Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2021-2022 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence (PD)

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{H}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{k}} = k_x \hat{i}_x + k_y \hat{i}_y + k_z \hat{i}_z$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

$$\vec{\mathbf{r}} = x\hat{i}_x + y\hat{i}_y + z\hat{i}_z$$

Source-free

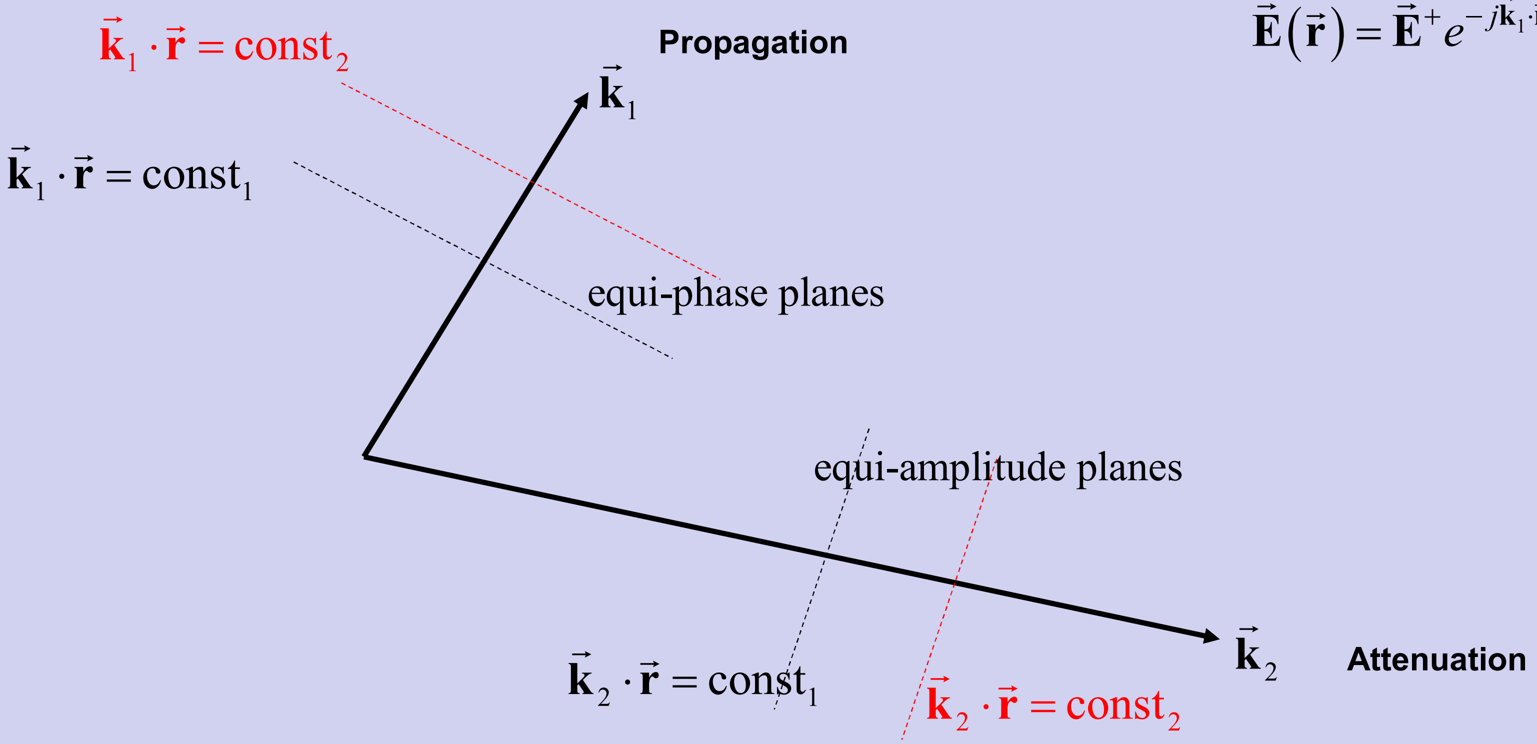
$$\begin{cases} \vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu\overrightarrow{\mathbf{H}} \\ \vec{\mathbf{k}} \times \overrightarrow{\mathbf{H}} = -\omega\varepsilon\vec{\mathbf{E}} \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}} \cdot \overrightarrow{\mathbf{H}} = 0 \end{cases}$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{k}} = \omega^2 \mu\varepsilon$$

$\vec{\mathbf{k}}$:propagation vector

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}_1 \cdot \vec{\mathbf{r}}} e^{-\vec{\mathbf{k}}_2 \cdot \vec{\mathbf{r}}}$$



When $\vec{\mathbf{k}}_1$ and $\vec{\mathbf{k}}_2$ are proportional, equi-amplitude and equi-phase planes become coincident: the plane wave is said HOMOGENEOUS

More generally, equi-amplitude and equi-phase planes may be not coincident: in this the plane wave is said NOT-HOMOGENEOUS

Plane Waves

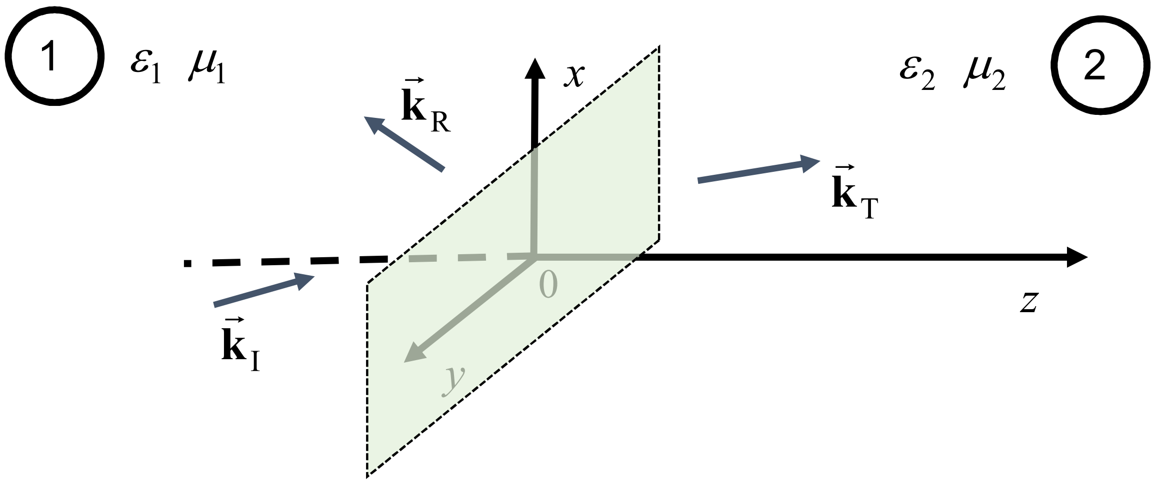
Incidence on a dielectric half-space

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x} e^{-jk_{\mathrm{I}y}y} e^{-jk_{\mathrm{I}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}\hat{i}_x + k_{\mathrm{I}y}\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}^2 + k_{\mathrm{I}y}^2 + k_{\mathrm{I}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x} e^{-jk_{\mathrm{R}y}y} e^{-jk_{\mathrm{R}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}\hat{i}_x + k_{\mathrm{R}y}\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}^2 + k_{\mathrm{R}y}^2 + k_{\mathrm{R}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x} e^{-jk_{\mathrm{T}y}y} e^{-jk_{\mathrm{T}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}\hat{i}_x + k_{\mathrm{T}y}\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}^2 + k_{\mathrm{T}y}^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$



$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

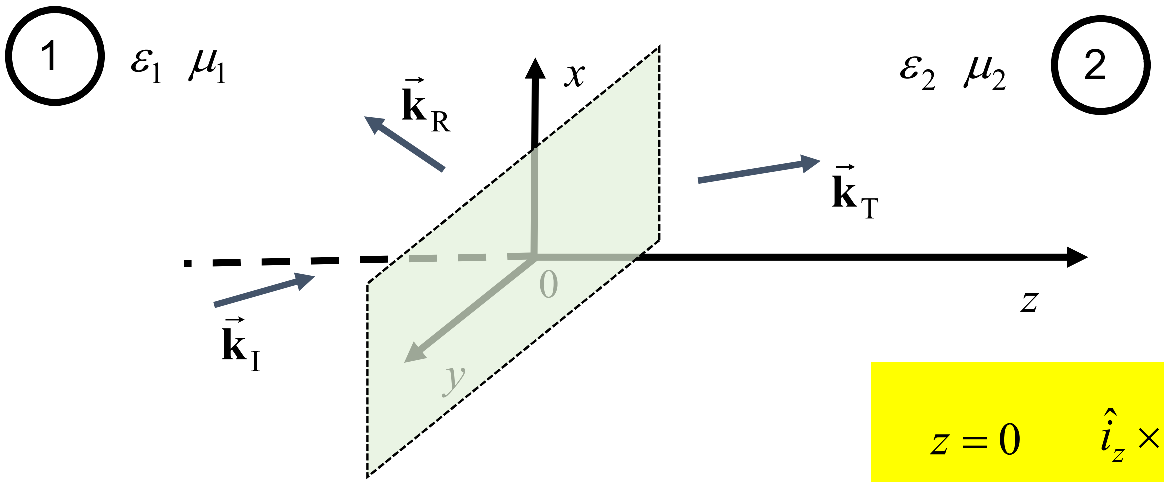
$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x} e^{-jk_{\mathrm{I}y}y} e^{-jk_{\mathrm{I}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}\hat{i}_x + k_{\mathrm{I}y}\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}^2 + k_{\mathrm{I}y}^2 + k_{\mathrm{I}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x} e^{-jk_{\mathrm{R}y}y} e^{-jk_{\mathrm{R}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}\hat{i}_x + k_{\mathrm{R}y}\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}^2 + k_{\mathrm{R}y}^2 + k_{\mathrm{R}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

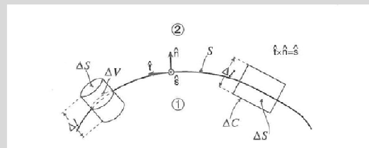
$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x} e^{-jk_{\mathrm{T}y}y} e^{-jk_{\mathrm{T}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}\hat{i}_x + k_{\mathrm{T}y}\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}^2 + k_{\mathrm{T}y}^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$



MEMO

Fields at boundaries

$$\hat{\mathbf{n}}\times(\vec{\mathbf{e}}_2 - \vec{\mathbf{e}}_1) = 0$$
$$\hat{\mathbf{n}}\times(\vec{\mathbf{h}}_2 - \vec{\mathbf{h}}_1) = \vec{\mathbf{j}}_s$$
$$(\vec{\mathbf{d}}_2 - \vec{\mathbf{d}}_1)\cdot\hat{\mathbf{n}} = \rho_s$$
$$(\vec{\mathbf{b}}_2 - \vec{\mathbf{b}}_1)\cdot\hat{\mathbf{n}} = 0$$
$$(\vec{\mathbf{j}}_2 - \vec{\mathbf{j}}_1)\cdot\hat{\mathbf{n}} = -\frac{\partial\rho_s}{\partial t}$$



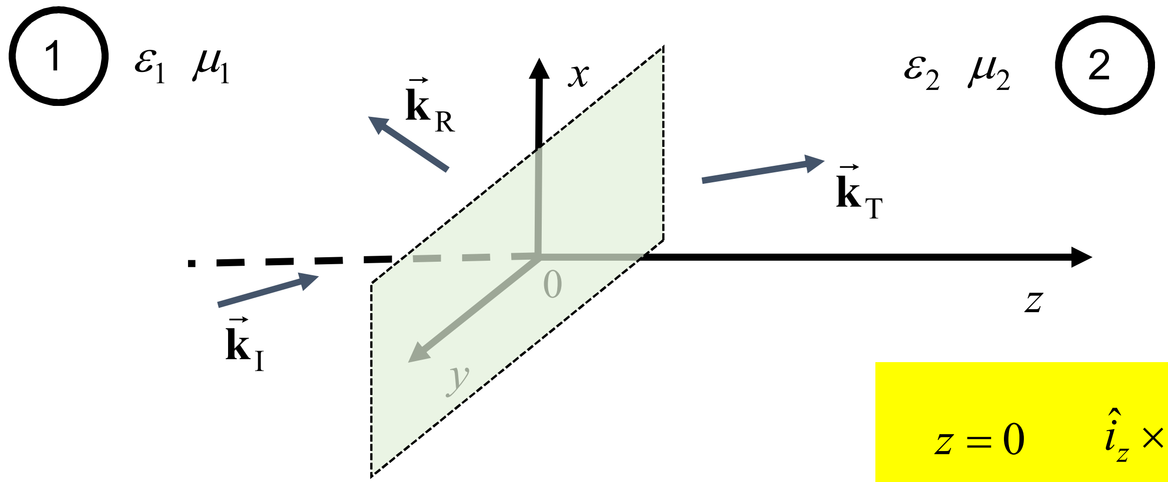
$$z = 0 \qquad \hat{i}_z\times\left(\vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1\right) = \mathbf{0} \qquad \hat{i}_z\times\vec{\mathbf{E}}_1 = \hat{i}_z\times\vec{\mathbf{E}}_2$$

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x} e^{-jk_{\mathrm{I}y}y} e^{-jk_{\mathrm{I}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}\hat{i}_x + k_{\mathrm{I}y}\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}^2 + k_{\mathrm{I}y}^2 + k_{\mathrm{I}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x} e^{-jk_{\mathrm{R}y}y} e^{-jk_{\mathrm{R}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}\hat{i}_x + k_{\mathrm{R}y}\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}^2 + k_{\mathrm{R}y}^2 + k_{\mathrm{R}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x} e^{-jk_{\mathrm{T}y}y} e^{-jk_{\mathrm{T}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}\hat{i}_x + k_{\mathrm{T}y}\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}^2 + k_{\mathrm{T}y}^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$



$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$z = 0 \qquad \hat{i}_z \times \left(\vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1\right) = \mathbf{0} \qquad \hat{i}_z \times \vec{\mathbf{E}}_1 = \hat{i}_z \times \vec{\mathbf{E}}_2$$

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x} e^{-jk_{\mathrm{I}y}y} e^{-jk_{\mathrm{I}z}z}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x} e^{-jk_{\mathrm{R}y}y} e^{-jk_{\mathrm{R}z}z}$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x} e^{-jk_{\mathrm{T}y}y} e^{-jk_{\mathrm{T}z}z}$$

$$z = 0 \qquad \hat{i}_z \times \vec{\mathbf{E}}_1 = \hat{i}_z \times \vec{\mathbf{E}}_2$$

$$z = 0 \qquad y = 0$$

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x}$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x}$$

$$\hat{i}_z \times \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x} + \hat{i}_z \times \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x} = \hat{i}_z \times \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x} \Rightarrow k_{\mathrm{I}x} = k_{Rx} = k_{\mathrm{T}x} = k_x$$

$$z = 0 \qquad x = 0$$

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}y}y}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}y}y}$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}z}z}$$

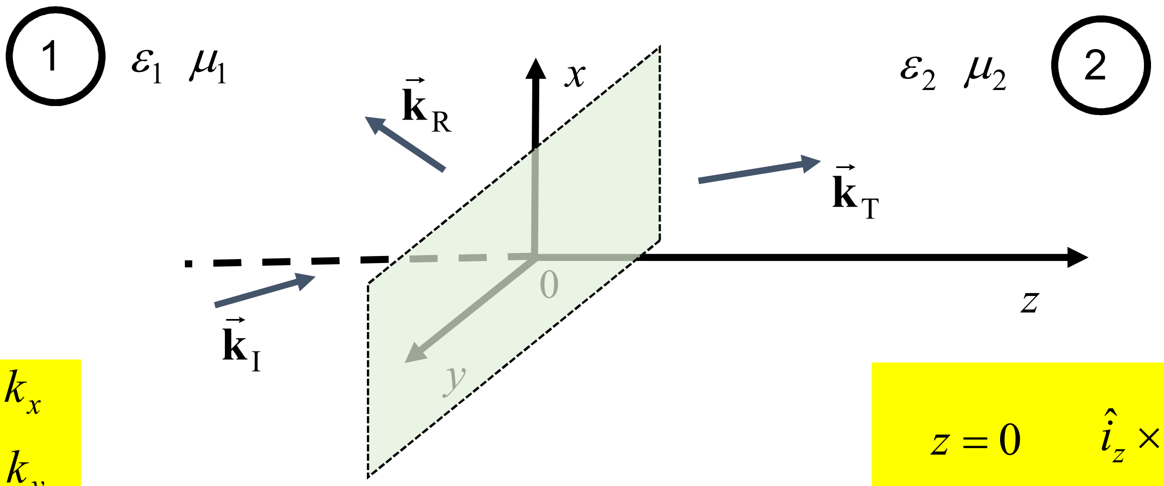
$$\hat{i}_z \times \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}y}y} + \hat{i}_z \times \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}y}y} = \hat{i}_z \times \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}y}y} \Rightarrow k_{\mathrm{I}y} = k_{Ry} = k_{\mathrm{T}y} = k_y$$

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x} e^{-jk_{\mathrm{I}y}y} e^{-jk_{\mathrm{I}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}\hat{i}_x + k_{\mathrm{I}y}\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}^2 + k_{\mathrm{I}y}^2 + k_{\mathrm{I}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x} e^{-jk_{\mathrm{R}y}y} e^{-jk_{\mathrm{R}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}\hat{i}_x + k_{\mathrm{R}y}\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}^2 + k_{\mathrm{R}y}^2 + k_{\mathrm{R}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x} e^{-jk_{\mathrm{T}y}y} e^{-jk_{\mathrm{T}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}\hat{i}_x + k_{\mathrm{T}y}\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}^2 + k_{\mathrm{T}y}^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$



$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$k_{\mathrm{I}x} = k_{Rx} = k_{\mathrm{T}x} = k_x$$

$$k_{\mathrm{I}y} = k_{Ry} = k_{\mathrm{T}y} = k_y$$

$$z = 0 \qquad \hat{i}_z \times \left(\vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1\right) = \mathbf{0} \qquad \hat{i}_z \times \vec{\mathbf{E}}_1 = \hat{i}_z \times \vec{\mathbf{E}}_2$$

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\rm I} e^{-j\vec{\mathbf{k}}_{\rm I}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\rm I} e^{-jk_{\rm Ix}x} e^{-jk_{\rm Iy}y} e^{-jk_{\rm Iz}z} \qquad \vec{\mathbf{k}}_{\rm I} = k_{\rm Ix}\hat{i}_x + k_{\rm Iy}\hat{i}_y + k_{\rm Iz}\hat{i}_z \qquad \vec{\mathbf{k}}_{\rm I}\cdot\vec{\mathbf{k}}_{\rm I} = k_{\rm Ix}^2 + k_{\rm Iy}^2 + k_{\rm Iz}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\rm R} e^{-j\vec{\mathbf{k}}_{\rm R}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\rm R} e^{-jk_{\rm Rx}x} e^{-jk_{\rm Ry}y} e^{-jk_{\rm Rz}z} \qquad \vec{\mathbf{k}}_{\rm R} = k_{\rm Rx}\hat{i}_x + k_{\rm Ry}\hat{i}_y + k_{\rm Rz}\hat{i}_z \qquad \vec{\mathbf{k}}_{\rm R}\cdot\vec{\mathbf{k}}_{\rm R} = k_{\rm Rx}^2 + k_{\rm Ry}^2 + k_{\rm Rz}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\rm T} e^{-j\vec{\mathbf{k}}_{\rm T}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\rm T} e^{-jk_{\rm Tx}x} e^{-jk_{\rm Ty}y} e^{-jk_{\rm Tz}z} \qquad \vec{\mathbf{k}}_{\rm T} = k_{\rm Tx}\hat{i}_x + k_{\rm Ty}\hat{i}_y + k_{\rm Tz}\hat{i}_z \qquad \vec{\mathbf{k}}_{\rm T}\cdot\vec{\mathbf{k}}_{\rm T} = k_{\rm Tx}^2 + k_{\rm Ty}^2 + k_{\rm Tz}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$

$$k_{\rm Ix} = k_{Rx} = k_{\rm Tx} = k_x$$

$$k_{\rm Iy} = k_{Ry} = k_{\rm Ty} = k_y$$

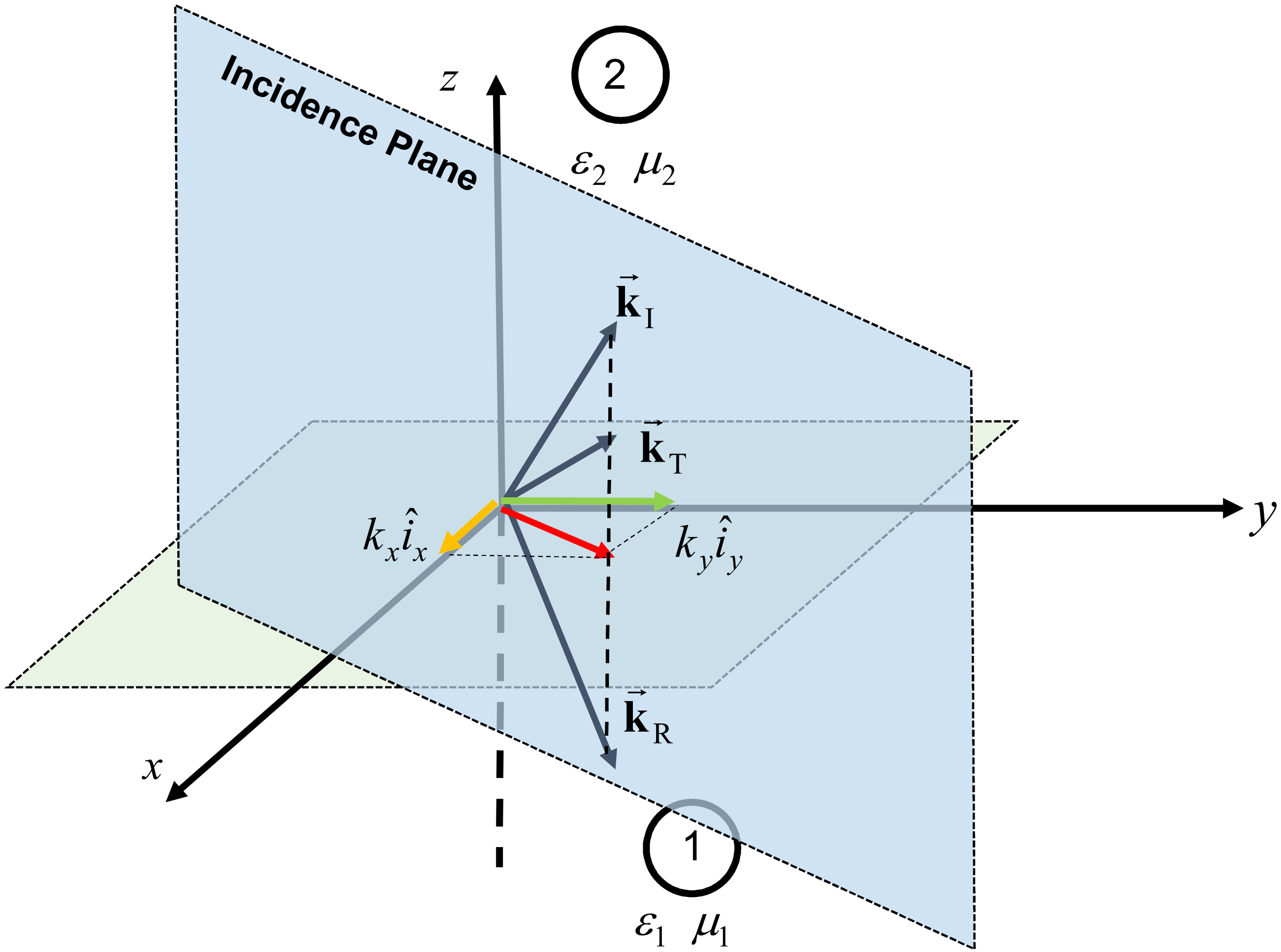
$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\rm I} e^{-j\vec{\mathbf{k}}_{\rm I}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\rm R} e^{-j\vec{\mathbf{k}}_{\rm R}\cdot\vec{\mathbf{r}}} \qquad \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\rm T} e^{-j\vec{\mathbf{k}}_{\rm T}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_{\rm I} e^{-j\vec{\mathbf{k}}_{\rm I}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\rm I} e^{-jk_x x} e^{-jk_y y} e^{-jk_{\rm Iz}z} \qquad \vec{\mathbf{k}}_{\rm I} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\rm Iz}\hat{i}_z \qquad \vec{\mathbf{k}}_{\rm I}\cdot\vec{\mathbf{k}}_{\rm I} = k_x^2 + k_y^2 + k_{\rm Iz}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\rm R} e^{-j\vec{\mathbf{k}}_{\rm R}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\rm R} e^{-jk_x x} e^{-jk_y y} e^{-jk_{\rm Rz}z} \qquad \vec{\mathbf{k}}_{\rm R} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\rm Rz}\hat{i}_z \qquad \vec{\mathbf{k}}_{\rm R}\cdot\vec{\mathbf{k}}_{\rm R} = k_x^2 + k_y^2 + k_{\rm Rz}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\rm T} e^{-j\vec{\mathbf{k}}_{\rm T}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\rm T} e^{-jk_x x} e^{-jk_y y} e^{-jk_{\rm Tz}z} \qquad \vec{\mathbf{k}}_{\rm T} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\rm Tz}\hat{i}_z \qquad \vec{\mathbf{k}}_{\rm T}\cdot\vec{\mathbf{k}}_{\rm T} = k_x^2 + k_y^2 + k_{\rm Tz}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$

Incidence on a dielectric half-space

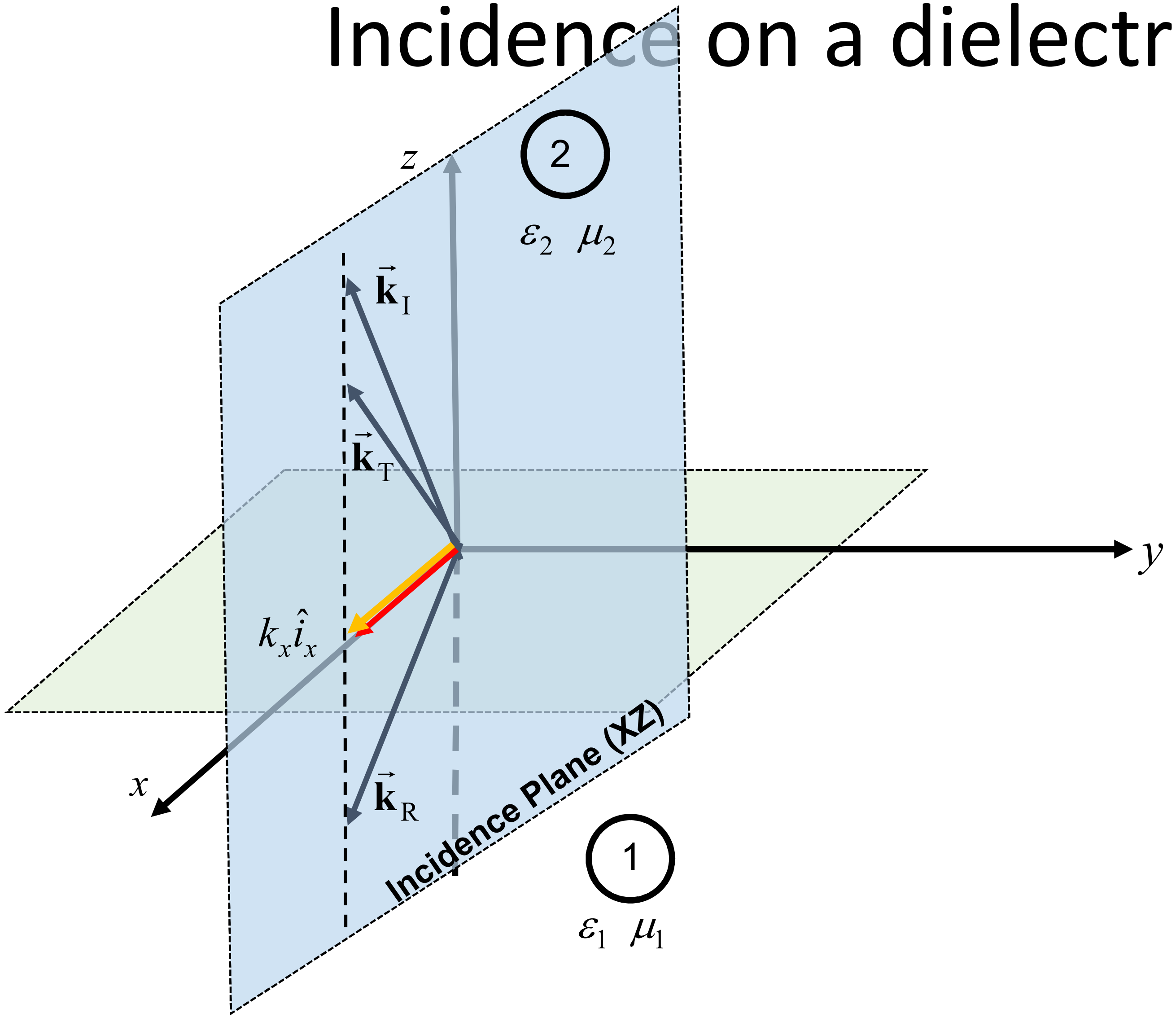


$$\vec{\mathbf{k}}_\mathrm{I} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_\mathrm{R} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_\mathrm{T} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z$$

Incidence on a dielectric half-space

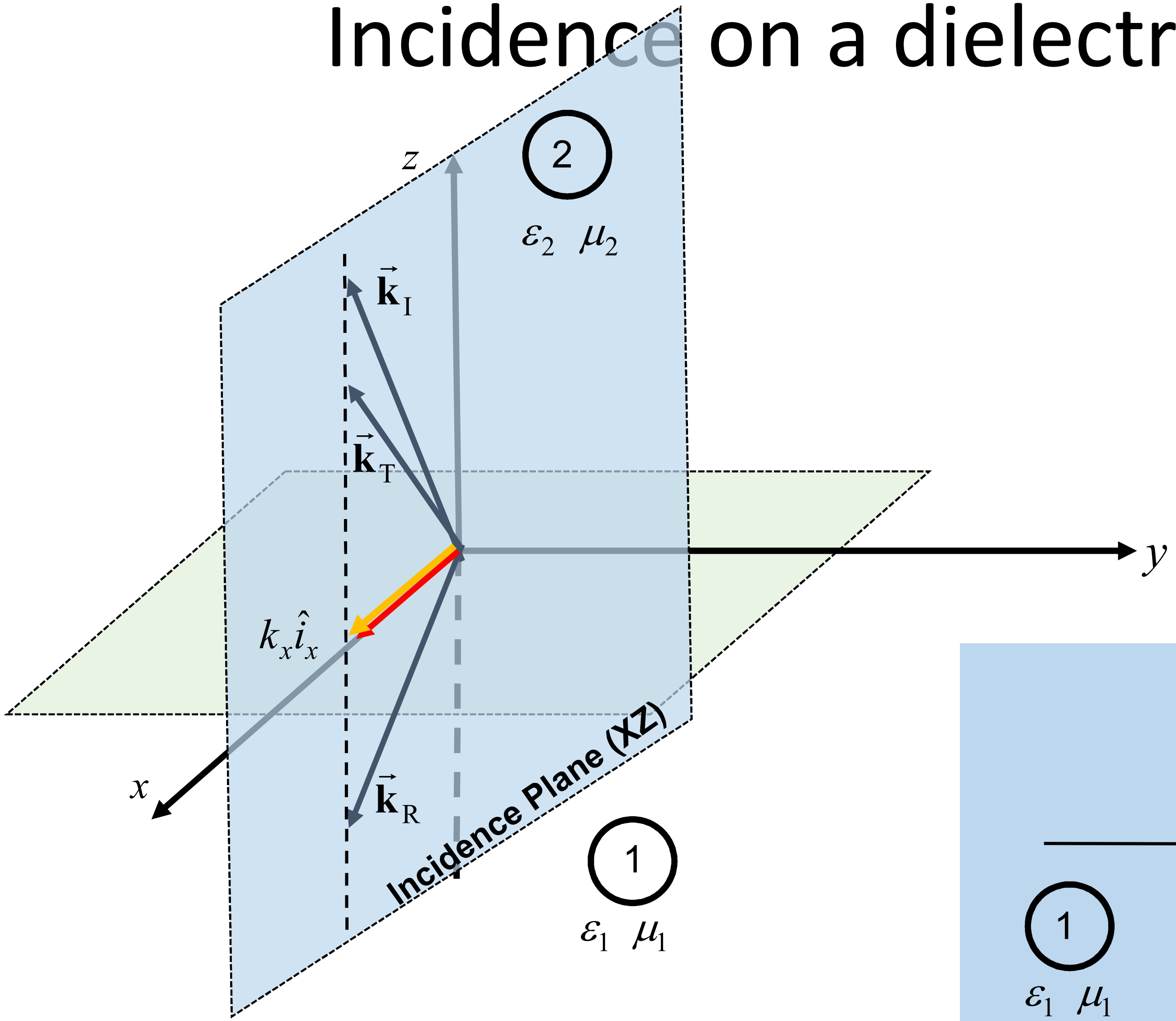


$$\vec{\mathbf{k}}_{\mathrm{I}} = k_x \hat{i}_x + k_y \hat{i}_y + k_{\mathrm{I}z} \hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{R}} = k_x \hat{i}_x + k_y \hat{i}_y + k_{\mathrm{R}z} \hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{T}} = k_x \hat{i}_x + k_y \hat{i}_y + k_{\mathrm{T}z} \hat{i}_z$$

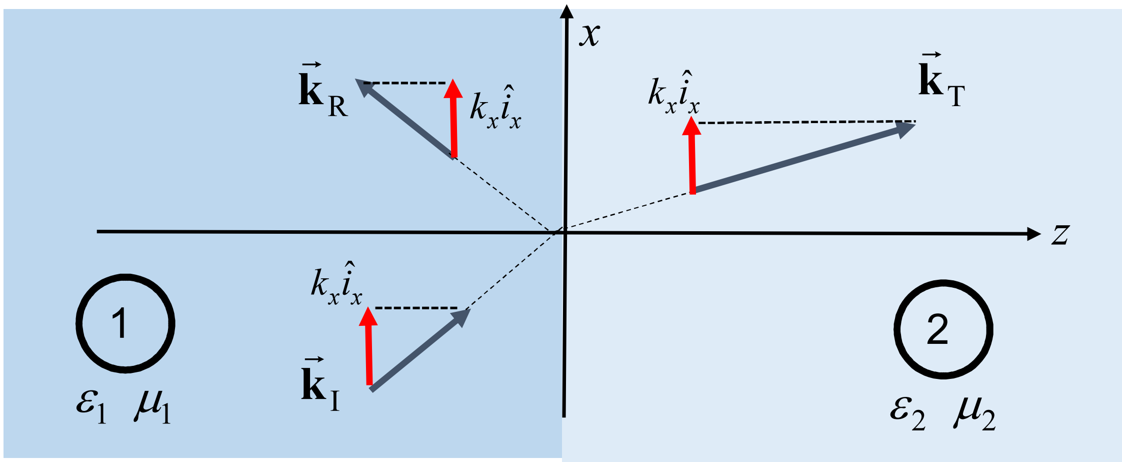
Incidence on a dielectric half-space



$$\vec{\mathbf{k}}_{\mathrm{I}} = k_x\hat{i}_x + k_{\mathrm{I}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{R}} = k_x\hat{i}_x + k_{\mathrm{R}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{T}} = k_x\hat{i}_x + k_{\mathrm{T}z}\hat{i}_z$$



Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{\mathrm{I}z} z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_x \hat{i}_x + k_{\mathrm{I}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_x^2 + k_{\mathrm{I}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{-jk_{\mathrm{R}z} z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_x \hat{i}_x + k_{\mathrm{R}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_x^2 + k_{\mathrm{R}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_{\mathrm{T}z} z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_x \hat{i}_x + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_x^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} \qquad \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

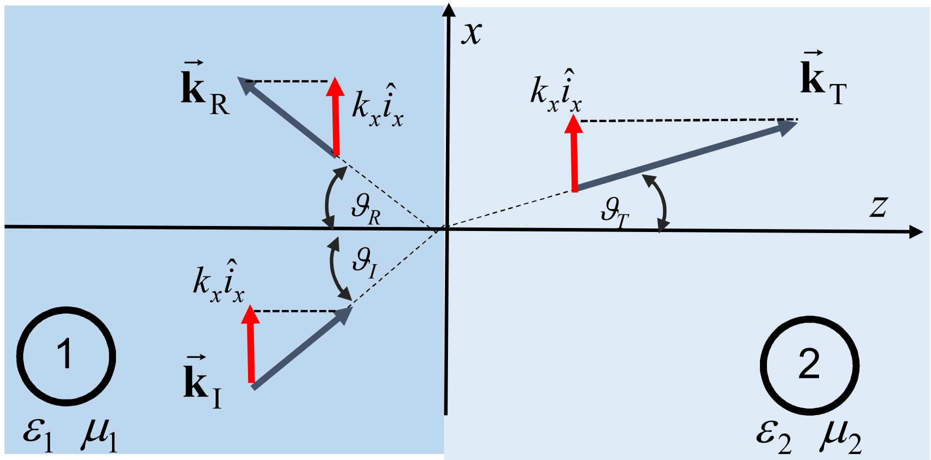
$$k_x = k_1 \sin\vartheta_I$$
$$k_x = k_1 \sin\vartheta_R$$
$$k_x = k_2 \sin\vartheta_T$$

$$\Rightarrow \vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$

Snellius Law

$$\sin\vartheta_{\mathrm{I}} = \frac{k_2}{k_1}\sin\vartheta_T \qquad \sin\vartheta_{\mathrm{I}} = \frac{\omega\sqrt{\mu_2\varepsilon_2}}{\omega\sqrt{\mu_1\varepsilon_1}}\sin\vartheta_T \qquad n = \frac{\sqrt{\mu_2\varepsilon_2}}{\sqrt{\mu_1\varepsilon_1}}$$



Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{\mathrm{Iz}} z}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{-jk_{\mathrm{Rz}} z}$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_{\mathrm{Tz}} z}$$

$$\vec{\mathbf{k}}_{\mathrm{I}} = k_x \hat{i}_x + k_{\mathrm{Iz}} \hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{R}} = k_x \hat{i}_x + k_{\mathrm{Rz}} \hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{T}} = k_x \hat{i}_x + k_{\mathrm{Tz}} \hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{I}} \cdot \vec{\mathbf{k}}_{\mathrm{I}} = k_x^2 + k_{\mathrm{Iz}}^2 = \omega^2 \mu_1 \varepsilon_1 = k_1^2$$

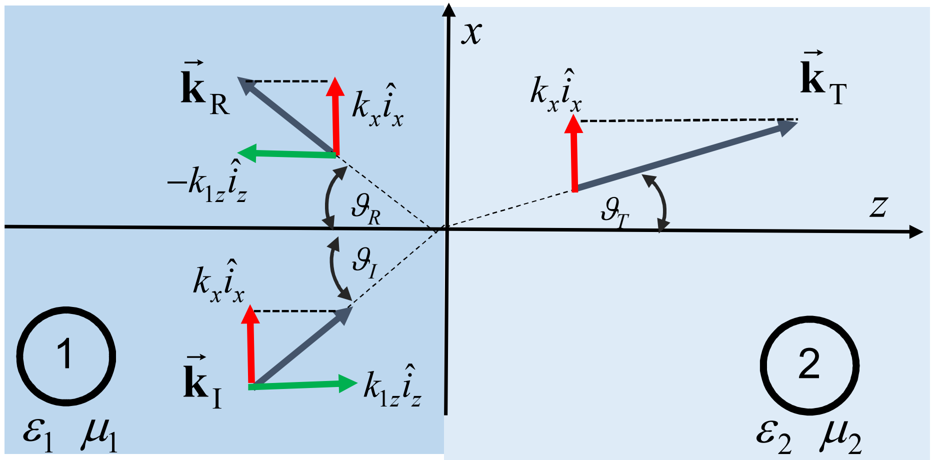
$$\vec{\mathbf{k}}_{\mathrm{R}} \cdot \vec{\mathbf{k}}_{\mathrm{R}} = k_x^2 + k_{\mathrm{Rz}}^2 = \omega^2 \mu_1 \varepsilon_1 = k_1^2$$

$$\vec{\mathbf{k}}_{\mathrm{T}} \cdot \vec{\mathbf{k}}_{\mathrm{T}} = k_x^2 + k_{\mathrm{Tz}}^2 = \omega^2 \mu_2 \varepsilon_2 = k_2^2$$

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} \qquad \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$k_x = k_1 \sin\vartheta_I$$
$$k_x = k_1 \sin\vartheta_R$$
$$k_x = k_2 \sin\vartheta_T$$

$$\vartheta_I = \vartheta_R$$
$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$



$$k_{\mathrm{Iz}}^2 = k_{\mathrm{Rz}}^2 \Rightarrow k_{\mathrm{Iz}} = -k_{\mathrm{Rz}} = k_{1z} \Rightarrow \begin{cases} \vec{\mathbf{k}}_{\mathrm{I}} = k_x \hat{i}_x + k_{1z} \hat{i}_z \\ \vec{\mathbf{k}}_{\mathrm{R}} = k_x \hat{i}_x - k_{1z} \hat{i}_z \end{cases}$$

Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_{\mathrm{T}z} z}$$

$$\vec{\mathbf{k}}_{\mathrm{I}} = k_x \hat{i}_x + k_{1z}\hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{R}} = k_x \hat{i}_x - k_{1z}\hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{T}} = k_x \hat{i}_x + k_{\mathrm{T}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_x^2 + k_{1z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_x^2 + k_{1z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_x^2 + k_{\mathrm{Tz}}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} \qquad \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$k_x = k_1 \sin\vartheta_I$$

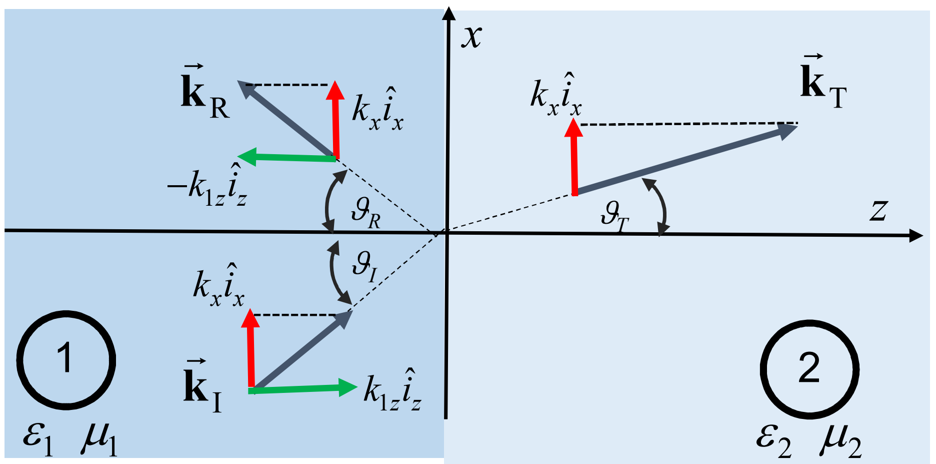
$$k_x = k_1 \sin\vartheta_R$$

$$k_x = k_2 \sin\vartheta_T$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$

$$k_{\mathrm{Iz}}^2 = k_{\mathrm{Rz}}^2 \Rightarrow k_{\mathrm{Iz}} = -k_{\mathrm{Rz}} = k_{1\mathrm{z}} \Rightarrow \begin{cases} \vec{\mathbf{k}}_{\mathrm{I}} = k_x \hat{i}_x + k_{1z}\hat{i}_z \\ \vec{\mathbf{k}}_{\mathrm{R}} = k_x \hat{i}_x - k_{1z}\hat{i}_z \end{cases}$$



Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{1z} z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_x \hat{i}_x + k_{1z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_x^2 + k_{1z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{jk_{1z} z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_x \hat{i}_x - k_{1z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_x^2 + k_{1z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

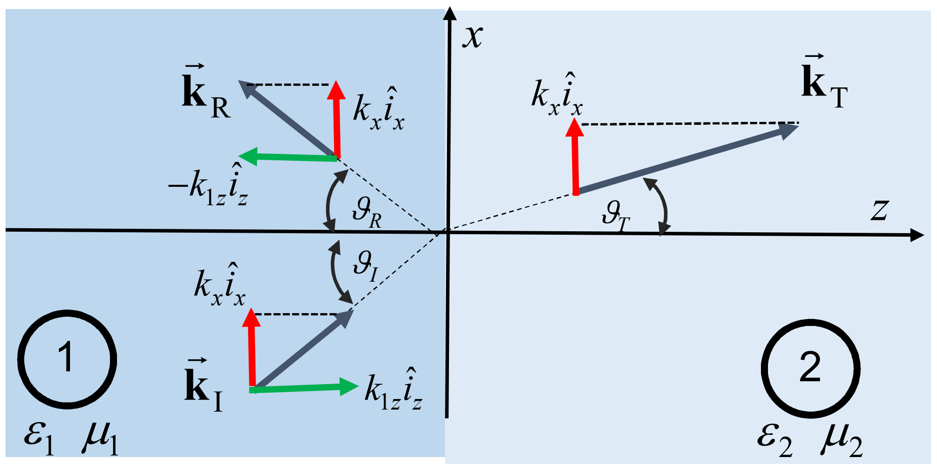
$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_{\mathrm{T}z} z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_x \hat{i}_x + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_x^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} \qquad \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$k_x = k_1 \sin\vartheta_I$$
$$k_x = k_1 \sin\vartheta_R$$
$$k_x = k_2 \sin\vartheta_T$$

$$k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$
$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$



Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{1z} z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_x \hat{i}_x + k_{1z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_x^2 + k_{1z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{jk_{1z} z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_x \hat{i}_x - k_{1z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_x^2 + k_{1z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_{2z} z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_x \hat{i}_x + k_{2z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_x^2 + k_{2z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} \qquad \vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

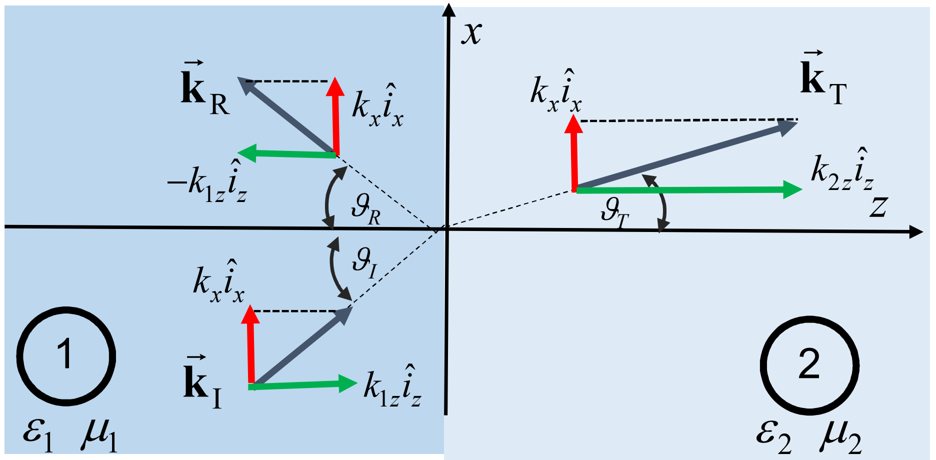
$$k_x = k_1 \sin\vartheta_I$$
$$k_x = k_1 \sin\vartheta_R$$
$$k_x = k_2 \sin\vartheta_T$$

$$k_{1z} = k_1 \cos\vartheta_I$$

$$\vartheta_I = \vartheta_R$$
$$k_1 \sin\vartheta_{\mathrm{I}} = k_2 \sin\vartheta_T$$

$$k_{2z} = k_2 \cos\vartheta_{\mathrm{T}}$$

$$k_{2z} = \sqrt{k_2^2 - k_x^2} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

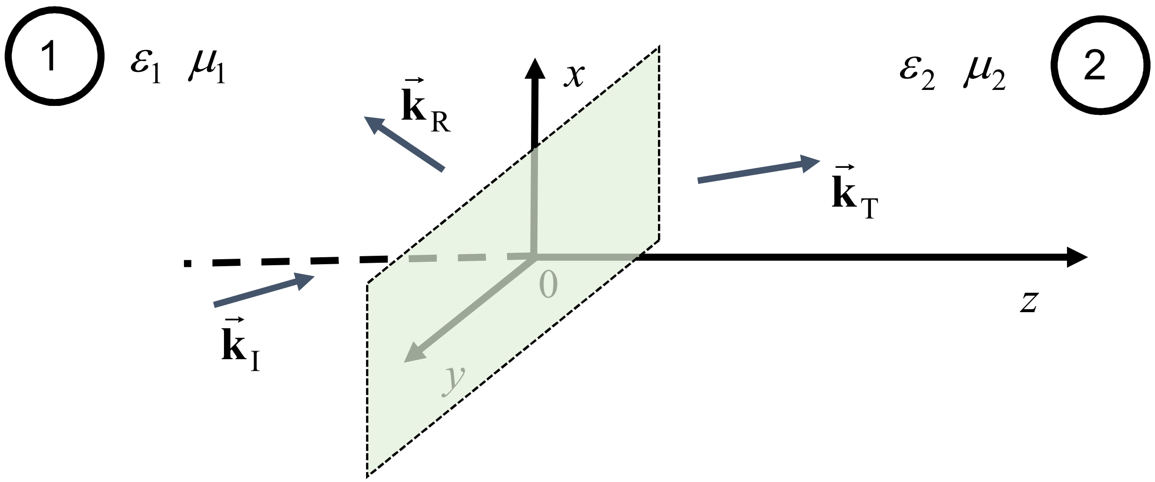


Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_{\mathrm{I}x}x} e^{-jk_{\mathrm{I}y}y} e^{-jk_{\mathrm{I}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}\hat{i}_x + k_{\mathrm{I}y}\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{k}}_{\mathrm{I}} = k_{\mathrm{I}x}^2 + k_{\mathrm{I}y}^2 + k_{\mathrm{I}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_{\mathrm{R}x}x} e^{-jk_{\mathrm{R}y}y} e^{-jk_{\mathrm{R}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}\hat{i}_x + k_{\mathrm{R}y}\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{k}}_{\mathrm{R}} = k_{\mathrm{R}x}^2 + k_{\mathrm{R}y}^2 + k_{\mathrm{R}z}^2 = \omega^2\mu_1\varepsilon_1 = k_1^2$$

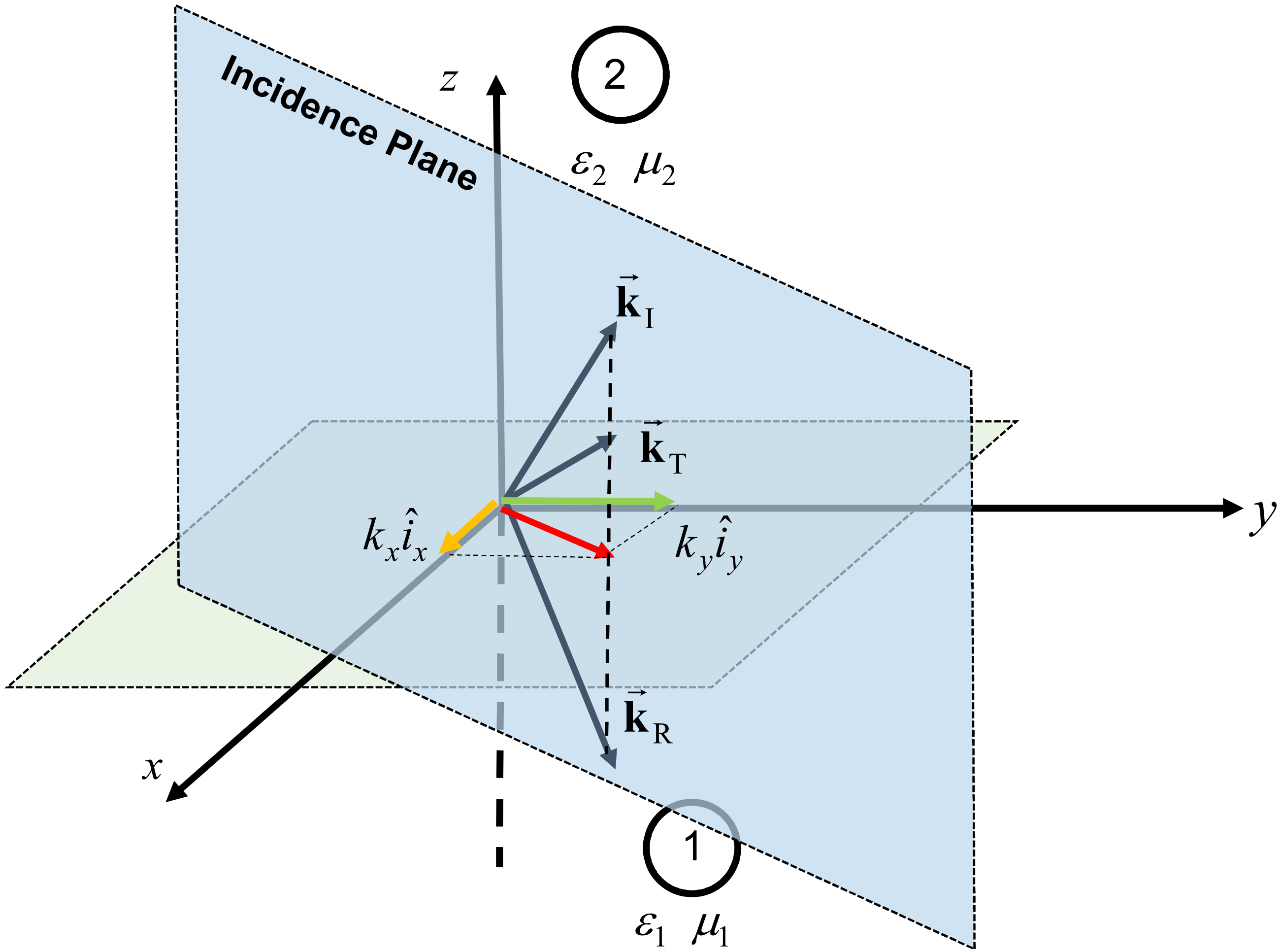
$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_{\mathrm{T}x}x} e^{-jk_{\mathrm{T}y}y} e^{-jk_{\mathrm{T}z}z} \qquad \vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}\hat{i}_x + k_{\mathrm{T}y}\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z \qquad \vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{k}}_{\mathrm{T}} = k_{\mathrm{T}x}^2 + k_{\mathrm{T}y}^2 + k_{\mathrm{T}z}^2 = \omega^2\mu_2\varepsilon_2 = k_2^2$$



$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

Incidence on a dielectric half-space

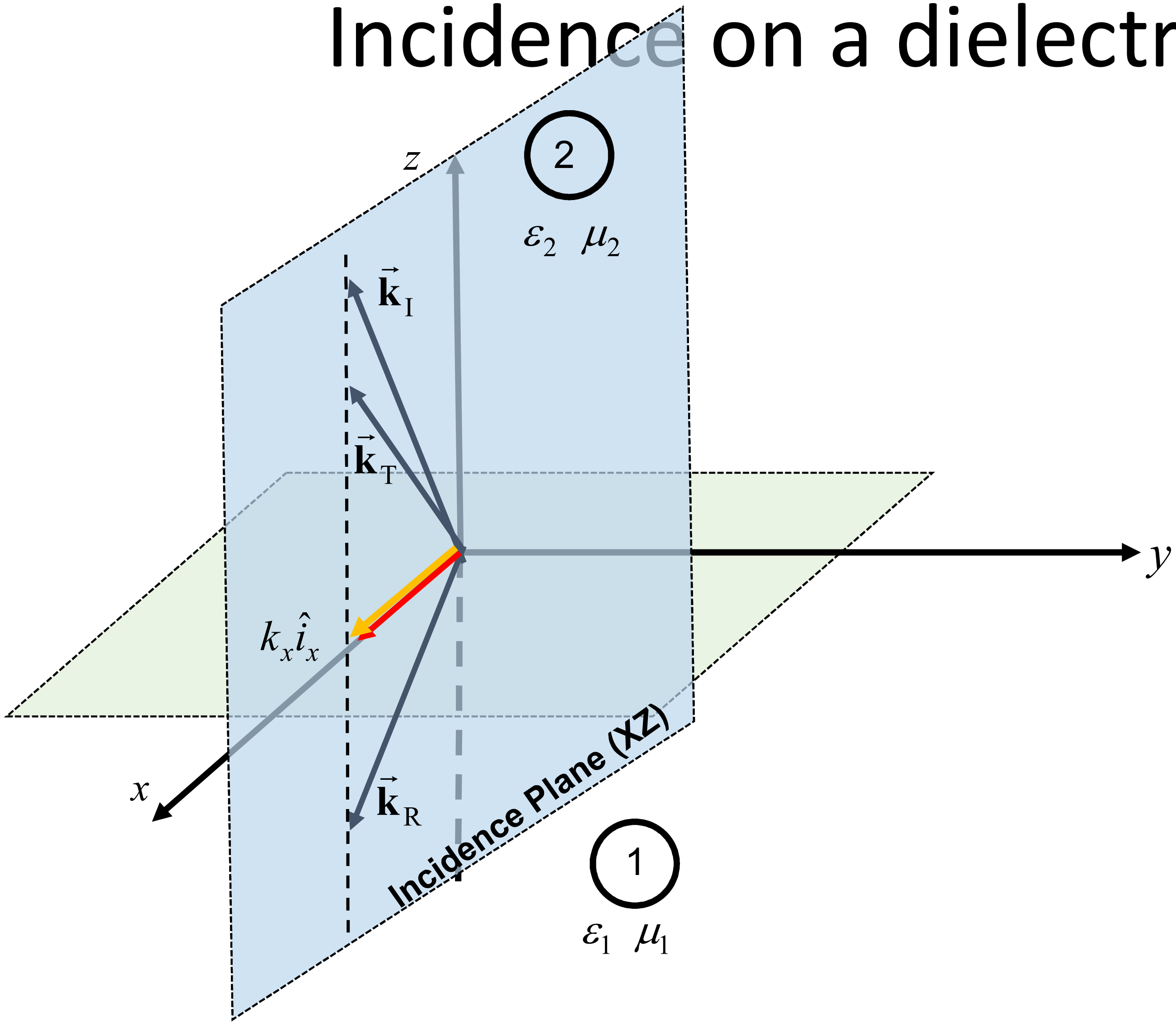


$$\vec{\mathbf{k}}_{\mathrm{I}} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{I}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{R}} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{R}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{T}} = k_x\hat{i}_x + k_y\hat{i}_y + k_{\mathrm{T}z}\hat{i}_z$$

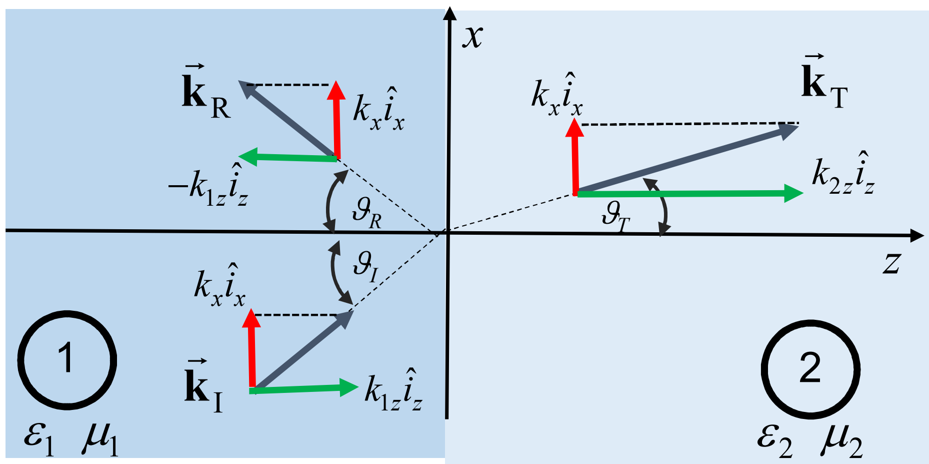
Incidence on a dielectric half-space



$$\vec{\mathbf{k}}_{\mathrm{I}} = k_x\hat{i}_x + k_{\mathrm{I}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{R}} = k_x\hat{i}_x + k_{\mathrm{R}z}\hat{i}_z$$

$$\vec{\mathbf{k}}_{\mathrm{T}} = k_x\hat{i}_x + k_{\mathrm{T}z}\hat{i}_z$$



Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{I} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{R} e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{R} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{T} e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

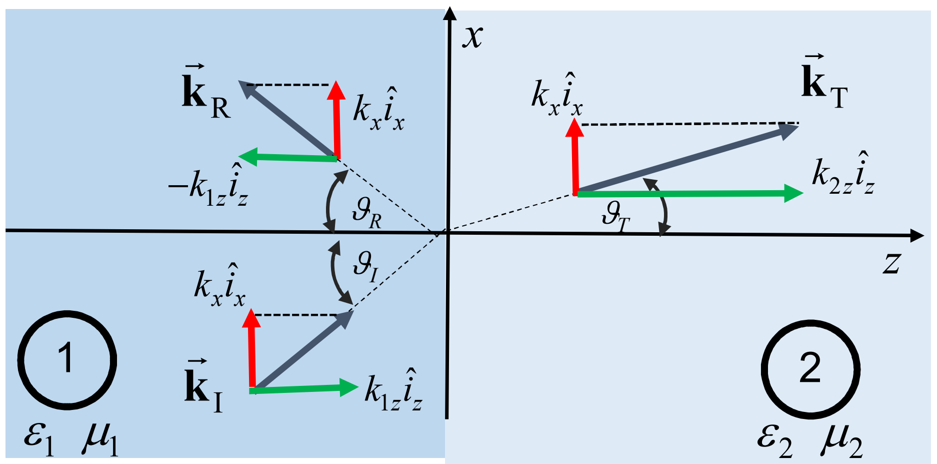
$$k_x = k_1 \sin\vartheta_\mathrm{I}$$

$$k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$



Incidence on a dielectric half-space

$$\frac{\partial}{\partial y} = 0$$

$$\frac{\partial}{\partial x} \to -jk_x$$

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{I} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{R} e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{R} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{T} e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I}$$

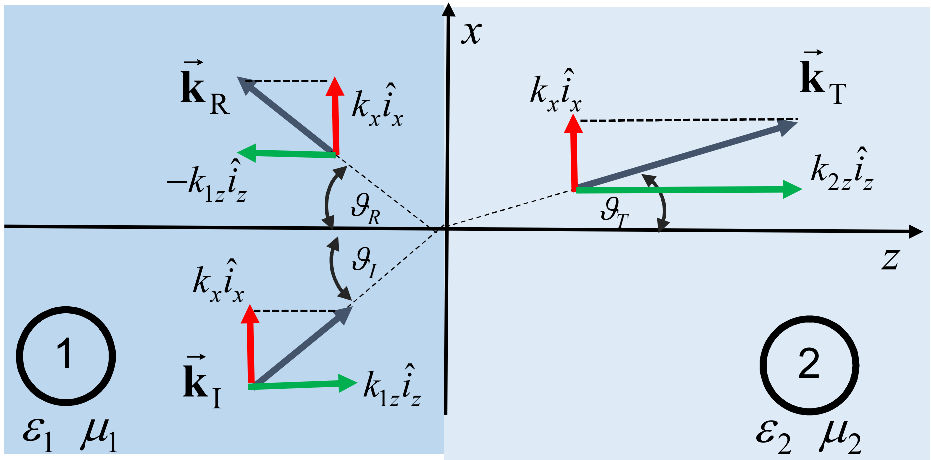
$$k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$$\frac{\partial}{\partial x}\vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} = \frac{\partial}{\partial x}\vec{\mathbf{E}}_\mathrm{I} e^{-jk_x x} e^{-jk_{1z} z} = -jk_x \vec{\mathbf{E}}_\mathrm{I} e^{-jk_x x} e^{-jk_{1z} z} = -jk_x \vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}}$$



Incidence on a dielectric half-space

$$\frac{\partial}{\partial y} = 0$$

$$\frac{\partial}{\partial x} \rightarrow -jk_x$$

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{I} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{R} e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{R} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{T} e^{-jk_x x} e^{-jk_{2z} z}$$

$$k_1 = \omega\sqrt{\mu_1\varepsilon_1}$$

$$k_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$k_x = k_1 \sin\vartheta_\mathrm{I}$$

$$k_{1z} = k_1 \cos\vartheta_\mathrm{I}$$

$$\vartheta_I = \vartheta_R$$

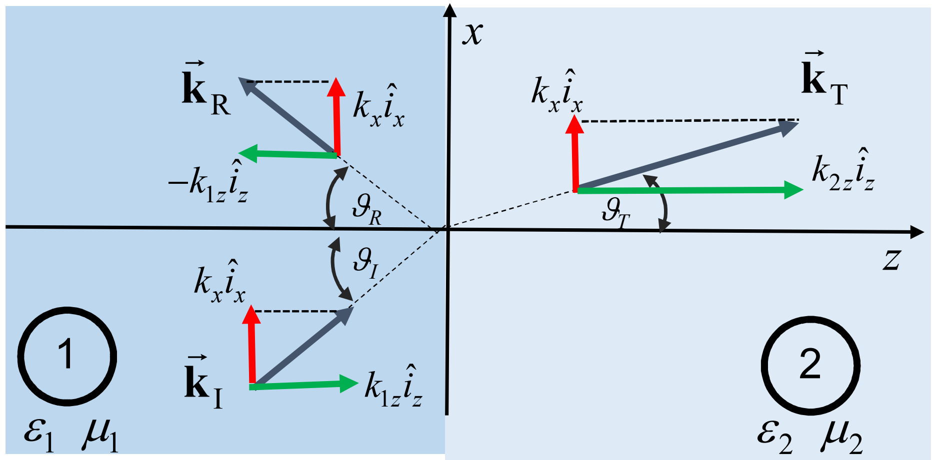
$$k_1 \sin\vartheta_\mathrm{I} = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_\mathrm{I}}$$

$$\nabla\times\vec{\mathbf{E}} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{i}_y + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{i}_z$$

$$\nabla\times\vec{\mathbf{E}} = \left(-\frac{\partial E_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial E_x}{\partial z} + jk_x E_z\right)\hat{i}_y + \left(-jk_x E_y\right)\hat{i}_z$$

$$\nabla\times\vec{\mathbf{H}} = \left(-\frac{\partial H_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial H_x}{\partial z} + jk_x H_z\right)\hat{i}_y + \left(-jk_x H_y\right)\hat{i}_z$$



Incidence on a dielectric half-space

$$\begin{cases} \nabla \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \end{cases}$$

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_{2z} z}$$

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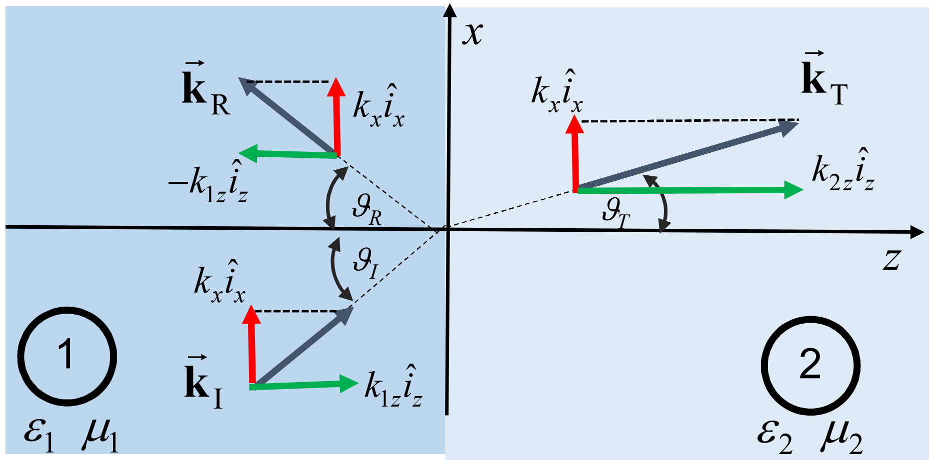
$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_{\mathrm{I}}}$$

$$\nabla \times \vec{\mathbf{E}} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{i}_y + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{i}_z$$

$$\nabla \times \vec{\mathbf{E}} = \left(-\frac{\partial E_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial E_x}{\partial z} + jk_x E_z\right)\hat{i}_y + \left(-jk_x E_y\right)\hat{i}_z$$

$$\nabla \times \vec{\mathbf{H}} = \left(-\frac{\partial H_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial H_x}{\partial z} + jk_x H_z\right)\hat{i}_y + \left(-jk_x H_y\right)\hat{i}_z$$



Incidence on a dielectric half-space

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$$\nabla\times\vec{\mathbf{E}} = -\frac{\partial E_y}{\partial z}\hat{i}_x + \left(\frac{\partial E_x}{\partial z} + jk_x E_z\right)\hat{i}_y - jk_x E_y \hat{i}_z$$

$$\nabla\times\vec{\mathbf{H}} = -\frac{\partial H_y}{\partial z}\hat{i}_x + \left(\frac{\partial H_x}{\partial z} + jk_x H_z\right)\hat{i}_y - jk_x H_y \hat{i}_z$$

Incidence on a dielectric half-space

$$\begin{cases}\nabla\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}}\\ \nabla\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}}\end{cases}$$

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$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_\mathrm{I} e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{I} e^{-jk_x x} e^{-jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{R} e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{R} e^{-jk_x x} e^{jk_{1z} z}$$

$$\vec{\mathbf{E}}_\mathrm{T} e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_\mathrm{T} e^{-jk_x x} e^{-jk_{2z} z}$$

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$$\nabla\times\vec{\mathbf{E}} = -\frac{\partial E_y}{\partial z}\hat{i}_x + \left(\frac{\partial E_x}{\partial z} + jk_x E_z\right)\hat{i}_y - jk_x E_y \hat{i}_z$$

$$-j\omega\mu\vec{\mathbf{H}} = -j\omega\mu H_x\hat{i}_x - j\omega\mu H_y\hat{i}_y - j\omega\mu H_z\hat{i}_z$$

$$-\frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial E_x}{\partial z} + jk_x E_z = -j\omega\mu H_y$$

$$-jk_x E_y = -j\omega\mu H_z$$

$$\nabla\times\vec{\mathbf{H}} = -\frac{\partial H_y}{\partial z}\hat{i}_x + \left(\frac{\partial H_x}{\partial z} + jk_x H_z\right)\hat{i}_y - jk_x H_y \hat{i}_z$$

$$j\omega\varepsilon\vec{\mathbf{E}} = j\omega\varepsilon E_x\hat{i}_x + j\omega\varepsilon E_y\hat{i}_y + j\omega\varepsilon E_z\hat{i}_z$$

$$-\frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x$$

$$\frac{\partial H_x}{\partial z} + jk_x H_z = j\omega\varepsilon E_y$$

$$-jk_x H_y = j\omega\varepsilon E_z$$

Incidence on a dielectric half-space

$$\nabla \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}}$$
$$\nabla \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}}$$

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$
$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{I}} e^{-jk_x x} e^{-jk_{1z} z}$$
$$\vec{\mathbf{E}}_{\mathrm{R}} e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{R}} e^{-jk_x x} e^{jk_{1z} z}$$
$$\vec{\mathbf{E}}_{\mathrm{T}} e^{-j\vec{\mathbf{k}}_{\mathrm{T}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}_{\mathrm{T}} e^{-jk_x x} e^{-jk_{2z} z}$$

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$$-\frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$
$$\frac{\partial E_x}{\partial z} + jk_x E_z = -j\omega\mu H_y$$
$$-jk_x E_y = -j\omega\mu H_z$$

$$-\frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x$$
$$\frac{\partial H_x}{\partial z} + jk_x H_z = j\omega\varepsilon E_y$$
$$-jk_x H_y = j\omega\varepsilon E_z$$

Incidence on a dielectric half-space

$$\begin{cases}\nabla \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}}\end{cases}$$

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_{\mathrm{I}} e^{-j\vec{\mathbf{k}}_{\mathrm{I}}\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_{\mathrm{R}}\cdot\vec{\mathbf{r}}}$$

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$[H_x, E_y, H_z]$ Perpendicular Polarization $\perp$

$$-\frac{\partial E_y}{\partial z} = -j\omega\mu H_x$$

$$\frac{\partial H_x}{\partial z} + jk_x H_z = j\omega\varepsilon E_y$$

$$-jk_x E_y = -j\omega\mu H_z$$

$[E_x, H_y, E_z]$ Parallel Polarization $\parallel$

$$-\frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x$$

$$\frac{\partial E_x}{\partial z} + jk_x E_z = -j\omega\mu H_y$$

$$-jk_x H_y = j\omega\varepsilon E_z$$

Incidence on a dielectric half-space

$$\begin{cases}\vec{\mathbf{k}}\times\vec{\mathbf{E}}=\omega\mu\vec{\mathbf{H}}\\ \vec{\mathbf{k}}\times\vec{\mathbf{H}}=-\omega\varepsilon\vec{\mathbf{E}}\\ \vec{\mathbf{k}}\cdot\vec{\mathbf{E}}=0\\ \vec{\mathbf{k}}\cdot\vec{\mathbf{H}}=0\end{cases}$$

$$\vec{\mathbf{E}}_1=\vec{\mathbf{E}}_\mathrm{I}e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}}+\vec{\mathbf{E}}_Re^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2=\vec{\mathbf{E}}_\mathrm{T}e^{-j\vec{\mathbf{k}}_\mathrm{T}\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_\mathrm{I}e^{-j\vec{\mathbf{k}}_\mathrm{I}\cdot\vec{\mathbf{r}}}=\vec{\mathbf{E}}_\mathrm{I}e^{-jk_xx}e^{-jk_{1z}z}$$

$$\vec{\mathbf{E}}_\mathrm{R}e^{-j\vec{\mathbf{k}}_\mathrm{R}\cdot\vec{\mathbf{r}}}=\vec{\mathbf{E}}_\mathrm{R}e^{-jk_xx}e^{jk_{1z}z}$$

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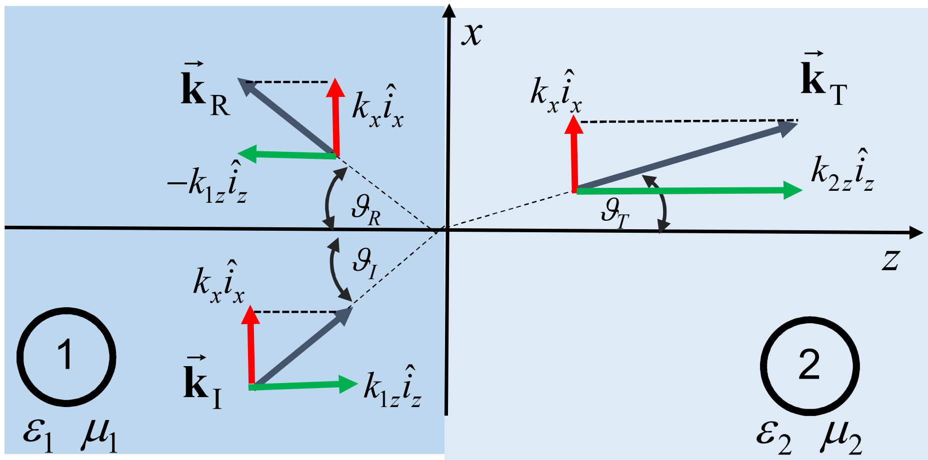
Perpendicular Polarization ⊥

$$\left[H_x,E_y,H_z\right]$$

Parallel Polarization ||

$$\left[E_x,H_y,E_z\right]$$



Incidence on a dielectric half-space

$$\vec{\mathbf{E}}_1 = \vec{\mathbf{E}}_I e^{-j\vec{\mathbf{k}}_I\cdot\vec{\mathbf{r}}} + \vec{\mathbf{E}}_R e^{-j\vec{\mathbf{k}}_R\cdot\vec{\mathbf{r}}}$$

$$\vec{\mathbf{E}}_2 = \vec{\mathbf{E}}_T e^{-j\vec{\mathbf{k}}_T\cdot\vec{\mathbf{r}}}$$

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$$\vartheta_I = \vartheta_R$$

$$k_1 \sin\vartheta_I = k_2 \sin\vartheta_T$$

$$k_{2z} = \sqrt{k_2^2 - k_1^2 \sin^2\vartheta_I}$$

Perpendicular Polarization $\perp$

$$\left[H_x, E_y, H_z\right]$$

Parallel Polarization ||

$$\left[E_x, H_y, E_z\right]$$