Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2021-2022 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

Phasor Domain (PD)

Fourier Domain (FD)

Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence (PD)

Plane Waves

General expression of plane waves (PD)

Plane Waves (Spectral Domains)

Source-free

Medium

- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

$$k = \omega\sqrt{\mu\varepsilon} \qquad k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Phasor Domain

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Fourier Domain

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega \varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z,\omega) = E_x^+(\omega) e^{-jkz} + E_x^-(\omega) e^{jkz}$$

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$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \end{cases}$$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

Plane Waves (Spectral Domains)

$$\left\{E_x, H_y\right\}$$

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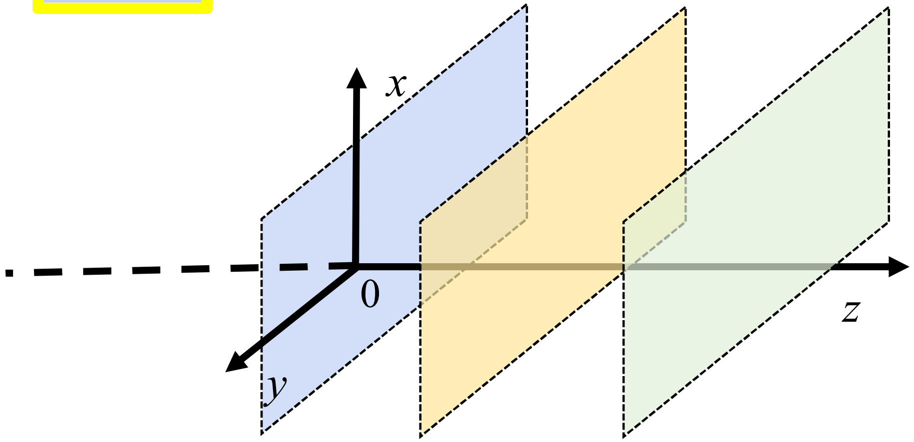

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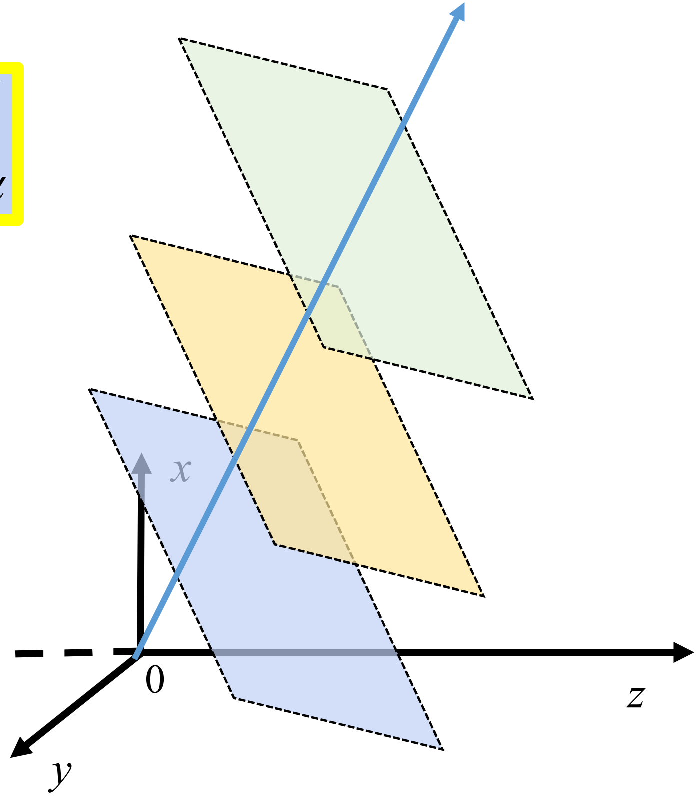
Phasor Domain



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$\{E_y, H_x\}$
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General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{H}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{k}} = k_x \hat{i}_x + k_y \hat{i}_y + k_z \hat{i}_z$$

$$\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

$$\vec{\mathbf{r}} = x\hat{i}_x + y\hat{i}_y + z\hat{i}_z$$

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$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

$$\frac{\partial}{\partial x} \to -jk_x \qquad \frac{\partial}{\partial y} \to -jk_y \qquad \frac{\partial}{\partial z} \to -jk_z$$

$$\frac{\partial}{\partial x}\vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \frac{\partial}{\partial x}\vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z} = -jk_x \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z} = -jk_x \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}}$$

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^+ e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^+ e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

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$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

Source-free

$$\begin{cases} \nabla\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \\ \nabla\cdot\vec{\mathbf{E}} = 0 \\ \nabla\cdot\vec{\mathbf{H}} = 0 \end{cases}$$

$$\frac{\partial}{\partial x} \to -jk_x \qquad \frac{\partial}{\partial y} \to -jk_y \qquad \frac{\partial}{\partial z} \to -jk_z$$

⬇

$$\nabla\times\vec{\mathbf{E}} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ -jk_x & -jk_y & -jk_z \\ E_x & E_y & E_z \end{vmatrix} = -j\vec{\mathbf{k}}\times\vec{\mathbf{E}}$$

$$\vec{\mathbf{a}}\times\vec{\mathbf{b}} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$\nabla\times\vec{\mathbf{E}} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^+ e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^+ e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

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Source-free

$$\begin{cases} \nabla\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \\ \nabla\cdot\vec{\mathbf{E}} = 0 \\ \nabla\cdot\vec{\mathbf{H}} = 0 \end{cases} \rightarrow \begin{cases} -j\vec{\mathbf{k}}\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ -j\vec{\mathbf{k}}\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \\ \ldots\ldots \\ \ldots\ldots\ldots \end{cases}$$

$$\frac{\partial}{\partial x} \rightarrow -jk_x \qquad \frac{\partial}{\partial y} \rightarrow -jk_y \qquad \frac{\partial}{\partial z} \rightarrow -jk_z$$

$$\Downarrow$$

$$\vec{\mathbf{a}}\times\vec{\mathbf{b}} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

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General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} \quad = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

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Source-free

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$$\vec{\mathbf{a}}\cdot\vec{\mathbf{b}} = a_x b_x + a_y b_y + a_z b_z$$

$$\nabla\cdot\vec{\mathbf{E}} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

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$$\begin{cases} \vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu \vec{\mathbf{H}} \\ \vec{\mathbf{k}} \times \vec{\mathbf{H}} = -\omega\varepsilon \vec{\mathbf{E}} \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{E}} = 0 \\ \vec{\mathbf{k}} \cdot \vec{\mathbf{H}} = 0 \end{cases}$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = \omega^2\mu\varepsilon$$

$$\vec{\mathbf{k}} \times (\vec{\mathbf{k}} \times \vec{\mathbf{E}}) = \omega\mu(\vec{\mathbf{k}} \times \vec{\mathbf{H}}) = \omega\mu(-\omega\varepsilon\vec{\mathbf{E}}) = -\omega^2\mu\varepsilon\vec{\mathbf{E}}$$

$$\vec{\mathbf{k}}(\vec{\mathbf{k}}\cdot\vec{\mathbf{E}}) - \vec{\mathbf{E}}(\vec{\mathbf{k}}\cdot\vec{\mathbf{k}})$$

$$-(\vec{\mathbf{k}}\cdot\vec{\mathbf{k}})\vec{\mathbf{E}} = -\omega^2\mu\varepsilon\vec{\mathbf{E}}$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = \omega^2\mu\varepsilon$$

$$\vec{\mathbf{A}} \times (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) = \vec{\mathbf{B}}(\vec{\mathbf{A}}\cdot\vec{\mathbf{C}}) - \vec{\mathbf{C}}(\vec{\mathbf{A}}\cdot\vec{\mathbf{B}})$$

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^+ e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^+ e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^+ e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{H}}^+ e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{k}} = k_x \hat{i}_x + k_y \hat{i}_y + k_z \hat{i}_z$$

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$$\vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_x x + k_y y + k_z z$$

Source-free

$$\vec{\mathbf{k}} \times \vec{\mathbf{E}} = \omega\mu\vec{\mathbf{H}}$$

$$\vec{\mathbf{k}} \times \vec{\mathbf{H}} = -\omega\varepsilon\vec{\mathbf{E}}$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = \omega^2\mu\varepsilon$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{E}} = 0$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{H}} = 0$$

$\vec{\mathbf{k}}$:propagation vector

Me

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MEMO

The electric and magnetic fields are related each other through the following relation:

$$\zeta\vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$$

where $\hat{i}_p$ **points to the propagation direction and** $\zeta = \sqrt{\mu/\varepsilon}$

$$\vec{\mathbf{k}} = k_z \hat{i}_z \Rightarrow \vec{\mathbf{k}}\cdot\vec{\mathbf{r}} = k_z z$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = k_z^2 \Rightarrow k_z^2 = \omega^2\mu\varepsilon \Rightarrow k_z = \omega\sqrt{\mu\varepsilon} = \beta - j\alpha$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^+ e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^+ e^{-jk_z z} = \vec{\mathbf{E}}^+ e^{-j\beta z} e^{-\alpha z}$$

$$\begin{cases} \vec{\mathbf{k}}\cdot\vec{\mathbf{E}} = k_z E_z = 0 \\ \vec{\mathbf{k}}\cdot\vec{\mathbf{H}} = k_z H_z = 0 \end{cases} \Rightarrow E_z = H_z = 0$$

$$\omega\mu\vec{\mathbf{H}} = \vec{\mathbf{k}} \times \vec{\mathbf{E}} = k_z \hat{i}_z \times \vec{\mathbf{E}} = \omega\sqrt{\mu\varepsilon}\,\hat{i}_z \times \vec{\mathbf{E}} \Rightarrow \sqrt{\frac{\mu}{\varepsilon}}\vec{\mathbf{H}} = \hat{i}_z \times \vec{\mathbf{E}}$$

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

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$\vec{\mathbf{k}}$:propagation vector

Medium

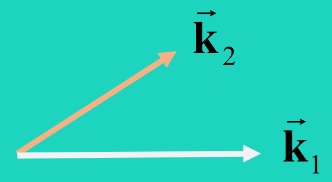
- Linear - Isotropic - Space nondispersive - Time dispersive - Lossy

- Homogeneous (Time-invariant & Space-invariant)

$$k = \omega\sqrt{\mu\varepsilon} = \beta - j\alpha$$

$$\vec{\mathbf{k}}\cdot\vec{\mathbf{k}} = \omega^2\mu\varepsilon = (\beta - j\alpha)^2 \Rightarrow \vec{\mathbf{k}} = \vec{\mathbf{k}}_1 - j\vec{\mathbf{k}}_2$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-j(\vec{\mathbf{k}}_1 - j\vec{\mathbf{k}}_2)\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}_1\cdot\vec{\mathbf{r}}} e^{-\vec{\mathbf{k}}_2\cdot\vec{\mathbf{r}}}$$



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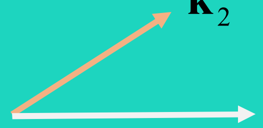
Propagation ($e^{-j\vec{\mathbf{k}}_1\cdot\vec{\mathbf{r}}}$)

Attenuation ($e^{-\vec{\mathbf{k}}_2\cdot\vec{\mathbf{r}}}$)



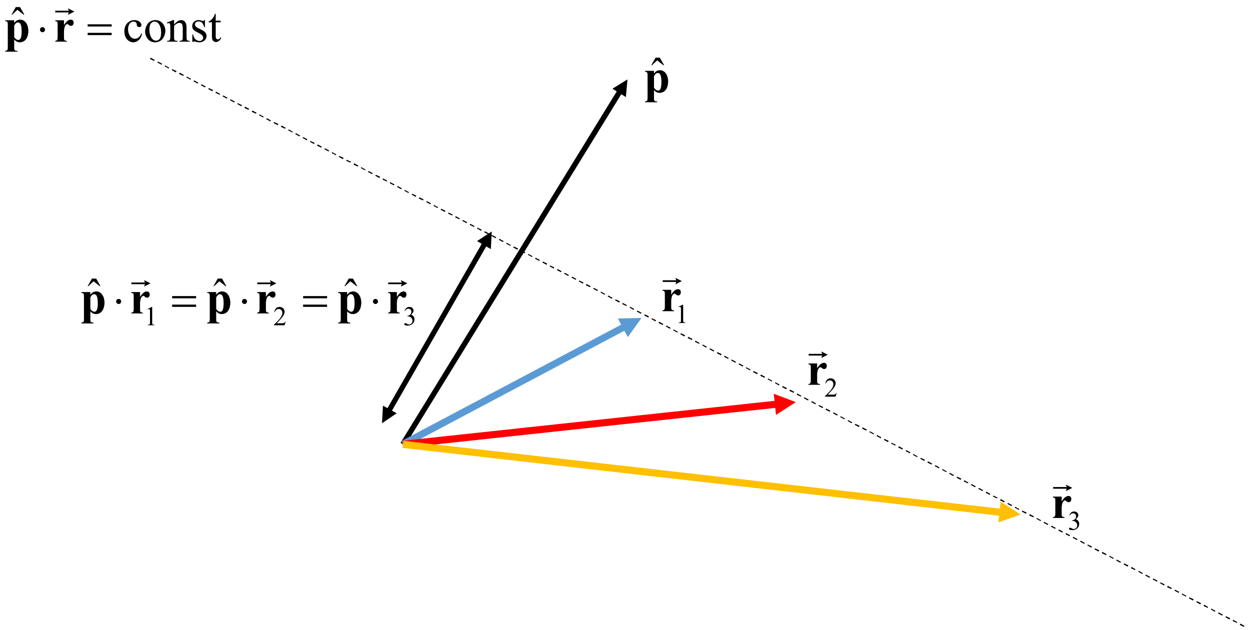
The wave propagates along $\vec{\mathbf{k}}_1$

The wave attenuates along $\vec{\mathbf{k}}_2$

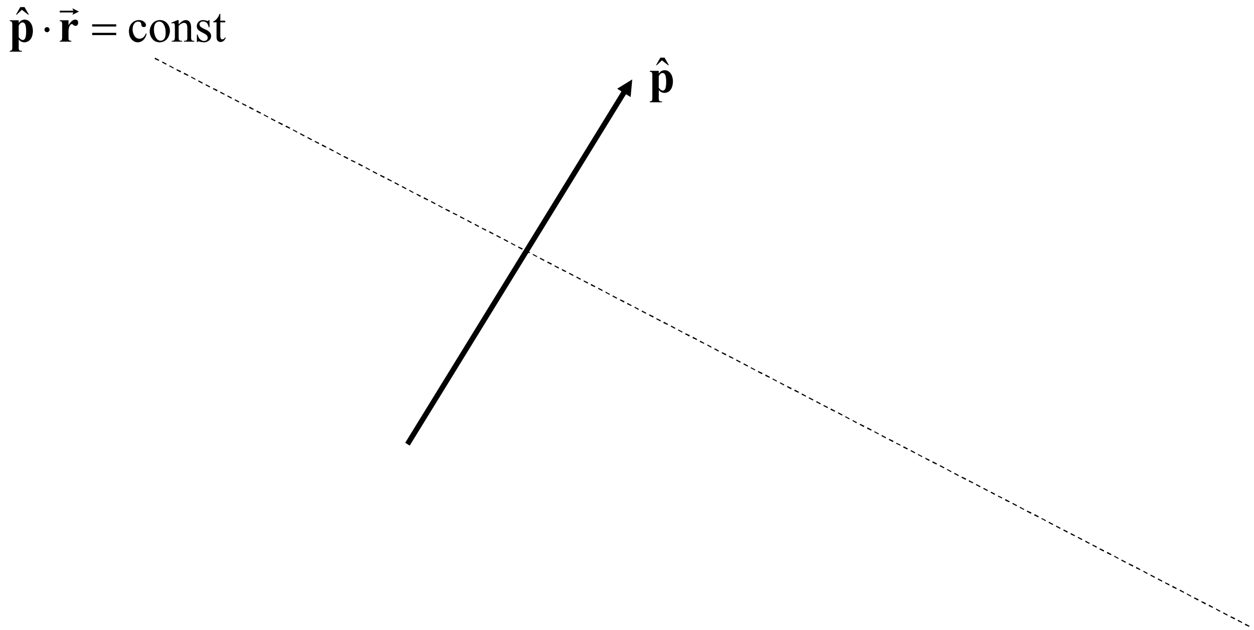
$\vec{\mathbf{k}}_1 \cdot \vec{\mathbf{r}} = \text{const} \Rightarrow$ equi-phase planes

$\vec{\mathbf{k}}_2 \cdot \vec{\mathbf{r}} = \text{const} \Rightarrow$ equi-amplitude planes

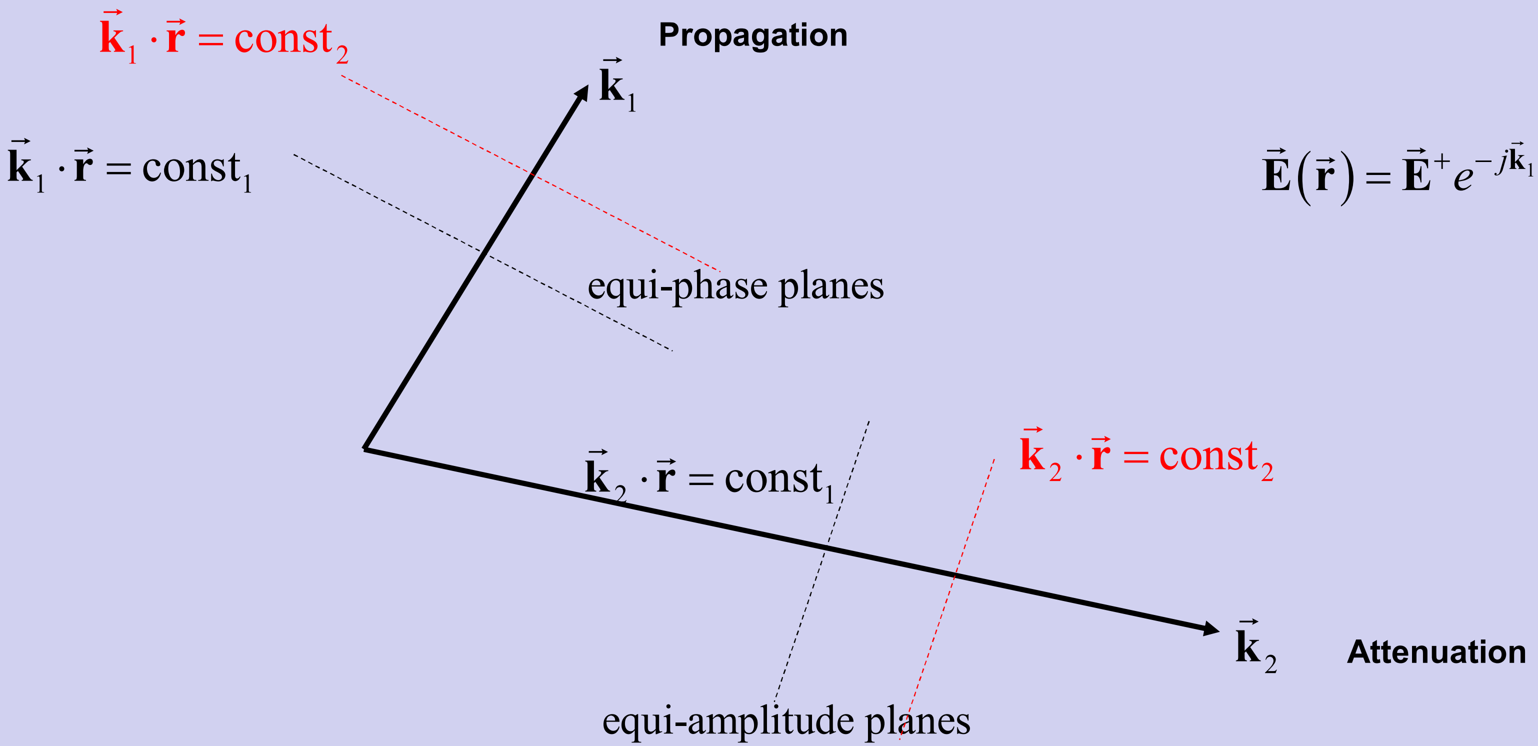
General expression of plane waves (PD)



General expression of plane waves (PD)



General expression of plane waves (PD)



$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}_1 \cdot \vec{\mathbf{r}}} e^{-\vec{\mathbf{k}}_2 \cdot \vec{\mathbf{r}}}$$

When $\vec{\mathbf{k}}_1$ and $\vec{\mathbf{k}}_2$ are proportional, equi-amplitude and equi-phase planes become coincident: the plane wave is said HOMOGENEOUS

More generally, equi-amplitude and equi-phase planes may be not coincident: in this the plane wave is said NOT-HOMOGENEOUS

General expression of plane waves (PD)

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{E}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^{+} e^{-j\vec{\mathbf{k}}\cdot\vec{\mathbf{r}}} = \vec{\mathbf{H}}^{+} e^{-jk_x x} e^{-jk_y y} e^{-jk_z z}$$

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Source-free

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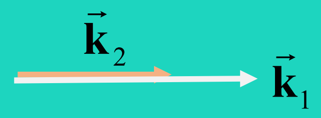
$$\vec{\mathbf{k}} = (\beta - j\alpha)\hat{i}_z = \beta\hat{i}_z - j\alpha\hat{i}_z = \vec{\mathbf{k}}_1 - j\vec{\mathbf{k}}_2$$



HOMOGENEOUS PLANE-WAVE