$$

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$$\Gamma \triangleq \frac{E_R}{E_I} \quad T \triangleq \frac{E_T}{E_I} \quad \Gamma = \frac{\cos \vartheta_1 - (\mu_1/\mu_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}{\cos \vartheta_1 + (\mu_1/\mu_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}} \quad T = 1 + \Gamma$$

$$E_{1y}(z, x) = [E_I e^{-jk_{1z}z} + E_R e^{jk_{1z}z}] e^{-jk_x x}$$

1

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Incidence on a dielectric half-space: \perp polarization

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Incidence on a dielectric half-space: \perp polarization

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2

Incidence on a dielectric half-space: \perp polarization

$$\Gamma_{\perp} \triangleq \frac{E_R}{E_I} \quad \Gamma_{\perp} = \frac{\cos \vartheta_1 - (\mu_1/\mu_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}{\cos \vartheta_1 + (\mu_1/\mu_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}} \quad T_{\perp} = 1 + \Gamma_{\perp}$$

$$T_{\perp} \triangleq \frac{E_T}{E_I}$$

$$k_1 = \omega \sqrt{\mu_1 \varepsilon_1}$$

$$k_2 = \omega \sqrt{\mu_2 \varepsilon_2}$$

$$k_x = k_1 \sin \vartheta_1$$

$$k_{1z} = k_1 \cos \vartheta_1$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_I = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_I}$$

$[H_x, E_y, H_z]$ Perpendicular Polarization \perp

$$H_x = \frac{1}{j\omega\mu} \frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu} E_y$$

$$E_{1y}(z, x) = [E_I e^{-jk_{1z}z} + E_R e^{jk_{1z}z}] e^{-jk_x x}$$

1

$$E_{2y}(z, x) = E_T e^{-jk_{2z}z} e^{-jk_x x}$$

2

Incidence on a dielectric half-space: \perp polarization

$$\Gamma_{\perp} \triangleq \frac{E_R}{E_I}$$

$$T_{\perp} \triangleq \frac{E_T}{E_I}$$

$$\Gamma_{\perp} = \frac{\cos \vartheta_1 - (\mu_1/\mu_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}{\cos \vartheta_1 + (\mu_1/\mu_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}$$

$$T_{\perp} = 1 + \Gamma_{\perp}$$

$$k_1 = \omega \sqrt{\mu_1 \varepsilon_1}$$

$$k_2 = \omega \sqrt{\mu_2 \varepsilon_2}$$

$$k_x = k_1 \sin \vartheta_1$$

$$k_{1z} = k_1 \cos \vartheta_1$$

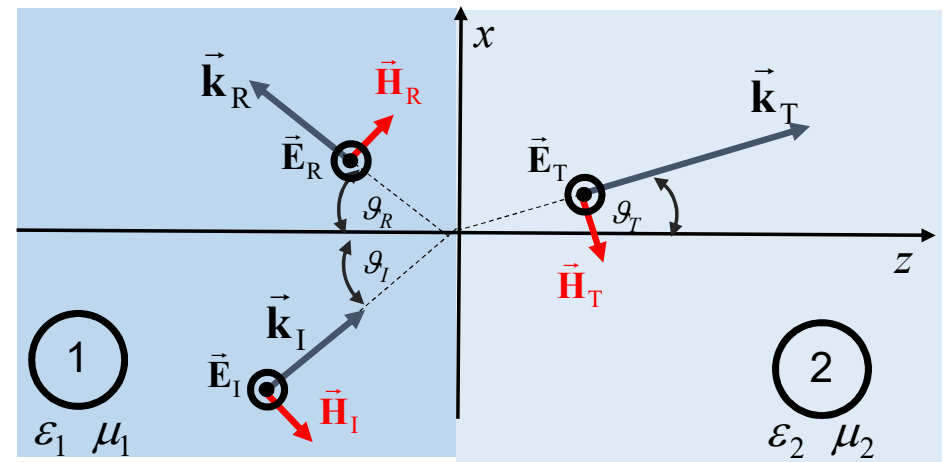
$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_I = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_I}$$

$[H_x, E_y, H_z]$ Perpendicular Polarization \perp

$$H_x = \frac{1}{j\omega\mu} \frac{\partial E_y}{\partial z}; \quad H_z = \frac{k_x}{\omega\mu} E_y$$



$$E_{1y}(z, x) = [E_I e^{-jk_{1z}z} + E_R e^{jk_{1z}z}] e^{-jk_x x} = E_I e^{-jk_x x} e^{-jk_{1z}z} + E_R e^{-jk_x x} e^{jk_{1z}z}$$

1

$$E_{2y}(z, x) = E_T e^{-jk_{2z}z} e^{-jk_x x}$$

2

Incidence on a dielectric half-space: \perp polarization

$$\begin{cases} \vec{k} \times \vec{E} = \omega\mu\vec{H} \\ \vec{k} \times \vec{H} = -\omega\varepsilon\vec{E} \\ \vec{k} \cdot \vec{E} = 0 \\ \vec{k} \cdot \vec{H} = 0 \end{cases}$$

$$\begin{aligned} \vec{E}_1 &= \vec{E}_I e^{-j\vec{k}_I \cdot \vec{r}} + \vec{E}_R e^{-j\vec{k}_R \cdot \vec{r}} & \vec{E}_I e^{-j\vec{k}_I \cdot \vec{r}} &= \vec{E}_I e^{-jk_x x} e^{-jk_{1z} z} \\ \vec{E}_2 &= \vec{E}_T e^{-j\vec{k}_T \cdot \vec{r}} & \vec{E}_R e^{-j\vec{k}_R \cdot \vec{r}} &= \vec{E}_R e^{-jk_x x} e^{jk_{1z} z} \\ & & \vec{E}_T e^{-j\vec{k}_T \cdot \vec{r}} &= \vec{E}_T e^{-jk_x x} e^{-jk_{2z} z} \end{aligned}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin \vartheta_1$$

$$k_{1z} = k_1 \cos \vartheta_1$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_1 = k_2 \sin \vartheta_T$$

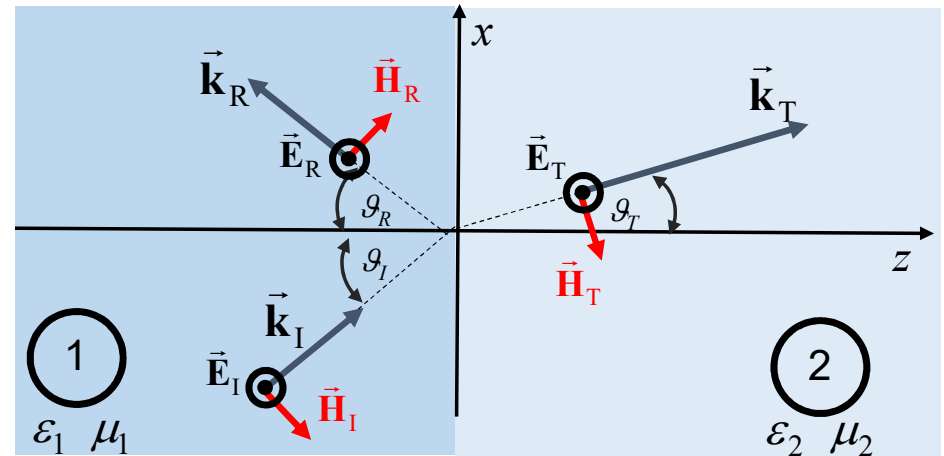
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}$$

$[H_x, E_y, H_z]$ Perpendicular Polarization \perp

$$-\frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial H_x}{\partial z} + jk_x H_z = j\omega\varepsilon E_y$$

$$-jk_x E_y = -j\omega\mu H_z$$



Incidence on a dielectric half-space: || polarization

$$\begin{cases} \vec{k} \times \vec{E} = \omega\mu\vec{H} \\ \vec{k} \times \vec{H} = -\omega\varepsilon\vec{E} \\ \vec{k} \cdot \vec{E} = 0 \\ \vec{k} \cdot \vec{H} = 0 \end{cases}$$

$$\begin{aligned} \vec{E}_1 &= \vec{E}_I e^{-j\vec{k}_I \cdot \vec{r}} + \vec{E}_R e^{-j\vec{k}_R \cdot \vec{r}} & \vec{E}_I e^{-j\vec{k}_I \cdot \vec{r}} &= \vec{E}_I e^{-jk_x x} e^{-jk_{1z} z} \\ \vec{E}_2 &= \vec{E}_T e^{-j\vec{k}_T \cdot \vec{r}} & \vec{E}_R e^{-j\vec{k}_R \cdot \vec{r}} &= \vec{E}_R e^{-jk_x x} e^{jk_{1z} z} \\ & & \vec{E}_T e^{-j\vec{k}_T \cdot \vec{r}} &= \vec{E}_T e^{-jk_x x} e^{-jk_{2z} z} \end{aligned}$$

$$\begin{aligned} k_1 &= \omega\sqrt{\mu_1\varepsilon_1} \\ k_2 &= \omega\sqrt{\mu_2\varepsilon_2} \end{aligned}$$

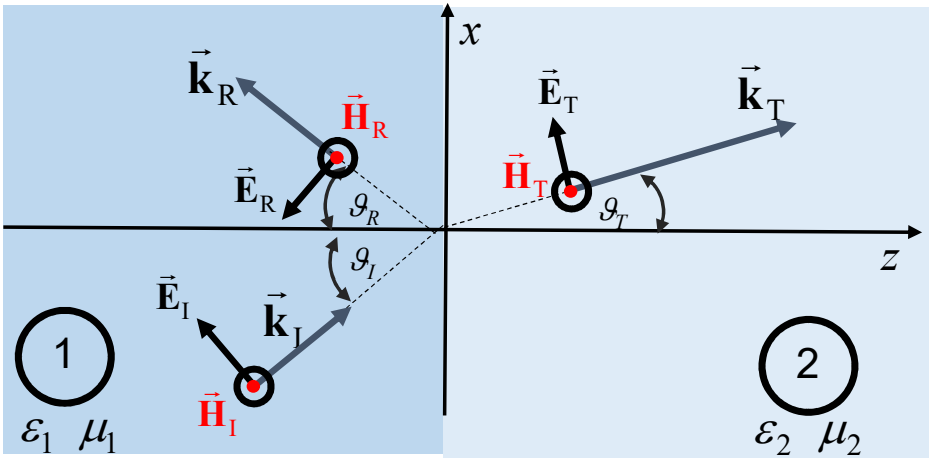
$$\begin{aligned} k_x &= k_1 \sin \vartheta_I \\ k_{1z} &= k_1 \cos \vartheta_I \end{aligned}$$

$$\begin{aligned} \vartheta_I &= \vartheta_R \\ k_1 \sin \vartheta_I &= k_2 \sin \vartheta_T \\ k_{2z} &= \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_I} \end{aligned}$$

$[E_x, H_y, E_z]$ **Parallel Polarization** ||

$$-\frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \quad \longrightarrow \quad E_x = -\frac{1}{j\omega\varepsilon} \frac{\partial H_y}{\partial z}$$

$$\frac{\partial E_x}{\partial z} + jk_x E_z = -j\omega\mu H_y$$

$$-jk_x H_y = j\omega\varepsilon E_z \quad \longrightarrow \quad E_z = -\frac{k_x}{\omega\varepsilon} H_y$$


Incidence on a dielectric half-space: || polarization

$$\begin{cases} \vec{k} \times \vec{E} = \omega\mu\vec{H} \\ \vec{k} \times \vec{H} = -\omega\varepsilon\vec{E} \\ \vec{k} \cdot \vec{E} = 0 \\ \vec{k} \cdot \vec{H} = 0 \end{cases}$$

$$\begin{aligned} \vec{E}_1 &= \vec{E}_I e^{-j\vec{k}_I \cdot \vec{r}} + \vec{E}_R e^{-j\vec{k}_R \cdot \vec{r}} & \vec{E}_I e^{-j\vec{k}_I \cdot \vec{r}} &= \vec{E}_I e^{-jk_x x} e^{-jk_{1z} z} \\ \vec{E}_2 &= \vec{E}_T e^{-j\vec{k}_T \cdot \vec{r}} & \vec{E}_R e^{-j\vec{k}_R \cdot \vec{r}} &= \vec{E}_R e^{-jk_x x} e^{jk_{1z} z} \\ & & \vec{E}_T e^{-j\vec{k}_T \cdot \vec{r}} &= \vec{E}_T e^{-jk_x x} e^{-jk_{2z} z} \end{aligned}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin \vartheta_1$$

$$k_{1z} = k_1 \cos \vartheta_1$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_1 = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}$$

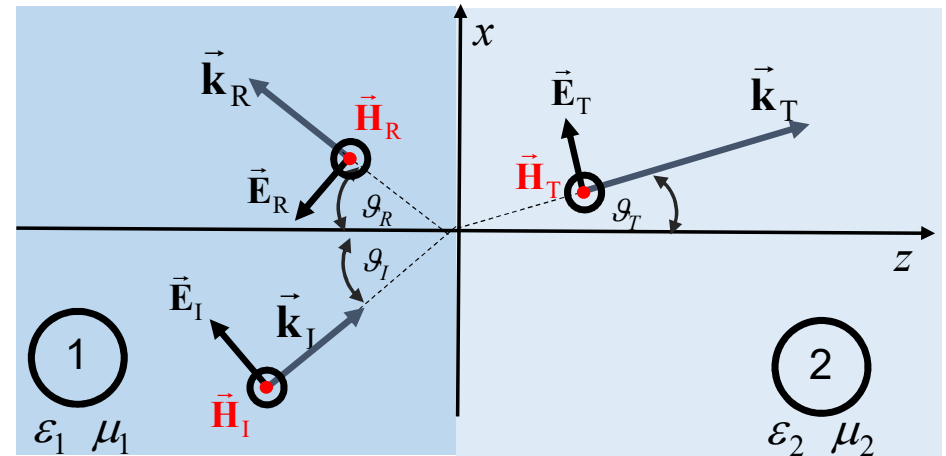
$[E_x, H_y, E_z]$ Parallel Polarization ||

$$E_x = -\frac{1}{j\omega\varepsilon} \frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\varepsilon} H_y$$

$$-\frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \quad \longrightarrow \quad E_x = -\frac{1}{j\omega\varepsilon} \frac{\partial H_y}{\partial z}$$

$$\frac{\partial E_x}{\partial z} + jk_x E_z = -j\omega\mu H_y$$

$$-jk_x H_y = j\omega\varepsilon E_z \quad \longrightarrow \quad E_z = -\frac{k_x}{\omega\varepsilon} H_y$$



Incidence on a dielectric half-space: || polarization

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} = \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega \sqrt{\mu_1 \epsilon_1}$$

$$k_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

$$k_x = k_1 \sin \vartheta_1$$

$$k_{1z} = k_1 \cos \vartheta_1$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_1 = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}$$

$$[E_x, H_y, E_z]$$

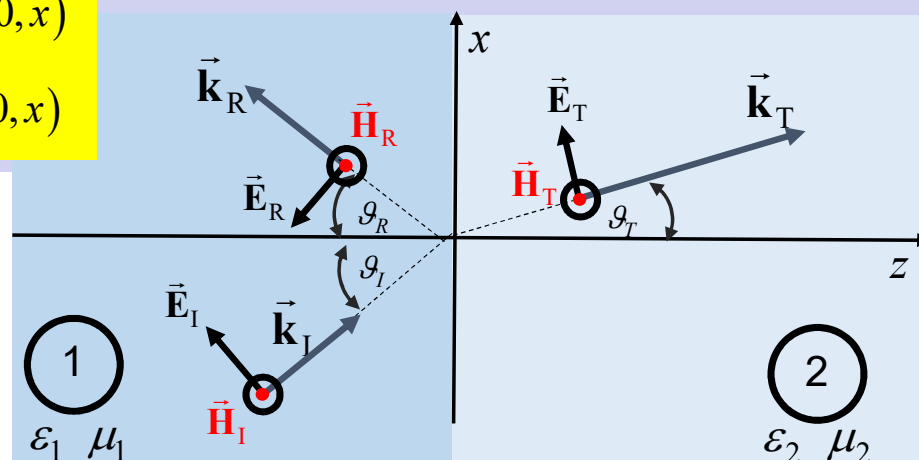
Parallel Polarization

||

$$E_x = -\frac{1}{j\omega\epsilon} \frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\epsilon} H_y$$

$$H_{1y}(z=0, x) = H_{2y}(z=0, x)$$

$$E_{1x}(z=0, x) = E_{2x}(z=0, x)$$



MEMO

Fields at boundaries

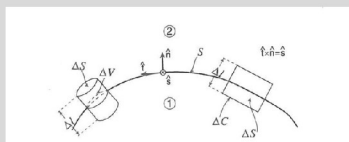
$$\hat{\mathbf{n}} \times (\mathbf{e}_2 - \mathbf{e}_1) = 0$$

$$\hat{\mathbf{n}} \times (\mathbf{h}_2 - \mathbf{h}_1) = \mathbf{j}$$

$$(\mathbf{d}_2 - \mathbf{d}_1) \cdot \hat{\mathbf{n}} = \rho_s$$

$$(\mathbf{b}_2 - \mathbf{b}_1) \cdot \hat{\mathbf{n}} = 0$$

$$(\hat{\mathbf{k}}_2 - \hat{\mathbf{k}}_1) \cdot \hat{\mathbf{n}} = -\frac{\partial \rho_s}{\partial t}$$



Incidence on a dielectric half-space: || polarization

$$\begin{aligned}\vec{\mathbf{E}}_1 &= \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z} \\ \vec{\mathbf{E}}_2 &= \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z} \\ & & \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}\end{aligned}$$

$$k_1 = \omega \sqrt{\mu_1 \epsilon_1}$$

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$$k_x = k_1 \sin \vartheta_1$$

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$$\left[E_x, H_y, E_z \right]$$

Parallel Polarization

||

$$E_x = -\frac{1}{j\omega\epsilon} \frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\epsilon} H_y$$

$$H_{1y}(z=0, x) = H_{2y}(z=0, x)$$

$$E_{1x}(z=0, x) = E_{2x}(z=0, x)$$

$$\vec{\mathbf{H}}_I e^{-j\vec{\mathbf{k}}_I \cdot \vec{\mathbf{r}}} = H_I e^{-jk_x x} e^{-jk_{1z} z} \hat{\mathbf{i}}_y \quad \vec{\mathbf{H}}_R e^{-j\vec{\mathbf{k}}_R \cdot \vec{\mathbf{r}}} = H_R e^{-jk_x x} e^{jk_{1z} z} \hat{\mathbf{i}}_y$$

1

$$\vec{\mathbf{H}}_1(z, x) = \left[H_I e^{-jk_{1z} z} + H_R e^{jk_{1z} z} \right] e^{-jk_x x} \hat{\mathbf{i}}_y$$

$$H_{1y}(z, x) = \left[H_I e^{-jk_{1z} z} + H_R e^{jk_{1z} z} \right] e^{-jk_x x}$$

$$E_{1x}(z, x) = -\frac{1}{j\omega\epsilon_1} \left[-jk_{1z} H_I e^{-jk_{1z} z} + jk_{1z} H_R e^{jk_{1z} z} \right] e^{-jk_x x}$$

$$\vec{\mathbf{H}}_I = H_I \hat{\mathbf{i}}_y$$

$$\vec{\mathbf{H}}_R = H_R \hat{\mathbf{i}}_y$$

$$\vec{\mathbf{H}}_T = H_T \hat{\mathbf{i}}_y$$

$$\vec{\mathbf{H}}_T e^{-j\vec{\mathbf{k}}_T \cdot \vec{\mathbf{r}}} = H_T e^{-jk_x x} e^{-jk_{2z} z} \hat{\mathbf{i}}_y$$

2

$$\vec{\mathbf{H}}_2(z, x) = H_T e^{-jk_{2z} z} e^{-jk_x x} \hat{\mathbf{i}}_y$$

$$H_{2y}(z, x) = H_T e^{-jk_{2z} z} e^{-jk_x x}$$

$$E_{2x}(z, x) = -\frac{1}{j\omega\epsilon_2} \left[-jk_{2z} H_T e^{-jk_{2z} z} e^{-jk_x x} \right]$$

Incidence on a dielectric half-space: || polarization

$$\begin{aligned}\vec{\mathbf{E}}_1 &= \vec{\mathbf{E}}_I e^{-j\vec{k}_I \cdot \vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{k}_R \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_1 e^{-j\vec{k}_1 \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z} \\ \vec{\mathbf{E}}_2 &= \vec{\mathbf{E}}_T e^{-j\vec{k}_T \cdot \vec{\mathbf{r}}} & \vec{\mathbf{E}}_R e^{-j\vec{k}_R \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z} \\ & & \vec{\mathbf{E}}_T e^{-j\vec{k}_T \cdot \vec{\mathbf{r}}} &= \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}\end{aligned}$$

$$k_1 = \omega \sqrt{\mu_1 \varepsilon_1}$$

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$$k_x = k_1 \sin \vartheta_I$$

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$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_I = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_I}$$

$[E_x, H_y, E_z]$ Parallel Polarization ||

$$E_x = -\frac{1}{j\omega\varepsilon} \frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\varepsilon} H_y$$

$$H_{1y}(z=0, x) = H_{2y}(z=0, x)$$

$$E_{1x}(z=0, x) = E_{2x}(z=0, x)$$

$$H_{1y}(z=0, x) = [H_I + H_R] e^{-jk_x x}$$

$$E_{1x}(z=0, x) = \frac{k_{1z}}{\omega\varepsilon_1} [H_I - H_R] e^{-jk_x x}$$

$$H_{1y}(z, x) = [H_I e^{-jk_{1z} z} + H_R e^{jk_{1z} z}] e^{-jk_x x}$$

$$E_{1x}(z, x) = -\frac{1}{j\omega\varepsilon_1} [-jk_{1z} H_I e^{-jk_{1z} z} + jk_{1z} H_R e^{jk_{1z} z}] e^{-jk_x x}$$

1

$$Z_1 = \frac{k_{1z}}{\omega\varepsilon_1}$$

$$H_{2y}(z=0, x) = H_T e^{-jk_x x}$$

$$E_{2x}(z=0, x) = \frac{k_{2z}}{\omega\varepsilon_2} H_T e^{-jk_x x}$$

$$H_{2y}(z, x) = H_T e^{-jk_{2z} z} e^{-jk_x x}$$

$$E_{2x}(z, x) = -\frac{1}{j\omega\varepsilon_2} [-jk_{2z} H_T e^{-jk_{2z} z} e^{-jk_x x}]$$

2

$$Z_2 = \frac{k_{2z}}{\omega\varepsilon_2}$$

Incidence on a dielectric half-space: || polarization

$$\begin{aligned}\vec{\mathbf{E}}_1 &= \vec{\mathbf{E}}_I e^{-j\vec{k}_I \cdot \vec{r}} + \vec{\mathbf{E}}_R e^{-j\vec{k}_R \cdot \vec{r}} & \vec{\mathbf{E}}_1 e^{-j\vec{k}_1 \cdot \vec{r}} &= \vec{\mathbf{E}}_I e^{-jk_x x} e^{-jk_{1z} z} \\ \vec{\mathbf{E}}_2 &= \vec{\mathbf{E}}_T e^{-j\vec{k}_T \cdot \vec{r}} & \vec{\mathbf{E}}_R e^{-j\vec{k}_R \cdot \vec{r}} &= \vec{\mathbf{E}}_R e^{-jk_x x} e^{jk_{1z} z} \\ & & \vec{\mathbf{E}}_T e^{-j\vec{k}_T \cdot \vec{r}} &= \vec{\mathbf{E}}_T e^{-jk_x x} e^{-jk_{2z} z}\end{aligned}$$

$$k_1 = \omega \sqrt{\mu_1 \epsilon_1}$$

$$k_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

$$k_x = k_1 \sin \vartheta_I$$

$$k_{1z} = k_1 \cos \vartheta_I$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_I = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_I}$$

$[E_x, H_y, E_z]$ Parallel Polarization ||

$$E_x = -\frac{1}{j\omega\epsilon} \frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\epsilon} H_y$$

$$H_{1y}(z=0, x) = H_{2y}(z=0, x)$$

$$E_{1x}(z=0, x) = E_{2x}(z=0, x)$$

$$H_{1y}(z=0, x) = [H_I + H_R] e^{-jk_x x}$$

$$E_{1x}(z=0, x) = Z_1 [H_I - H_R] e^{-jk_x x}$$

$$H_{1y}(z, x) = [H_I e^{-jk_{1z} z} + H_R e^{jk_{1z} z}] e^{-jk_x x}$$

$$E_{1x}(z, x) = -\frac{1}{j\omega\epsilon_1} [-jk_{1z} H_I e^{-jk_{1z} z} + jk_{1z} H_R e^{jk_{1z} z}] e^{-jk_x x}$$

1

$$Z_1 = \frac{k_{1z}}{\omega\epsilon_1}$$

$$H_{2y}(z=0, x) = H_T e^{-jk_x x}$$

$$E_{2x}(z=0, x) = Z_2 H_T e^{-jk_x x}$$

$$H_{2y}(z, x) = H_T e^{-jk_{2z} z} e^{-jk_x x}$$

$$E_{2x}(z, x) = -\frac{1}{j\omega\epsilon_2} [-jk_{2z} H_T e^{-jk_{2z} z} e^{-jk_x x}]$$

2

$$Z_2 = \frac{k_{2z}}{\omega\epsilon_2}$$

Incidence on a dielectric half-space: || polarization

$$\begin{cases} 1 - \Gamma = \frac{Z_1}{Z_2} T \\ 1 + \Gamma = T \end{cases}$$

$$\Gamma \triangleq -\frac{H_R}{H_I}$$

$$T \triangleq \frac{Z_2 H_T}{Z_1 H_I}$$

$$Z_1 = \frac{k_{1z}}{\omega \epsilon_1}$$

$$Z_2 = \frac{k_{2z}}{\omega \epsilon_2}$$

$$k_1 = \omega \sqrt{\mu_1 \epsilon_1}$$

$$k_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

$$k_x = k_1 \sin \theta_1$$

$$k_{1z} = k_1 \cos \theta_1$$

$$\theta_I = \theta_R$$

$$k_1 \sin \theta_1 = k_2 \sin \theta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \theta_1}$$

$[E_x, H_y, E_z]$ **Parallel Polarization** ||

$$E_x = -\frac{1}{j\omega\epsilon} \frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\epsilon} H_y$$

$$H_{1y}(z=0, x) = H_{2y}(z=0, x)$$

$$E_{1x}(z=0, x) = E_{2x}(z=0, x)$$

$$\begin{cases} H_I + H_R = H_T \\ H_I - H_R = \frac{Z_2}{Z_1} H_T \end{cases} \quad \begin{cases} 1 + \frac{H_R}{H_I} = \frac{H_T}{H_I} \\ 1 - \frac{H_R}{H_I} = \frac{Z_2}{Z_1} \frac{H_T}{H_I} \end{cases}$$

$$H_{1y}(z=0, x) = [H_I + H_R] e^{-jk_x x} \quad \textcircled{1}$$

$$E_{1x}(z=0, x) = Z_1 [H_I - H_R] e^{-jk_x x}$$

$$H_{1y}(z, x) = [H_I e^{-jk_{1z} z} + H_R e^{jk_{1z} z}] e^{-jk_x x}$$

$$E_{1x}(z, x) = -\frac{1}{j\omega\epsilon_1} [-jk_{1z} H_I e^{-jk_{1z} z} + jk_{1z} H_R e^{jk_{1z} z}] e^{-jk_x x}$$

$$Z_1 = \frac{k_{1z}}{\omega\epsilon_1}$$

$$H_{2y}(z=0, x) = H_T e^{-jk_x x} \quad \textcircled{2}$$

$$E_{2x}(z=0, x) = Z_2 H_T e^{-jk_x x}$$

$$H_{2y}(z, x) = H_T e^{-jk_{2z} z} e^{-jk_x x}$$

$$E_{2x}(z, x) = -\frac{1}{j\omega\epsilon_2} [-jk_{2z} H_T e^{-jk_{2z} z} e^{-jk_x x}]$$

$$Z_2 = \frac{k_{2z}}{\omega\epsilon_2}$$

Incidence on a dielectric half-space: || polarization

| | | | | | |
|--|---|---|---|--|--|
| $\begin{cases} 1 - \Gamma = \frac{Z_1}{Z_2} T \\ 1 + \Gamma = T \end{cases}$ | $\Gamma \triangleq -\frac{H_R}{H_I}$ $T \triangleq \frac{Z_2 H_T}{Z_1 H_I}$ | $\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$ $T = \frac{2Z_2}{Z_1 + Z_2}$ | $Z_1 = \frac{k_{1z}}{\omega \epsilon_1}$ $Z_2 = \frac{k_{2z}}{\omega \epsilon_2}$ | $k_1 = \omega \sqrt{\mu_1 \epsilon_1}$ $k_2 = \omega \sqrt{\mu_2 \epsilon_2}$ $k_x = k_1 \sin \vartheta_I$ $k_{1z} = k_1 \cos \vartheta_I$ | $\vartheta_I = \vartheta_R$ $k_1 \sin \vartheta_I = k_2 \sin \vartheta_T$ $k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_I}$ |
|--|---|---|---|--|--|

$[E_x, H_y, E_z]$ **Parallel Polarization** ||

| | |
|---|---|
| $E_x = -\frac{1}{j\omega\epsilon} \frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\epsilon} H_y$ | $H_{1y}(z=0, x) = H_{2y}(z=0, x)$ $E_{1x}(z=0, x) = E_{2x}(z=0, x)$ |
|---|---|

①

$$H_{1y}(z=0, x) = [H_I + H_R] e^{-jk_x x}$$

$$E_{1x}(z=0, x) = Z_1 [H_I - H_R] e^{-jk_x x}$$

$$H_{1y}(z, x) = [H_I e^{-jk_{1z} z} + H_R e^{jk_{1z} z}] e^{-jk_x x}$$

$$E_{1x}(z, x) = -\frac{1}{j\omega\epsilon_1} [-jk_{1z} H_I e^{-jk_{1z} z} + jk_{1z} H_R e^{jk_{1z} z}] e^{-jk_x x}$$

$Z_1 = \frac{k_{1z}}{\omega\epsilon_1}$

②

$$H_{2y}(z=0, x) = H_T e^{-jk_x x}$$

$$E_{2x}(z=0, x) = Z_2 H_T e^{-jk_x x}$$

$$H_{2y}(z, x) = H_T e^{-jk_{2z} z} e^{-jk_x x}$$

$$E_{2x}(z, x) = -\frac{1}{j\omega\epsilon_2} [-jk_{2z} H_T e^{-jk_{2z} z} e^{-jk_x x}]$$

$Z_2 = \frac{k_{2z}}{\omega\epsilon_2}$

Incidence on a dielectric half-space: || polarization

$$\begin{cases} 1 - \Gamma = \frac{Z_1}{Z_2} T \\ 1 + \Gamma = T \end{cases} \quad \Gamma \triangleq -\frac{H_R}{H_I} \quad T \triangleq \frac{Z_2 H_T}{Z_1 H_I}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \quad T = \frac{2Z_2}{Z_1 + Z_2}$$

$$Z_1 = \frac{k_{1z}}{\omega \epsilon_1} \quad Z_2 = \frac{k_{2z}}{\omega \epsilon_2}$$

$$k_1 = \omega \sqrt{\mu_1 \epsilon_1} \quad k_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

$$k_x = k_1 \sin \vartheta_1 \quad k_{1z} = k_1 \cos \vartheta_1$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_1 = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}$$

$[E_x, H_y, E_z]$ Parallel Polarization ||

$$E_x = -\frac{1}{j\omega\epsilon} \frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\epsilon} H_y$$

1

$$H_{1y}(z, x) = [H_I e^{-jk_{1z}z} + H_R e^{jk_{1z}z}] e^{-jk_x x}$$

2

$$H_{2y}(z, x) = H_T e^{-jk_{2z}z} e^{-jk_x x}$$

Incidence on a dielectric half-space: || polarization

$$\begin{cases} 1 - \Gamma = \frac{Z_1}{Z_2} T \\ 1 + \Gamma = T \end{cases} \quad \begin{cases} \Gamma \triangleq -\frac{H_R}{H_I} \\ T \triangleq \frac{Z_2 H_T}{Z_1 H_I} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$T = \frac{2Z_2}{Z_1 + Z_2}$$

$$Z_1 = \frac{k_{1z}}{\omega \epsilon_1}$$

$$Z_2 = \frac{k_{2z}}{\omega \epsilon_2}$$

$$k_1 = \omega \sqrt{\mu_1 \epsilon_1}$$

$$k_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

$$k_x = k_1 \sin \vartheta_1$$

$$k_{1z} = k_1 \cos \vartheta_1$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_I = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}$$

$[E_x, H_y, E_z]$ Parallel Polarization ||

$$E_x = -\frac{1}{j\omega\epsilon} \frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\epsilon} H_y$$

$$\Gamma_{||} = -\frac{\cos \vartheta_1 - (\epsilon_1/\epsilon_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}{\cos \vartheta_1 + (\epsilon_1/\epsilon_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}$$

$$H_{1y}(z, x) = [H_I e^{-jk_{1z}z} + H_R e^{jk_{1z}z}] e^{-jk_x x}$$

1

$$H_{2y}(z, x) = H_T e^{-jk_{2z}z} e^{-jk_x x}$$

2

Incidence on a dielectric half-space: \parallel polarization

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$Z_1 = \frac{k_{1z}}{\omega \varepsilon_1}$$

$$Z_2 = \frac{k_{2z}}{\omega \varepsilon_2}$$

$$k_{1z} = k_1 \cos \vartheta_1$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}$$

Fresnel reflection coefficient

$$\begin{aligned} \Gamma_{\parallel} &= \frac{\frac{k_{2z}}{\omega \varepsilon_2} - \frac{k_{1z}}{\omega \varepsilon_1}}{\frac{k_{2z}}{\omega \varepsilon_2} + \frac{k_{1z}}{\omega \varepsilon_1}} = \frac{k_{2z} \varepsilon_1 - k_{1z} \varepsilon_2}{k_{2z} \varepsilon_1 + k_{1z} \varepsilon_2} = -\frac{k_{1z} - k_{2z} (\varepsilon_1 / \varepsilon_2)}{k_{1z} + k_{2z} (\varepsilon_1 / \varepsilon_2)} = -\frac{k_1 \cos \vartheta_1 - (\varepsilon_1 / \varepsilon_2) \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}}{k_1 \cos \vartheta_1 + (\varepsilon_1 / \varepsilon_2) \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}} \\ &= -\frac{k_1 \cos \vartheta_1 - (\varepsilon_1 / \varepsilon_2) k_1 \sqrt{(k_2 / k_1)^2 - \sin^2 \vartheta_1}}{k_1 \cos \vartheta_1 + (\varepsilon_1 / \varepsilon_2) k_1 \sqrt{(k_2 / k_1)^2 - \sin^2 \vartheta_1}} = -\frac{\cos \vartheta_1 - (\varepsilon_1 / \varepsilon_2) \sqrt{(k_2 / k_1)^2 - \sin^2 \vartheta_1}}{\cos \vartheta_1 + (\varepsilon_1 / \varepsilon_2) \sqrt{(k_2 / k_1)^2 - \sin^2 \vartheta_1}} \end{aligned}$$

Incidence on a dielectric half-space: || polarization

$$\begin{cases} 1 - \Gamma = \frac{Z_1}{Z_2} T \\ 1 + \Gamma = T \end{cases} \quad \begin{cases} \Gamma \triangleq -\frac{H_R}{H_I} \\ T \triangleq \frac{Z_2 H_T}{Z_1 H_I} \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2} \quad T = \frac{2Z_2}{Z_1 + Z_2}$$

$$Z_1 = \frac{k_{1z}}{\omega \epsilon_1} \quad Z_2 = \frac{k_{2z}}{\omega \epsilon_2}$$

$$k_1 = \omega \sqrt{\mu_1 \epsilon_1} \quad k_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

$$k_x = k_1 \sin \vartheta_1 \quad k_{1z} = k_1 \cos \vartheta_1$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_I = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_I}$$

$[E_x, H_y, E_z]$ Parallel Polarization ||

$$E_x = -\frac{1}{j\omega\epsilon} \frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\epsilon} H_y$$

$$\Gamma_{||} = -\frac{\cos \vartheta_1 - (\epsilon_1/\epsilon_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}{\cos \vartheta_1 + (\epsilon_1/\epsilon_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}} \quad T_{||} = 1 + \Gamma_{||}$$

$$H_{1y}(z, x) = [H_I e^{-jk_{1z}z} + H_R e^{jk_{1z}z}] e^{-jk_x x}$$

1

$$H_{2y}(z, x) = H_T e^{-jk_{2z}z} e^{-jk_x x}$$

2

Incidence on a dielectric half-space: || polarization

$$\begin{cases} 1 - \Gamma = \frac{Z_1}{Z_2} T \\ 1 + \Gamma = T \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$T = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega \sqrt{\mu_1 \epsilon_1}$$

$$k_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

$$k_x = k_1 \sin \vartheta_1$$

$$k_{1z} = k_1 \cos \vartheta_1$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_1 = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}$$

$[E_x, H_y, E_z]$ Parallel Polarization ||

$$E_x = -\frac{1}{j\omega\epsilon} \frac{\partial H_y}{\partial z}; \quad E_z = -\frac{k_x}{\omega\epsilon} H_y$$

$$\Gamma \triangleq -\frac{H_R}{H_I}$$

$$T \triangleq \frac{Z_2 H_T}{Z_1 H_I}$$

$$Z_1 = \frac{k_{1z}}{\omega\epsilon_1}$$

$$Z_2 = \frac{k_{2z}}{\omega\epsilon_2}$$

$$\Gamma_{\parallel} = -\frac{\cos \vartheta_1 - (\epsilon_1/\epsilon_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}{\cos \vartheta_1 + (\epsilon_1/\epsilon_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}$$

$$T_{\parallel} = 1 + \Gamma_{\parallel}$$

$$H_{1y}(z, x) = [H_I e^{-jk_{1z}z} + H_R e^{jk_{1z}z}] e^{-jk_x x}$$

1

$$H_{2y}(z, x) = H_T e^{-jk_{2z}z} e^{-jk_x x}$$

2

Incidence on a dielectric half-space: || polarization

$$\begin{cases} 1 - \Gamma = \frac{Z_1}{Z_2} T \\ 1 + \Gamma = T \end{cases}$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$T = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega \sqrt{\mu_1 \epsilon_1}$$

$$k_2 = \omega \sqrt{\mu_2 \epsilon_2}$$

$$k_x = k_1 \sin \vartheta_1$$

$$k_{1z} = k_1 \cos \vartheta_1$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_1 = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}$$

$[E_x, H_y, E_z]$ **Parallel Polarization** ||

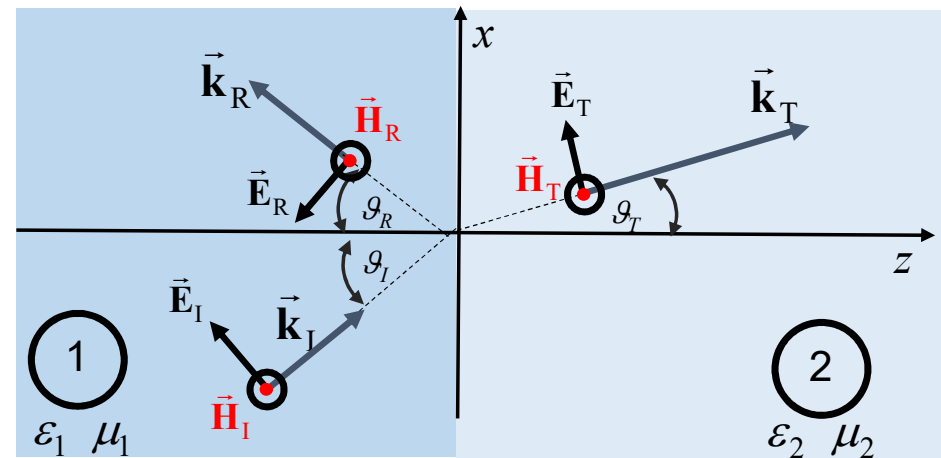
$$H_{1y}(z, x) = [H_I e^{-jk_{1z}z} + H_R e^{jk_{1z}z}] e^{-jk_x x} \quad (1)$$

$$H_{2y}(z, x) = H_T e^{-jk_{2z}z} e^{-jk_x x} \quad (2)$$

$$\Gamma_{||} \triangleq -\frac{H_R}{H_I} \quad Z_1 = \frac{k_{1z}}{\omega \epsilon_1}$$

$$T_{||} \triangleq \frac{Z_2 H_T}{Z_1 H_I} \quad Z_2 = \frac{k_{2z}}{\omega \epsilon_2}$$

$$\Gamma_{||} = -\frac{\cos \vartheta_1 - (\epsilon_1/\epsilon_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}{\cos \vartheta_1 + (\epsilon_1/\epsilon_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}$$



Incidence on a dielectric half-space: \perp polarization

$$\begin{cases} 1 - \Gamma = \frac{Z_1}{Z_2} T \\ 1 + \Gamma = T \end{cases}$$

$$\begin{aligned} \Gamma &= \frac{Z_2 - Z_1}{Z_1 + Z_2} \\ T &= \frac{2Z_2}{Z_1 + Z_2} \end{aligned}$$

$$\begin{aligned} k_1 &= \omega \sqrt{\mu_1 \varepsilon_1} \\ k_2 &= \omega \sqrt{\mu_2 \varepsilon_2} \end{aligned}$$

$$\begin{aligned} k_x &= k_1 \sin \vartheta_1 \\ k_{1z} &= k_1 \cos \vartheta_1 \end{aligned}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_1 = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** \perp

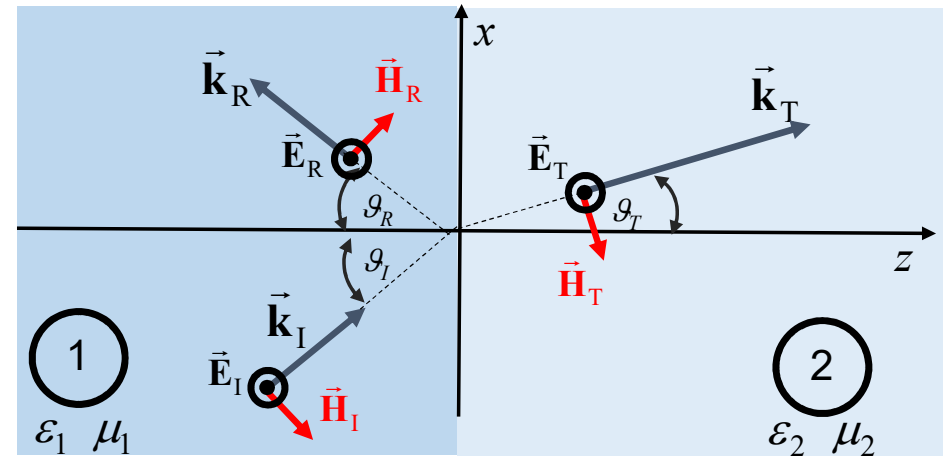
$$E_{1y}(z, x) = [E_I e^{-jk_{1z}z} + E_R e^{jk_{1z}z}] e^{-jk_x x} \quad (1)$$

$$E_{2y}(z, x) = E_T e^{-jk_{2z}z} e^{-jk_x x} \quad (2)$$

$$\Gamma_{\perp} \triangleq \frac{E_R}{E_I} \quad Z_1 = \frac{\omega \mu_1}{k_{1z}}$$

$$T_{\perp} \triangleq \frac{E_T}{E_I} \quad Z_2 = \frac{\omega \mu_2}{k_{2z}}$$

$$\Gamma_{\perp} = \frac{\cos \vartheta_1 - (\mu_1/\mu_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}{\cos \vartheta_1 + (\mu_1/\mu_2) \sqrt{(k_2/k_1)^2 - \sin^2 \vartheta_1}}$$



Incidence on a dielectric half-space

$$\begin{cases} 1 - \Gamma = \frac{Z_1}{Z_2} T \\ 1 + \Gamma = T \end{cases} \quad \Gamma = \frac{Z_2 - Z_1}{Z_1 + Z_2}$$

$$T = \frac{2Z_2}{Z_1 + Z_2}$$

$$k_1 = \omega \sqrt{\mu_1 \varepsilon_1}$$

$$k_2 = \omega \sqrt{\mu_2 \varepsilon_2}$$

$$k_x = k_1 \sin \vartheta_1$$

$$k_{1z} = k_1 \cos \vartheta_1$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin \vartheta_1 = k_2 \sin \vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2 \vartheta_1}$$

$[H_x, E_y, H_z]$ **Perpendicular Polarization** \perp

$$E_{1y}(z, x) = [E_I e^{-jk_{1z}z} + E_R e^{jk_{1z}z}] e^{-jk_x x} \quad (1)$$

$$E_{2y}(z, x) = E_T e^{-jk_{2z}z} e^{-jk_x x} \quad (2)$$

$$\Gamma_{\perp} \triangleq \frac{E_R}{E_I} \quad Z_1 = \frac{\omega \mu_1}{k_{1z}}$$

$$T_{\perp} \triangleq \frac{E_T}{E_I} \quad Z_2 = \frac{\omega \mu_2}{k_{2z}}$$

$$\Gamma_{\perp} = \frac{\cos \vartheta_1 - (\mu_1 / \mu_2) \sqrt{(k_2 / k_1)^2 - \sin^2 \vartheta_1}}{\cos \vartheta_1 + (\mu_1 / \mu_2) \sqrt{(k_2 / k_1)^2 - \sin^2 \vartheta_1}}$$

$[E_x, H_y, E_z]$ **Parallel Polarization** \parallel

$$H_{1y}(z, x) = [H_I e^{-jk_{1z}z} + H_R e^{jk_{1z}z}] e^{-jk_x x} \quad (1)$$

$$H_{2y}(z, x) = H_T e^{-jk_{2z}z} e^{-jk_x x} \quad (2)$$

$$\Gamma_{\parallel} \triangleq -\frac{H_R}{H_I} \quad Z_1 = \frac{k_{1z}}{\omega \varepsilon_1}$$

$$T_{\parallel} \triangleq \frac{Z_2 H_T}{Z_1 H_I} \quad Z_2 = \frac{k_{2z}}{\omega \varepsilon_2}$$

$$\Gamma_{\parallel} = -\frac{\cos \vartheta_1 - (\varepsilon_1 / \varepsilon_2) \sqrt{(k_2 / k_1)^2 - \sin^2 \vartheta_1}}{\cos \vartheta_1 + (\varepsilon_1 / \varepsilon_2) \sqrt{(k_2 / k_1)^2 - \sin^2 \vartheta_1}}$$