

# **Campi Elettromagnetici**

**Corso di Laurea in Ingegneria Informatica,  
Biomedica e delle Telecomunicazioni**

**a.a. 2021-2022 - Laurea “Triennale” – Secondo semestre - Secondo anno**

**Università degli Studi di Napoli “Parthenope”**

**Stefano Perna**

# Color legend

New formulas, important considerations,  
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

# Plane Waves

Time domain (TD)

Spectral domains

Phasor Domain (PD)

Fourier Domain (FD)

Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

# Plane Waves

## Time domain

# Plane Waves (TD)

$$\begin{cases} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t} \end{cases} \quad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e_x}{\partial t^2} = 0$$

$\{e_x, h_y\}$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$e_x(z, t) = e_x^+(z - ct) + e_x^-(z + ct)$$

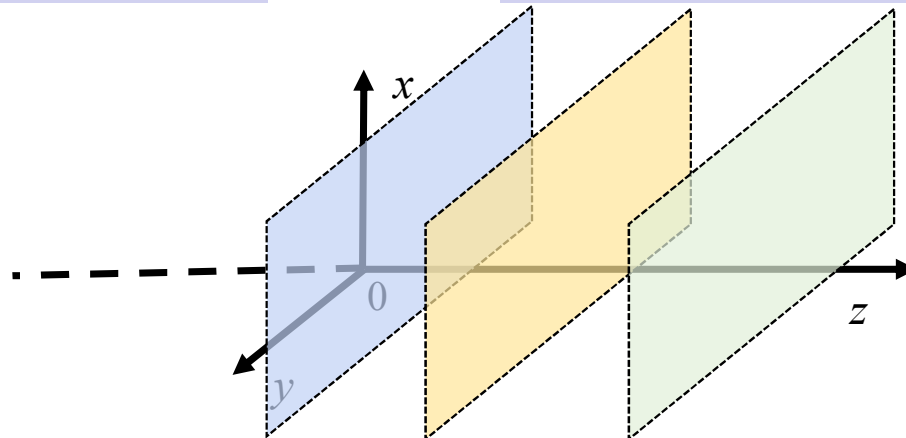
$$\zeta h_y(z, t) = e_x^+(z - ct) - e_x^-(z + ct)$$

$$\begin{cases} \frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t} \\ \frac{\partial h_x}{\partial z} = \varepsilon \frac{\partial e_y}{\partial t} \end{cases} \quad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e_y}{\partial t^2} = 0$$

$\{e_y, h_x\}$

$$e_y(z, t) = e_y^+(z - ct) + e_y^-(z + ct)$$

$$-\zeta h_x(z, t) = e_y^+(z - ct) - e_y^-(z + ct)$$



Source-free

Medium

- Linear
- Local (TND & SND)
- Isotropic
- Homogeneous (TI - SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$e_z(z, t) = h_z(z, t) = 0$$

$\{e_y, h_x\}$   
 $\{e_x, h_y\}$

Independent  
each other

# Plane Waves (TD)

$$\{e_x^+, h_y^+\}$$

$$\begin{cases} e_x^+(z, t) = e_x^+(z - ct) \\ \zeta h_y^+(z, t) = e_x^+(z - ct) \end{cases}$$

$$\{e_y^+, h_x^+\}$$

$$\begin{cases} e_y^+(z, t) = e_y^+(z - ct) \\ \zeta h_x^+(z, t) = -e_y^+(z - ct) \end{cases}$$

the e.m. field propagates along  $\hat{i}_p = \hat{i}_z$

$$\{e_x^-, h_y^-\}$$

$$\begin{cases} e_x^-(z, t) = e_x^-(z + ct) \\ \zeta h_y^-(z, t) = -e_x^-(z + ct) \end{cases}$$

$$\{e_y^-, h_x^-\}$$

$$\begin{cases} e_y^-(z, t) = e_y^-(z + ct) \\ \zeta h_x^-(z, t) = e_y^-(z + ct) \end{cases}$$

the e.m. field propagates along  $\hat{i}_p = -\hat{i}_z$

- the e.m. field lies on the plane xy orthogonal to the propagation direction
- The Poynting vector is directed along the direction of propagation
- $|\vec{e}|$  and  $|\vec{h}|$  are proportional through  $\zeta$
- $\zeta \vec{h} = \hat{i}_p \times \vec{e}$

Source-free

Medium

- Linear
- Time non-dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$e_z(z, t) = h_z(z, t) = 0$$

$$\{e_y, h_x\}$$

$$\{e_x, h_y\}$$

Independent  
each other

# Plane Waves

## Spectral domains

# Plane Waves (Spectral Domains)

$$\begin{cases} \frac{dE_x}{dz} = -j\omega\mu H_y \\ \frac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \quad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$\begin{aligned} k &= \omega\sqrt{\mu\varepsilon} \\ k &= \beta - j\alpha \end{aligned}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

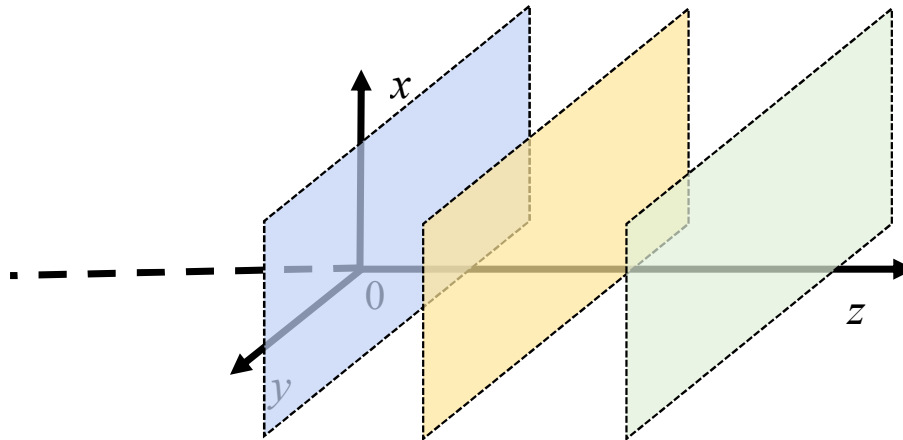
$$\zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \frac{dE_y}{dz} = j\omega\mu H_x \\ \frac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \quad \{E_y, H_x\}$$

$$\frac{d^2 E_y}{dz^2} + k^2 E_y = 0$$

$$E_y = E_y^+ e^{-jkz} + E_y^- e^{jkz}$$

$$-\zeta H_x = E_y^+ e^{-jkz} - E_y^- e^{jkz}$$



Source-free

Medium

- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- ~~- Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

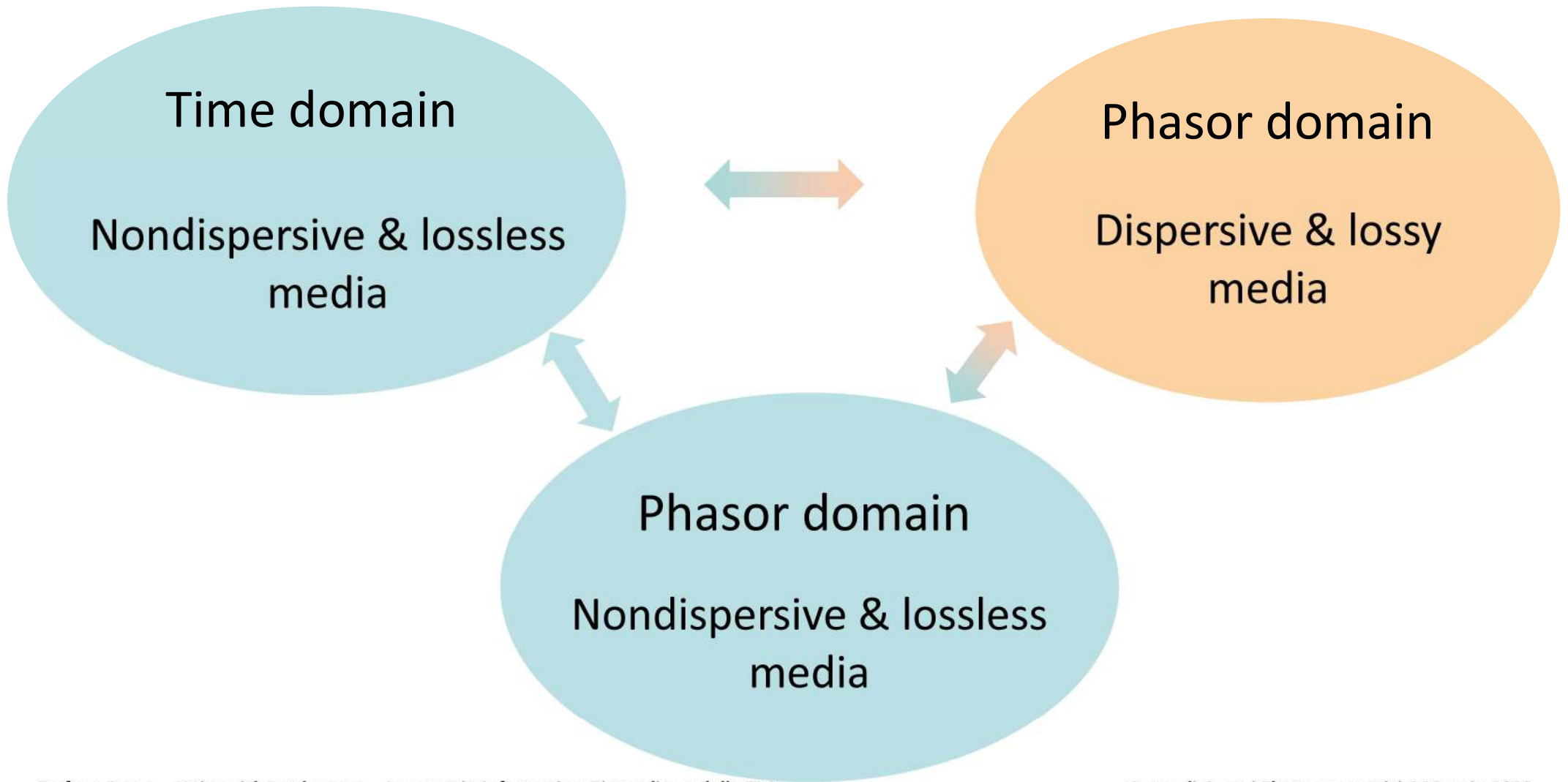


$$E_z = H_z = 0$$

$$\begin{aligned} &\{E_y, H_x\} \\ &\{E_x, H_y\} \end{aligned} \quad \text{Independent each other}$$



# Razionale



# Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$E_x^-(z) = E^- e^{j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^-(z) = -E^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

**Progressive plane wave**

$$E^- e^{j\beta z} \rightarrow e_x^-(z, t) = |E^-| \cos(\omega_0 t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

**Regressive plane wave**

**Time nondispersive & lossless**

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\varepsilon_{eq} = \varepsilon$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

**Source-free**

**Medium**

- Linear
- **Time nondispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

**Independent**

$$\{E_x, H_y\}$$

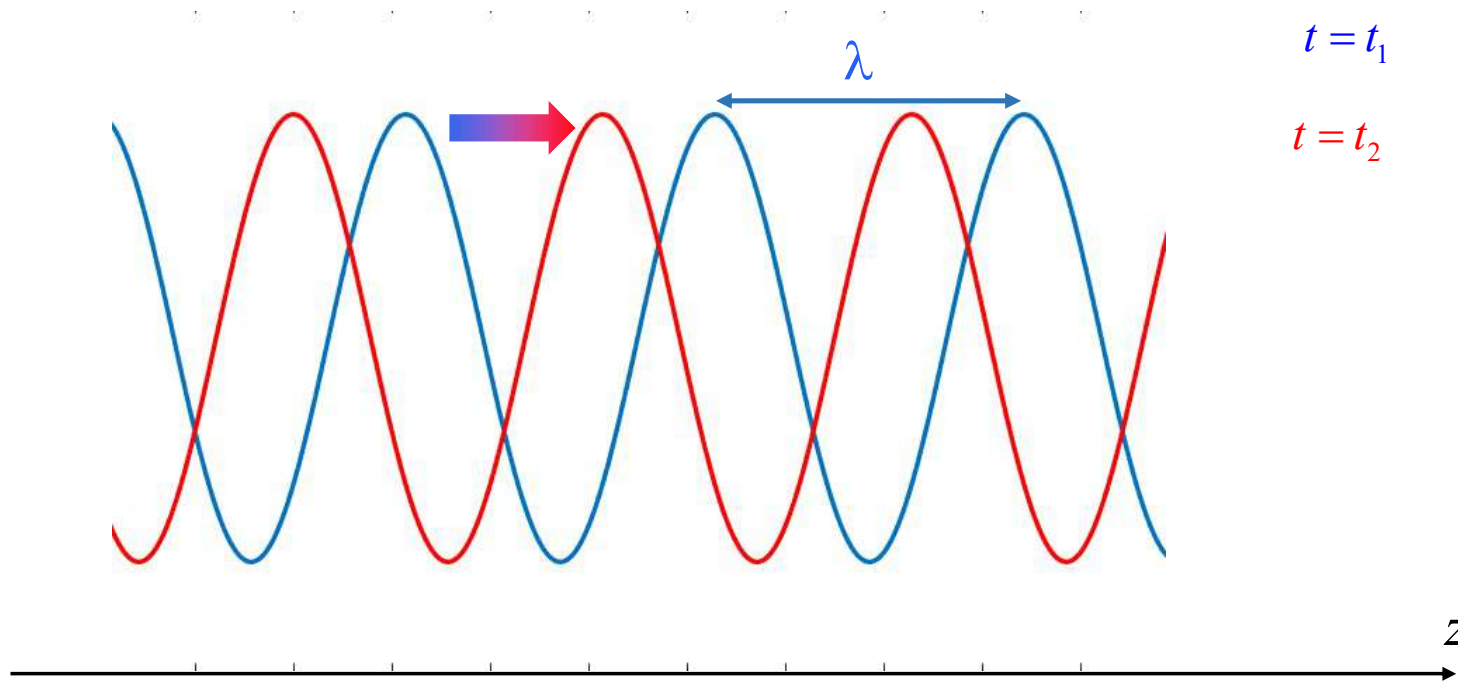
**each other**

# Plane Waves (Phasor Domain)

Time nondispersive & lossless medium

$$e_x^+(z,t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

$$v_p = \frac{\omega_0}{\beta}$$



# Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_0 = 2\pi f_0$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

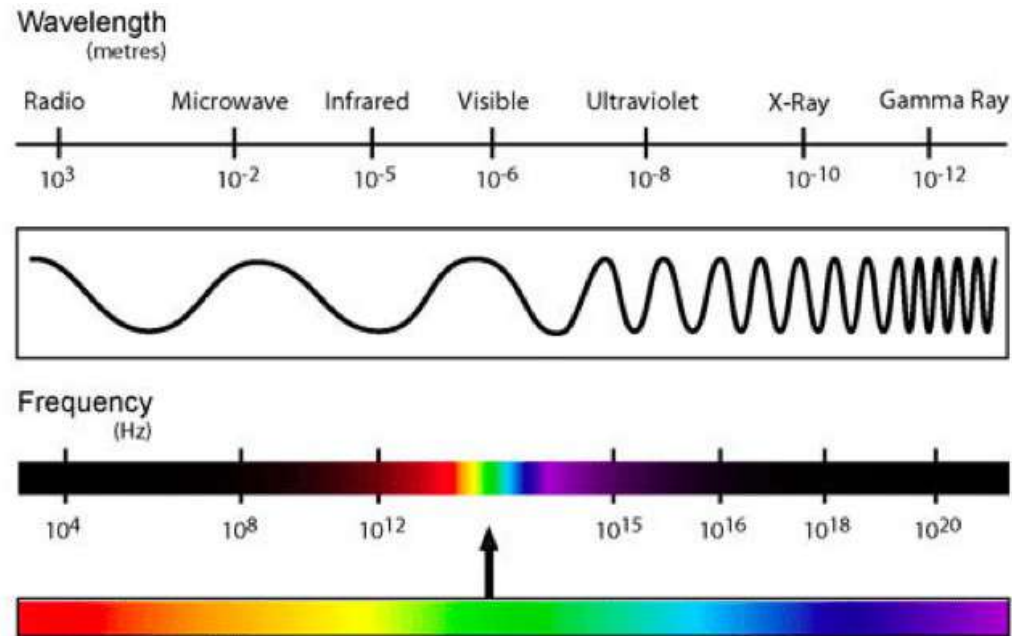
$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

**Progressive plane wave**

- The term  $e^{-j\beta z}$  is related to the propagation along the (positive sense of the) z-axis
- When z is small with respect to  $\lambda$ , the propagation effects become negligible

# Electromagnetic spectrum



**Free space (Linear, isotropic, local, homogeneous)**

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f}$$

*f*: frequency

*λ*: wavelength

*c*: light speed

*β*: propagation constant

# Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f_0}$$

$$E_x^+(z) = E^+ e^{-jkz}$$

$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\omega_0 = 2\pi f_0$$

$$E^+ e^{-jkz} \rightarrow e^+(z, t) = e^{-\alpha z} |E^+| \cos(\omega_0 t - \beta z + \varphi^+)$$

**Time dispersive (lossy)**

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \text{real} \end{cases}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu \varepsilon} = \beta(\omega_0)$$

$$\omega_0 = 2\pi f_0$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f_0}$$

**Time nondispersive & lossless**

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu \varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

**Source-free**

**Medium**

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

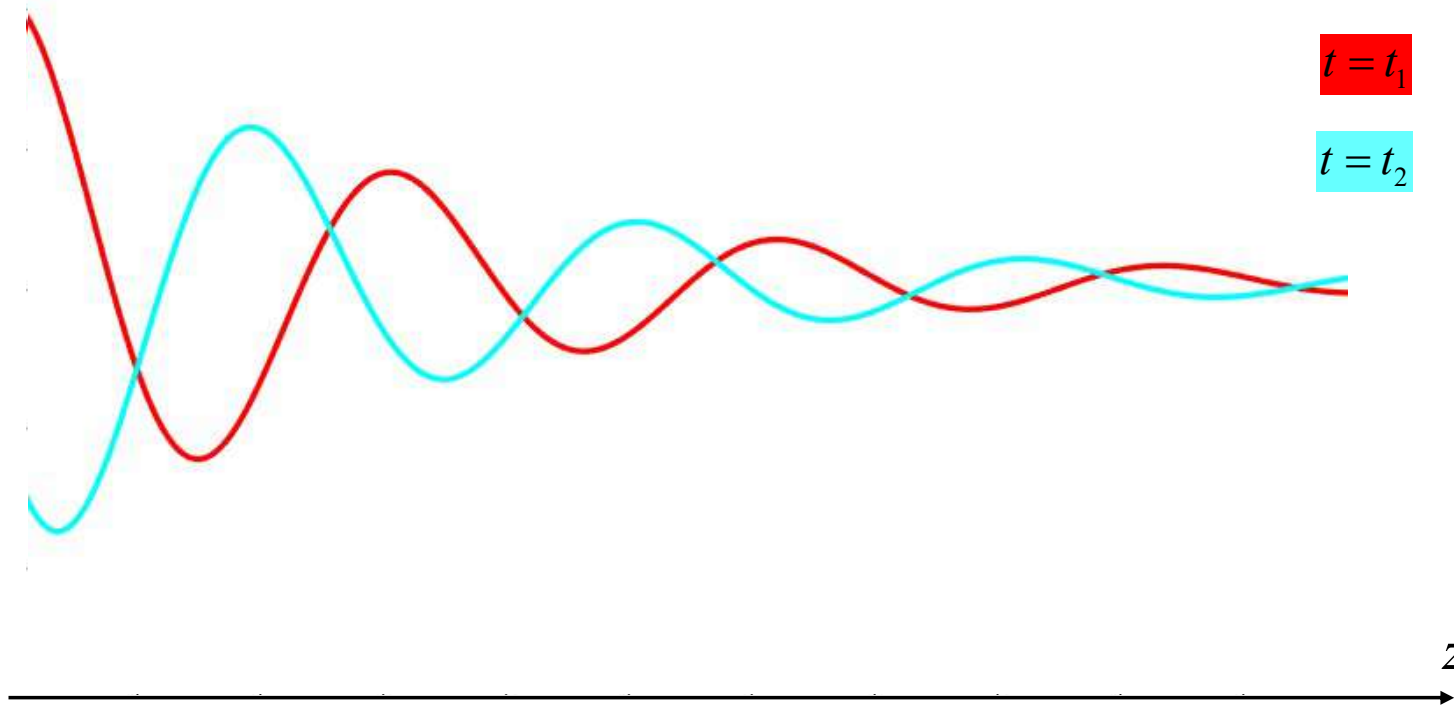
**Independent each other**

# Plane Waves (Phasor Domain)

Time dispersive & lossy medium

$$e_x^+(z,t) = |E^+| e^{-\alpha z} \cos(\omega_0 t - \beta z + \varphi^+) = e^{-\alpha z} e_x^+(z - v_p t)$$

$$v_p = \frac{\omega_0}{\beta}$$



# Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x^+(z) = E^+ e^{-jkz}$$

$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\operatorname{Re}\{\omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)}\}} = \frac{1}{\operatorname{Re}\{\sqrt{\mu(\omega_0) \varepsilon(\omega_0)}\}} = v_p(\omega_0)$$

$$E^+ e^{-jkz} \rightarrow e^+(z, t) = e^{-\alpha z} |E^+| \cos(\omega_0 t - \beta z + \varphi^+)$$

**Time dispersive (lossy)**

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \text{real} \end{cases}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu \varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu \varepsilon}} = c$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

**Time nondispersive & lossless**

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases} \quad c = \frac{1}{\sqrt{\mu \varepsilon}}$$

**Source-free**

**Medium**

- Linear
- **Time dispersive**
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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$   
 $\{E_x, H_y\}$  **Independent each other**

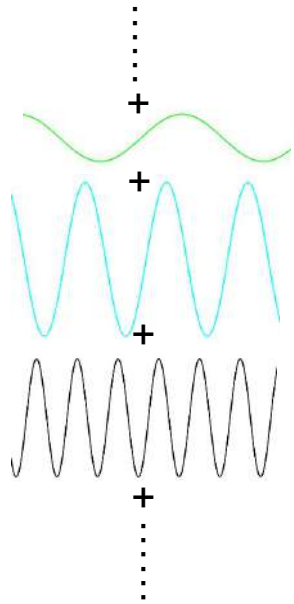
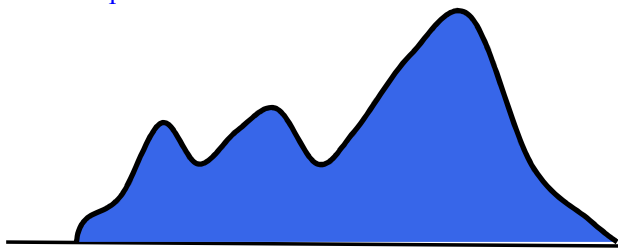


# Plane Waves

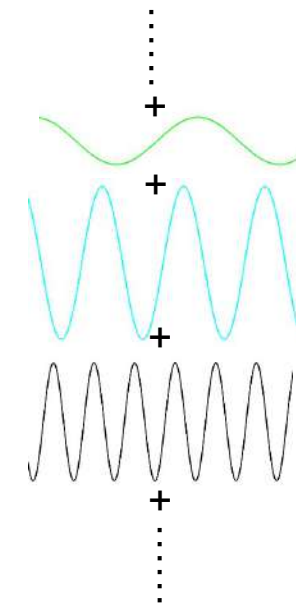
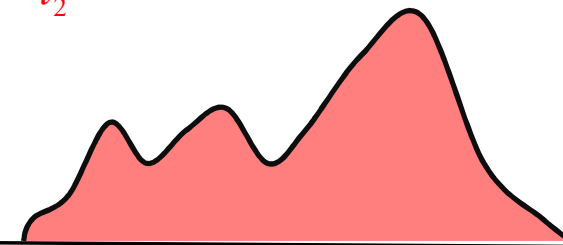
Time nondispersive & lossless medium

$$v_p = c$$

$t = t_1$



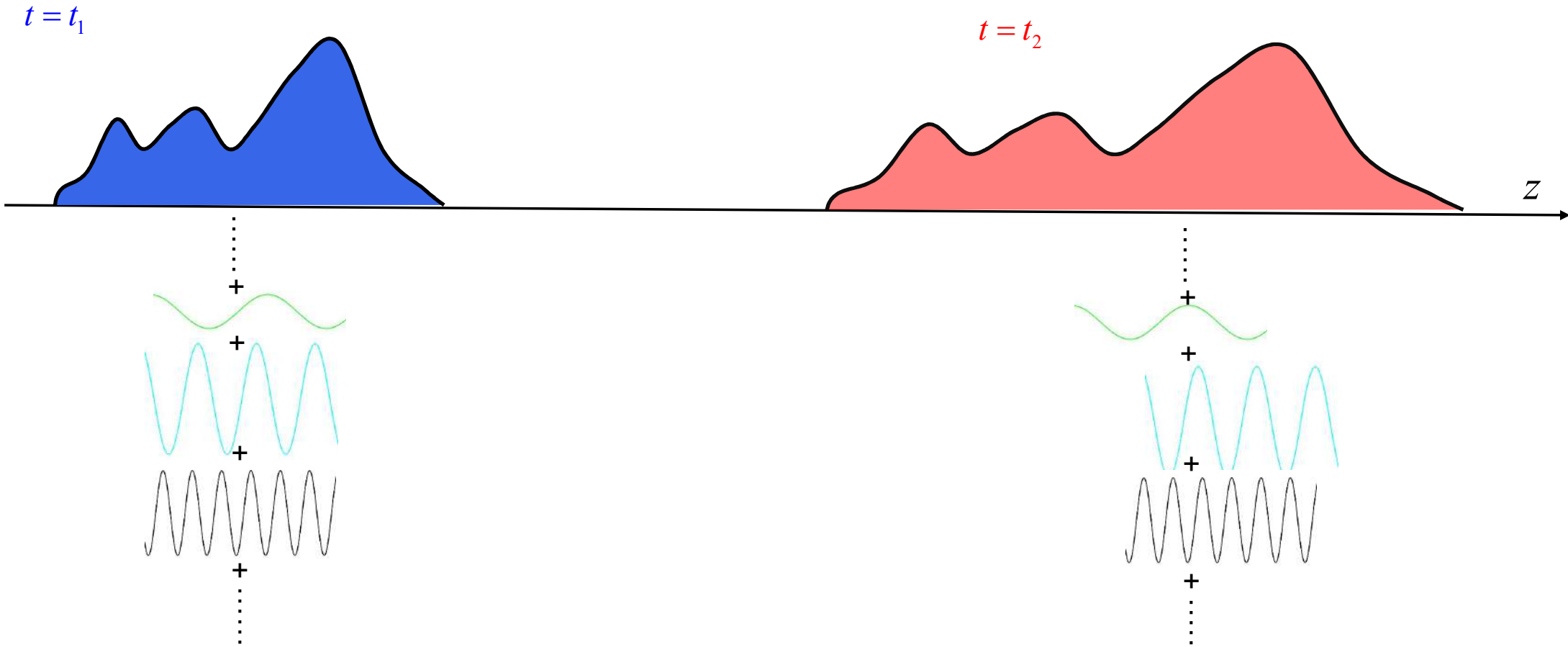
$t = t_2$



# Plane Waves

Time dispersive medium

$$v_p = v_p(\omega_0)$$



# Plane Waves (Phasor Domain)

## Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu \varepsilon} = \beta(\omega_0)$$

$$c = \frac{1}{\sqrt{\mu \varepsilon}}$$

$$v_p = \frac{\omega_0}{\beta} = c$$

## Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \text{real} \end{cases}$$

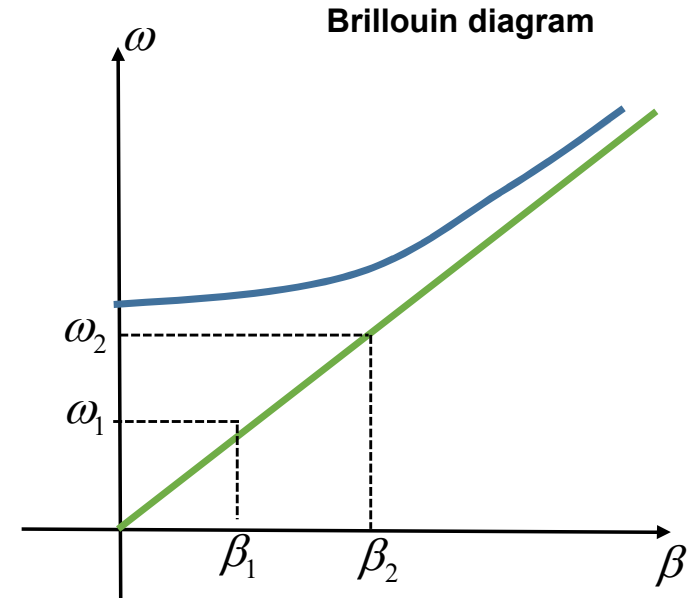
$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

Attenuation

$$\alpha \neq 0$$

Distortion

$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$



nondispersive

dispersive

$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = c$$

# Plane Waves (Phasor Domain)

## Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu \varepsilon} = \beta(\omega_0)$$

$$c = \frac{1}{\sqrt{\mu \varepsilon}}$$

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$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \text{real} \end{cases}$$

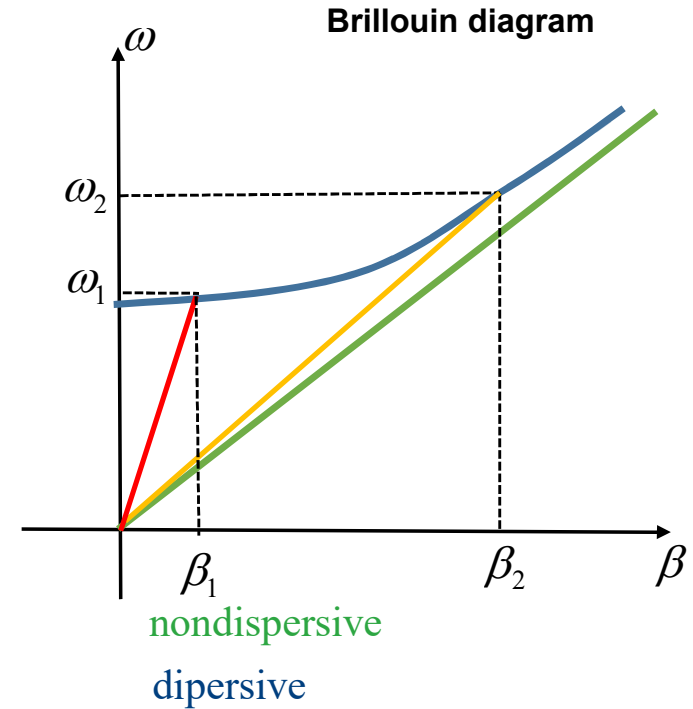
$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

Attenuation

$$\alpha \neq 0$$

Distortion

$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$



$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = c$$

# Plane Waves (Spectral Domains)

$$\begin{cases} \frac{dE_x}{dz} = -j\omega_0\mu H_y \\ \frac{dH_y}{dz} = -j\omega_0\varepsilon E_x \end{cases} \quad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$\begin{cases} k = \omega\sqrt{\mu\varepsilon} \\ k = \beta - j\alpha \end{cases}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

**Phasor Domain**

$$\begin{cases} \frac{dE_x}{dz} = -j\omega\mu H_y \\ \frac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \quad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z, \omega) = E_x^+(\omega) e^{-jkz} + E_x^-(\omega) e^{jkz}$$

$$\zeta H_y(z, \omega) = E_x^+(\omega) e^{-jkz} - E_x^-(\omega) e^{jkz}$$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \end{cases}$$

$$k(\omega) = \omega \sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

**Fourier Domain**

**Source-free**

**Medium**

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

Independent

$$\{E_x, H_y\}$$

each other

# Plane Waves (**Fourier Domain**)

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\begin{cases} \frac{dE_x}{dz} = -j\omega\mu H_y \\ \frac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \quad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z, \omega) = E_x^+(\omega)e^{-jkz} + E_x^-(\omega)e^{jkz}$$

$$\zeta H_y(z, \omega) = E_x^+(\omega)e^{-jkz} - E_x^-(\omega)e^{jkz}$$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \end{cases}$$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

**Fourier Domain**

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$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

Independent

$$\{E_x, H_y\}$$

each other

# Plane Waves (Fourier Domain)

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\begin{cases} \frac{dE_x}{dz} = -j\omega\mu H_y \\ \frac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \quad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x^+(z, \omega) = E^+(\omega) e^{-jkz}$$

$$\zeta H_y^+(z, \omega) = E^+(\omega) e^{-jkz}$$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \end{cases}$$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

**Fourier Domain**

**Source-free**

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- Linear
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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

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$$E_z = H_z = 0$$

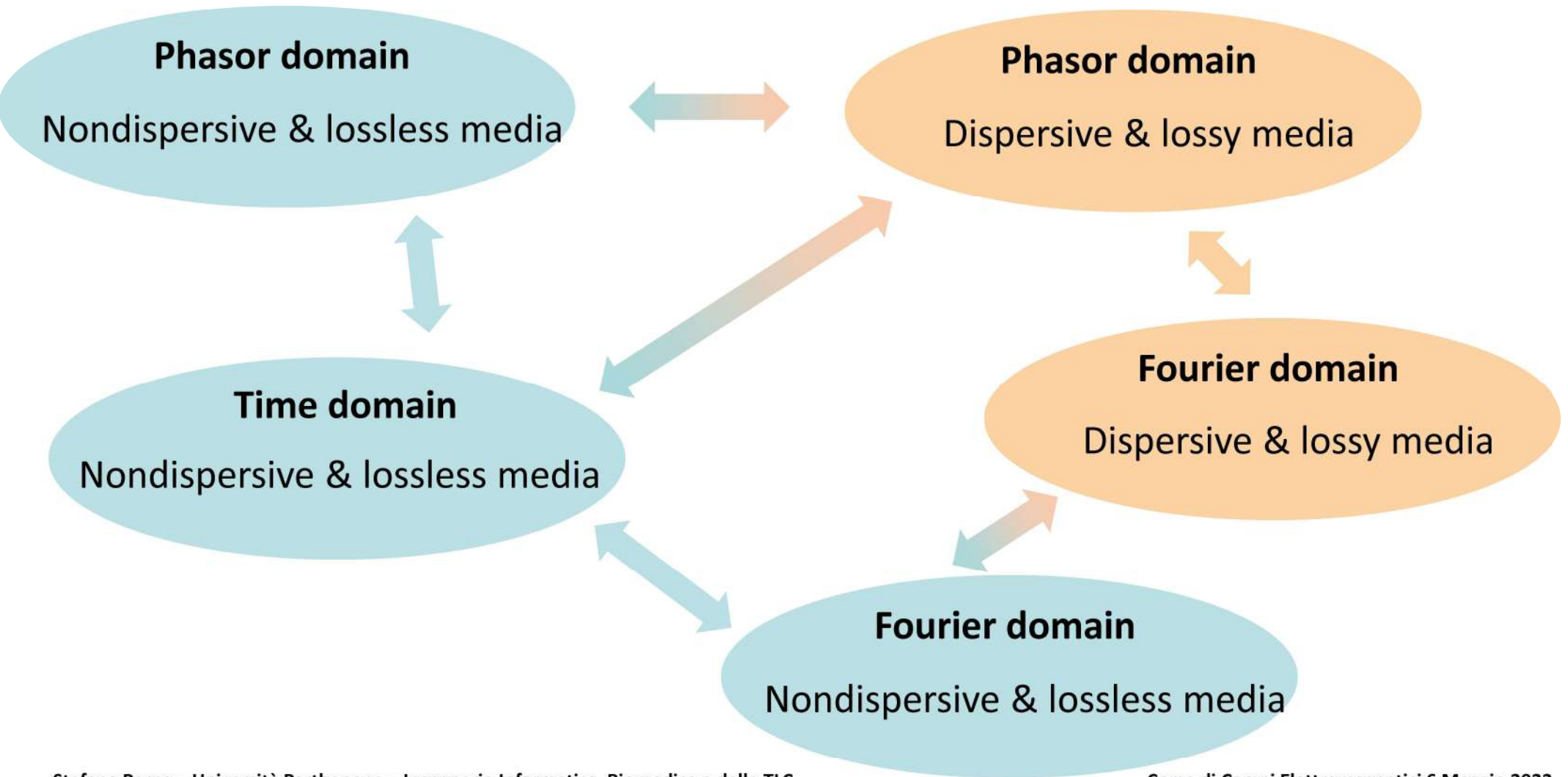
$$\{E_y, H_x\}$$

Independent

$$\{E_x, H_y\}$$

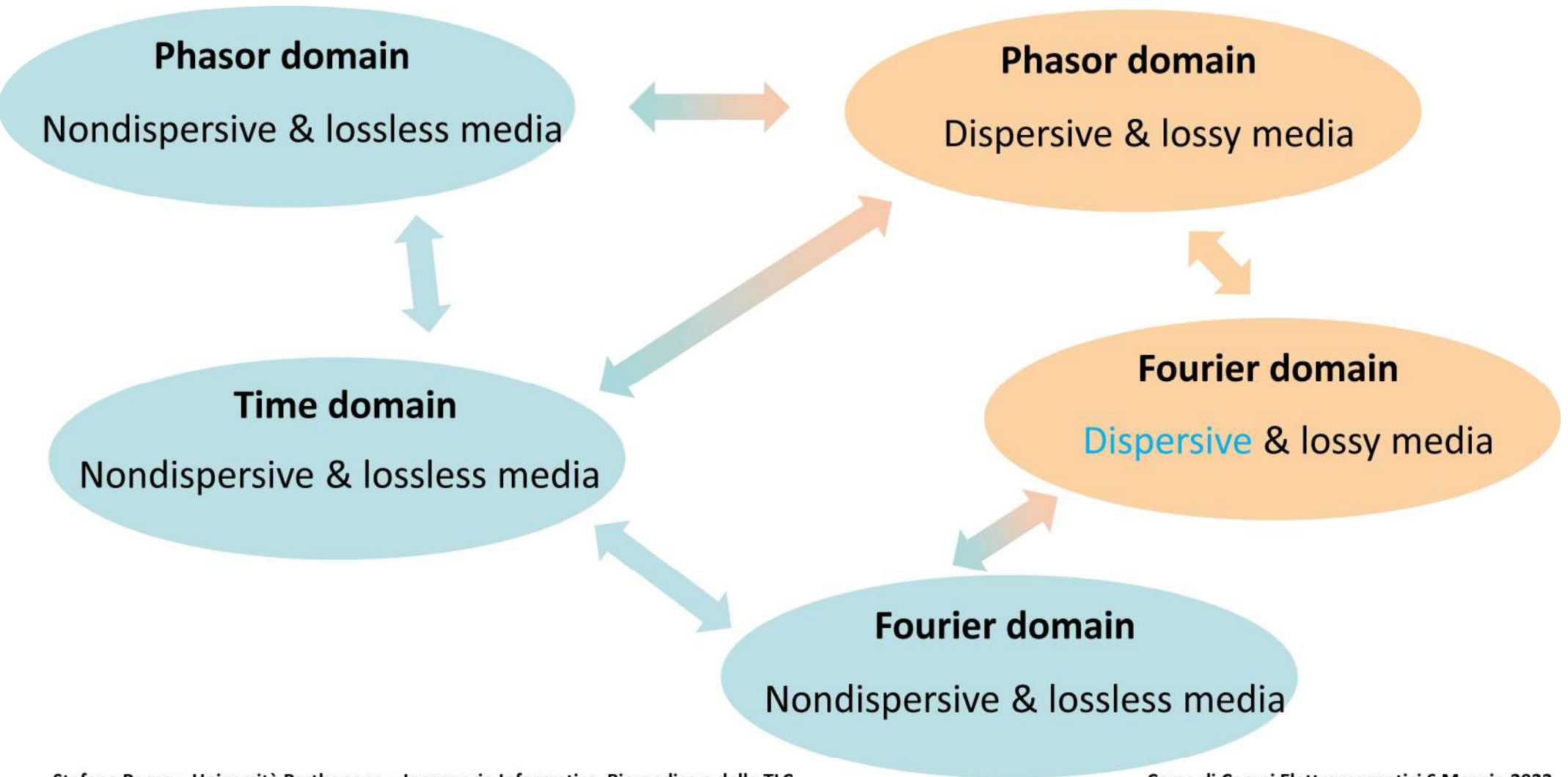
each other

# Razionale





# Razionale



# Plane Waves (Fourier Domain)

## Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \\ \sigma: \text{real} \end{cases}$$

$$\varepsilon_{eq}(\omega) = \varepsilon(\omega) \left[ 1 - \frac{j\sigma}{\omega\varepsilon(\omega)} \right]$$

$$\begin{aligned} k(\omega) &= \omega \sqrt{\mu(\omega)\varepsilon(\omega)} \\ k(\omega) &= \beta(\omega) - j\alpha(\omega) \end{aligned}$$

$$\begin{aligned} k &= \omega \sqrt{\mu\varepsilon} \\ k &= \beta - j\alpha \end{aligned}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\begin{cases} \frac{dE_x}{dz} = -j\omega\mu H_y \\ \frac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \quad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x^+(z, \omega) = E^+(\omega) e^{-jkz}$$

$$\zeta H_y^+(z, \omega) = E^+(\omega) e^{-jkz}$$

## Fourier Domain

## Source-free

### Medium

- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

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$$\begin{aligned} \{E_y, H_x\} \\ \{E_x, H_y\} \end{aligned} \quad \text{Independent each other}$$

# Plane Waves (Fourier Domain)

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## Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$k(\omega) = \omega\sqrt{\mu\varepsilon} = \beta(\omega)$$

# Plane Waves (Fourier Domain)

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# Plane Waves (Fourier Domain)

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$$= \frac{1}{2\pi} \int E^+(\omega) e^{-j\frac{\omega}{c} z} e^{j\omega t} d\omega = \frac{1}{2\pi} \int E^+(\omega) e^{j\omega \left(t - \frac{z}{c}\right)} d\omega$$

**Time nondispersive & lossless**

$$\begin{cases} \epsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu \epsilon}}$$

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$$e_x^+(z, t) = \frac{1}{2\pi} \int E^+(\omega) e^{j\omega \left(t - \frac{z}{c}\right)} d\omega$$

**Progressive plane wave**

$$e_x^+(z=0, t) = \frac{1}{2\pi} \int E^+(\omega) e^{j\omega t} d\omega = f(t)$$

$$e_x^+(z > 0, t) = \frac{1}{2\pi} \int E^+(\omega) e^{j\omega \left(t - \frac{z}{c}\right)} d\omega = f\left(t - \frac{z}{c}\right) = f\left[-\frac{1}{c}(z - ct)\right] = f[(z - ct)]$$

**Time nondispersive & lossless**

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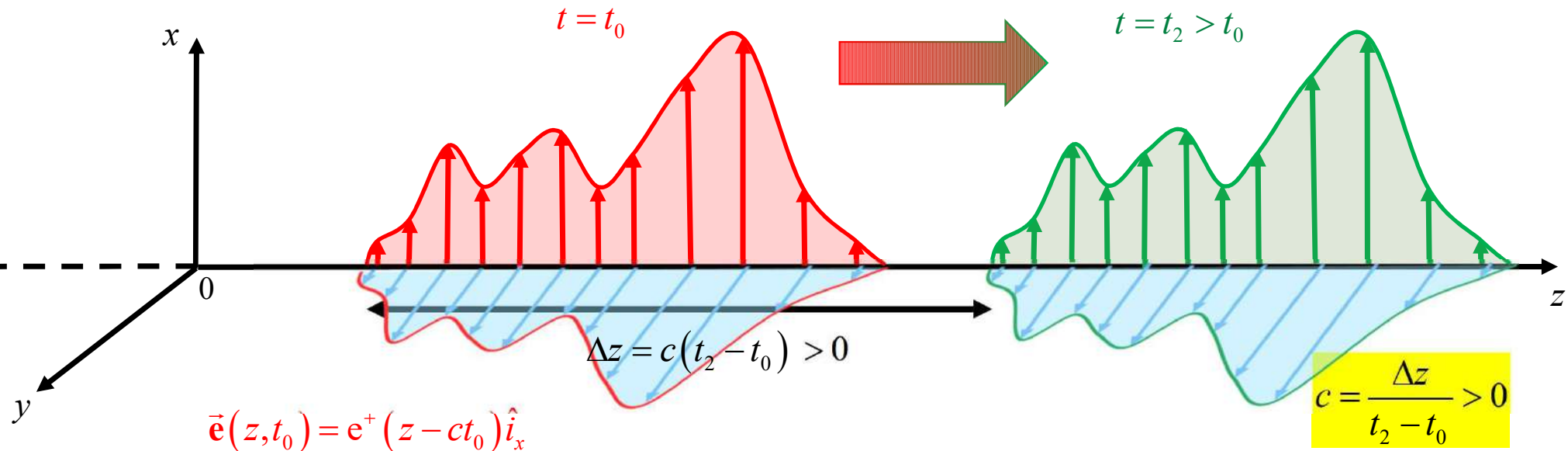


$$E_z = H_z = 0$$

$\{E_y, H_x\}$   
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**Independent each other**

# Plane Waves



$$\vec{e}(z, t_0) = e^+(z - ct_0) \hat{i}_x$$

$$\vec{e}(z, t_2) = e^+(z - ct_2) \hat{i}_x = e^+(z - ct_0 + ct_0 - ct_2) \hat{i}_x = e^+(z - ct_0 - c[t_2 - t_0]) \hat{i}_x$$

The electromagnetic perturbation **propagates** without deformation and with constant speed  $c$  along the positive sense of the  $z$ -axis

$\begin{cases} e^+(z - ct) \\ h^+(z - ct) \end{cases}$  is referred to as electromagnetic **progressive plane wave**



# Plane Waves (Fourier Domain)

$$\{E_x, H_y\}$$

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# Mathematical tools

$$f(t) \longrightarrow \boxed{\text{FT}} \longrightarrow F(\omega)$$

$$f(t) \text{ real} \Rightarrow \boxed{F(\omega) = F^*(-\omega)}$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \left[ \int_{-\infty}^0 F(\omega) e^{j\omega t} d\omega + \int_0^{\infty} F(\omega) e^{j\omega t} d\omega \right] = \frac{1}{2\pi} \left[ \int_0^{\infty} [F(\omega) e^{j\omega t}]^* d\omega + \int_0^{\infty} F(\omega) e^{j\omega t} d\omega \right]$$

$$\eta = -\omega$$

$$\int_{-\infty}^0 F(\omega) e^{j\omega t} d\omega = -\int_0^{\infty} F(-\eta) e^{-j\eta t} d\eta = \int_0^{\infty} F(-\eta) e^{-j\eta t} d\eta = \int_0^{\infty} F^*(\eta) [e^{j\eta t}]^* d\eta = \int_0^{\infty} [F(\eta) e^{j\eta t}]^* d\eta = \int_0^{\infty} [F(\omega) e^{j\omega t}]^* d\omega$$

# Mathematical tools

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$$\begin{aligned} f(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \left[ \int_{-\infty}^0 F(\omega) e^{j\omega t} d\omega + \int_0^{\infty} F(\omega) e^{j\omega t} d\omega \right] = \frac{1}{2\pi} \left[ \int_0^{\infty} [F(\omega) e^{j\omega t}]^* d\omega + \int_0^{\infty} F(\omega) e^{j\omega t} d\omega \right] \\ &= \frac{1}{2\pi} \int_0^{\infty} F(\omega) e^{j\omega t} + [F(\omega) e^{j\omega t}]^* d\omega = \frac{1}{2\pi} \left[ \int_0^{\infty} 2 \operatorname{Re}\{F(\omega) e^{j\omega t}\} d\omega \right] = \operatorname{Re} \left\{ \frac{1}{\pi} \int_0^{\infty} F(\omega) e^{j\omega t} d\omega \right\} \end{aligned}$$

$$q = a + jb$$

$$q + q^* = (a + jb) + (a - jb) = 2a = 2 \operatorname{Re}\{q\}$$

# Plane Waves : dispersion

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$$\alpha(\omega) \approx 0$$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) \\ \mu(\omega) = \mu_1(\omega) \\ \sigma = 0 \end{cases}$$

# Plane Waves : dispersion

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$$\zeta H_y^+(z, \omega) = E^+(\omega) e^{-jkz} = E^+(\omega) e^{-j\beta z} e^{-\alpha z}$$

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Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossless**

$$\alpha(\omega) \approx 0$$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) \\ \mu(\omega) = \mu_1(\omega) \\ \sigma = 0 \end{cases}$$

# Plane Waves : dispersion

$$\{E_x, H_y\}$$

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$$\frac{1}{\pi} \int_0^{\infty} E^+(\omega) e^{-j\beta(\omega)z} e^{j\omega t} d\omega$$

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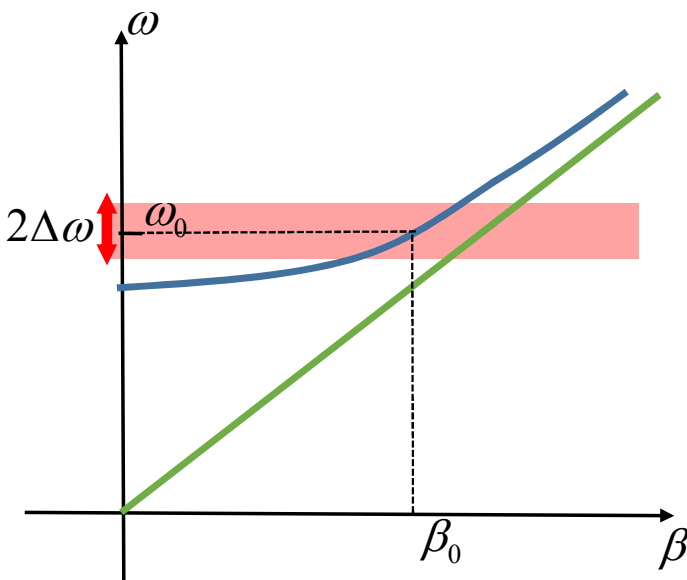
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nondispersive :  $\beta = \omega \sqrt{\mu\varepsilon}$

dispersive :  $\beta = \omega \sqrt{\mu(\omega)\varepsilon(\omega)}$

$$\frac{1}{\pi} \int_0^{\infty} E^+(\omega) e^{-j\beta(\omega)z} e^{j\omega t} d\omega \approx \frac{1}{\pi} \int_{\omega_0 - \Delta\omega}^{\omega_0 + \Delta\omega} E^+(\omega) e^{-j\beta(\omega)z} e^{j\omega t} d\omega$$

$$\beta(\omega) \approx \beta(\omega_0) + \beta'(\omega_0)(\omega - \omega_0) + \frac{1}{2} \beta''(\omega_0)(\omega - \omega_0)^2 + \dots$$

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# Plane Waves : dispersion

$$\begin{aligned} \frac{1}{\pi} \int_0^{\infty} E^+(\omega) e^{-j\beta(\omega)z} e^{j\omega t} d\omega &\approx \frac{1}{\pi} \int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega) e^{-j\beta(\omega)z} e^{j\omega t} d\omega \approx \frac{1}{\pi} \int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega) e^{-j\left[\beta_0 + \frac{(\omega-\omega_0)}{v_g}\right]z} e^{j\omega t} d\omega \\ &= \frac{1}{\pi} \int_{-\Delta\omega}^{\Delta\omega} E^+(\eta + \omega_0) e^{-j\left[\beta_0 + \frac{\eta}{v_g}\right]z} e^{j(\eta + \omega_0)t} d\eta = \frac{1}{\pi} e^{-j\beta_0 z} e^{j\omega_0 t} \int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0 + \eta) e^{-j\frac{\eta}{v_g}z} e^{j\eta t} d\eta \\ &= \frac{1}{\pi} e^{j\omega_0\left(t - \frac{z}{v_p}\right)} \int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0 + \eta) e^{j\left(t - \frac{z}{v_g}\right)\eta} d\eta \end{aligned}$$

$\eta = \omega - \omega_0$

$$e^{-j\beta_0 z} e^{j\omega_0 t} = e^{j\omega_0\left(t - \frac{\beta_0 z}{\omega_0}\right)} = e^{j\omega_0\left(t - \frac{z}{v_p}\right)}$$

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# Plane Waves : dispersion

$$\frac{1}{\pi} \int_0^{\infty} E^+(\omega) e^{-j\beta(\omega)z} e^{j\omega t} d\omega$$

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$$e_x^+(z=0, t) = \text{Re} \left\{ \frac{1}{\pi} e^{j\omega_0 t} \int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0 + \eta) e^{j\eta t} d\eta \right\} = \text{Re} \left\{ \frac{1}{\pi} e^{j\omega_0 t} f(t) \right\} = \frac{1}{\pi} f(t) \cos[\omega_0 t]$$

$f(t)$

$$e_x^+(z > 0, t) = \text{Re} \left\{ \frac{1}{\pi} e^{j\omega_0 \left( t - \frac{z}{v_p} \right)} \int_{-\Delta\omega}^{\Delta\omega} E^+(\omega_0 + \eta) e^{j \left( t - \frac{z}{v_g} \right) \eta} d\eta \right\} = \text{Re} \left\{ \frac{1}{\pi} e^{j\omega_0 \left( t - \frac{z}{v_p} \right)} f \left( t - \frac{z}{v_g} \right) \right\}$$

$$= \frac{1}{\pi} f \left( t - \frac{z}{v_g} \right) \cos \left[ \omega_0 \left( t - \frac{z}{v_p} \right) \right]$$

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$$e_x^+(z=0, t)$$

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$$e_x^+(z > 0, t)$$

$$= \frac{1}{\pi} f \left( t - \frac{z}{v_g} \right) \cos \left[ \omega_0 \left( t - \frac{z}{v_p} \right) \right]$$

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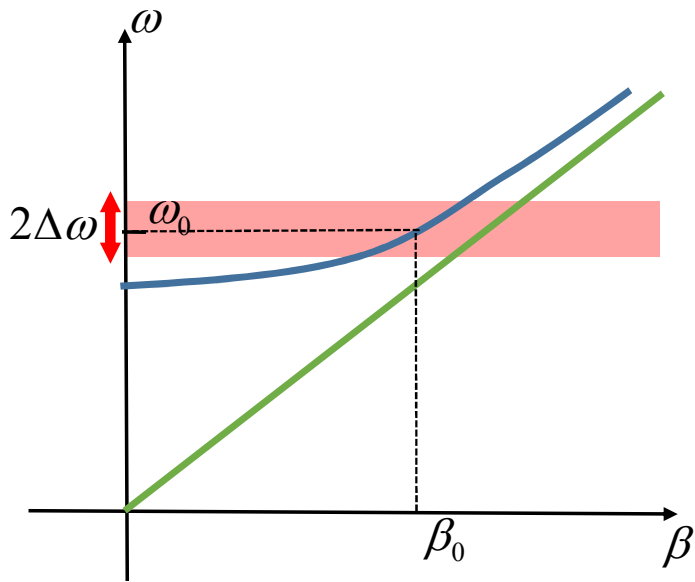
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# Plane Waves : dispersion



nondispersive :  $\beta = \omega\sqrt{\mu\varepsilon}$

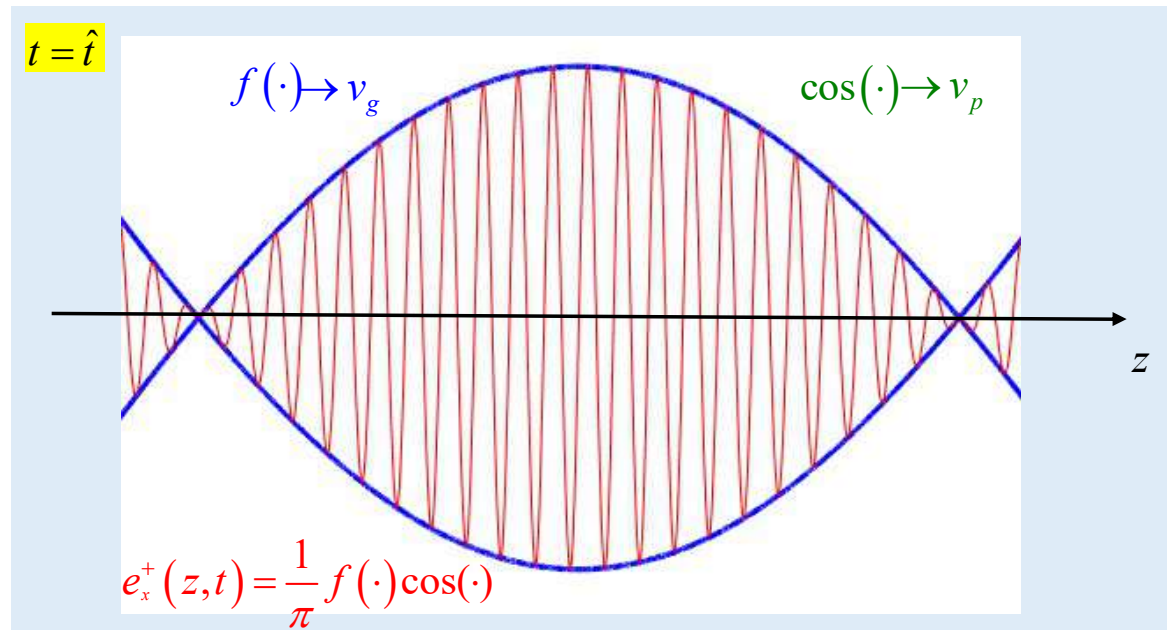
dispersive :  $\beta = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$

$$e_x^+(z,t) = \frac{1}{\pi} f\left(t - \frac{z}{v_g}\right) \cos\left[\omega_0\left(t - \frac{z}{v_p}\right)\right]$$

$$\beta_0 = \beta(\omega_0)$$

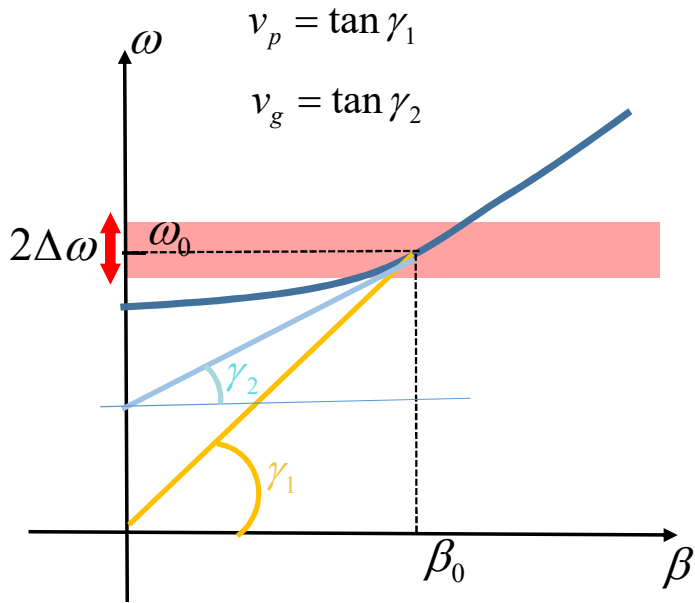
$$v_g = \frac{1}{\beta'(\omega_0)}$$

$$v_p = \frac{\omega_0}{\beta_0}$$



$$e_x^+(z,t) = \frac{1}{\pi} f(\cdot) \cos(\cdot)$$

# Plane Waves : dispersion



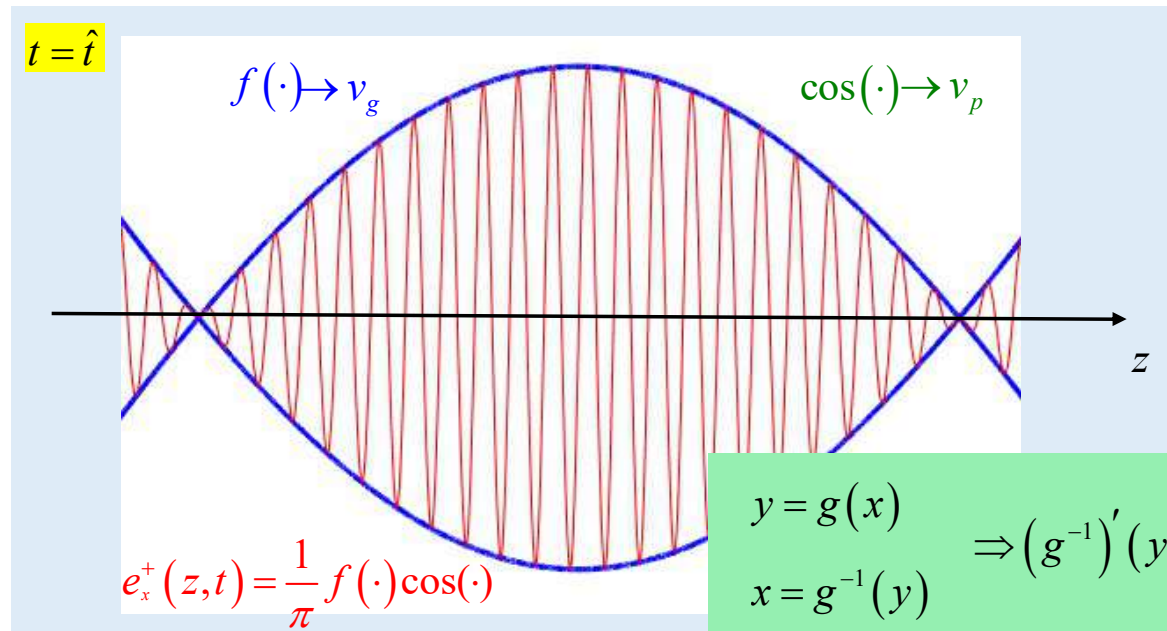
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$$\beta_0 = \beta(\omega_0)$$

$$v_g = \frac{1}{\beta'(\omega_0)} = v'(\beta_0)$$

$$v_p = \frac{\omega_0}{\beta_0}$$



$$y = g(x) \Rightarrow (g^{-1})'(y_0) = \frac{1}{g'(x_0)}$$

$$x = g^{-1}(y)$$

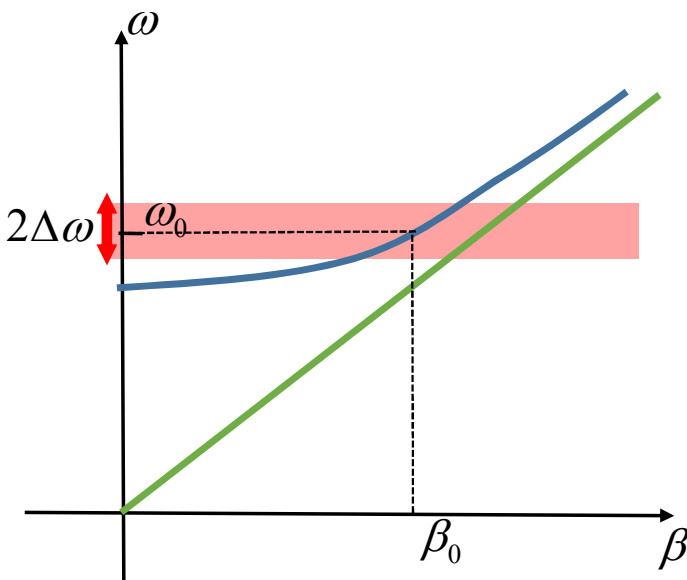
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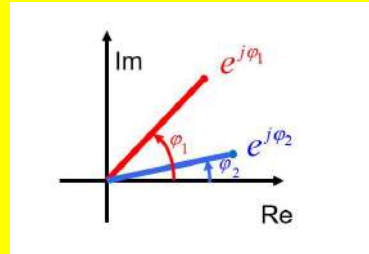
$$v_g = \frac{1}{\beta'(\omega_0)}$$



# Plane Waves : dispersion

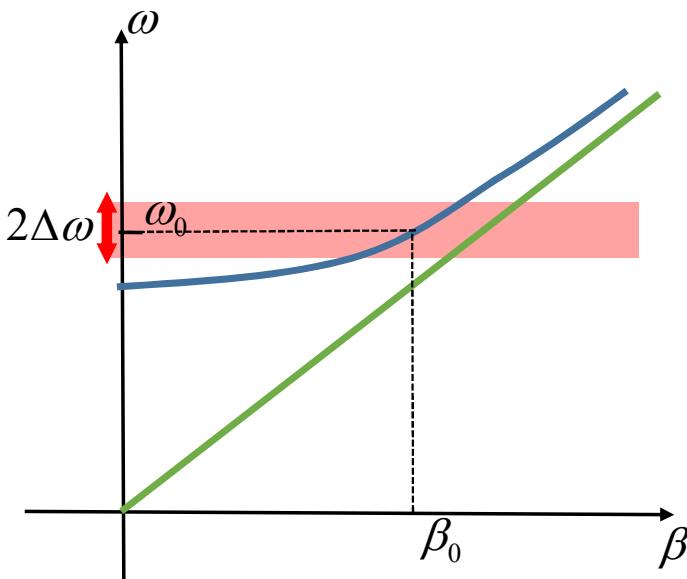
$$e^{-j\beta(\omega)z} = e^{-j\beta(\omega_0)z} e^{-j\beta'(\omega_0)(\omega-\omega_0)z} \cancel{e^{-j\frac{\beta''(\omega_0)}{2}(\omega-\omega_0)^2 z}} \dots$$

$$e^{-j\frac{\beta''(\omega_0)}{2}(\omega-\omega_0)^2 z} \approx 1$$



$$\frac{1}{2} \beta''(\omega_0) \Delta\omega^2 z \ll 2\pi$$

Channel & carrier frequency



nondispersive:  $\beta = \omega\sqrt{\mu\varepsilon}$

dispersive:  $\beta = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$

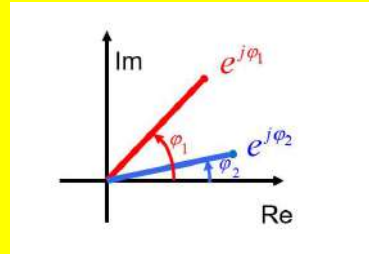
$$\frac{1}{\pi} \int_0^\infty E^+(\omega) e^{-j\beta(\omega)z} e^{j\omega t} d\omega \approx \frac{1}{\pi} \int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega) e^{-j\beta(\omega)z} e^{j\omega t} d\omega$$

$$\beta(\omega) \approx \beta(\omega_0) + \beta'(\omega_0)(\omega - \omega_0) + \cancel{\frac{1}{2} \beta''(\omega_0)(\omega - \omega_0)^2} + \dots$$

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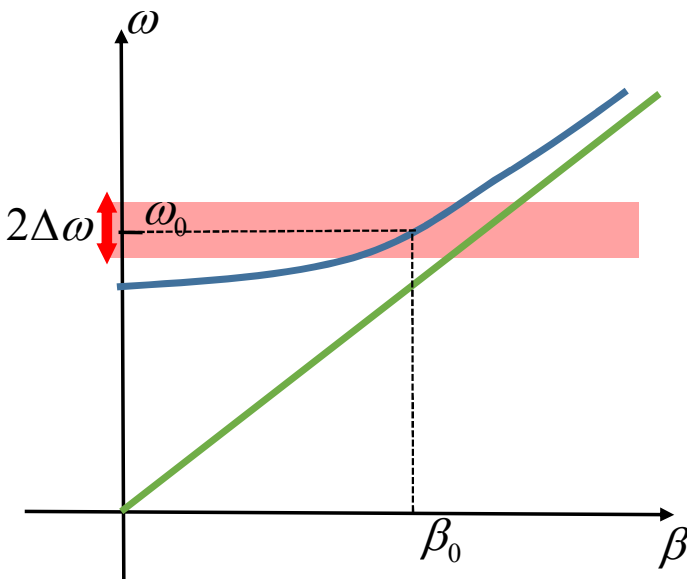
$$e^{-j\beta(\omega)z} = e^{-j\beta(\omega_0)z} e^{-j\beta'(\omega_0)(\omega-\omega_0)z} \cancel{e^{-j\frac{\beta''(\omega_0)}{2}(\omega-\omega_0)^2 z}} \dots$$

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Channel & carrier frequency      Bandwidth



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dispersive:  $\beta = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$

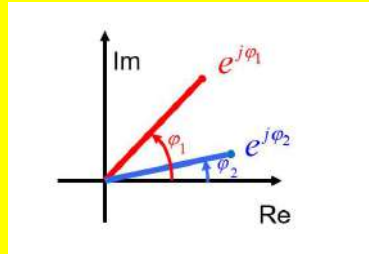
$$\frac{1}{\pi} \int_0^\infty E^+(\omega) e^{-j\beta(\omega)z} e^{j\omega t} d\omega \approx \frac{1}{\pi} \int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega) e^{-j\beta(\omega)z} e^{j\omega t} d\omega$$

$$\beta(\omega) \approx \beta(\omega_0) + \beta'(\omega_0)(\omega - \omega_0) + \cancel{\frac{1}{2}\beta''(\omega_0)(\omega - \omega_0)^2} + \dots$$

# Plane Waves : dispersion

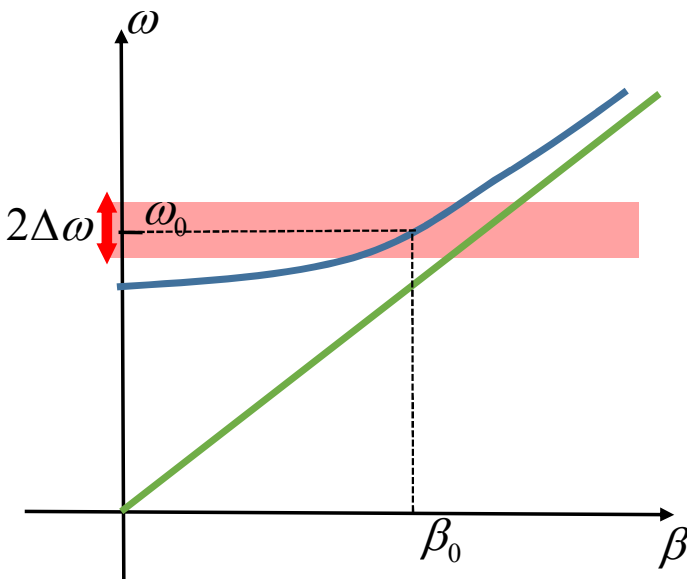
$$e^{-j\beta(\omega)z} = e^{-j\beta(\omega_0)z} e^{-j\beta'(\omega_0)(\omega-\omega_0)z} \cancel{e^{-j\frac{\beta''(\omega_0)}{2}(\omega-\omega_0)^2 z}} \dots$$

$$e^{-j\frac{\beta''(\omega_0)}{2}(\omega-\omega_0)^2 z} \approx 1$$



$$\frac{1}{2}\beta''(\omega_0)\Delta\omega^2 z \ll 2\pi$$

Channel & carrier frequency      Bandwidth      Distance



nondispersive :  $\beta = \omega\sqrt{\mu\varepsilon}$

dispersive :  $\beta = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$

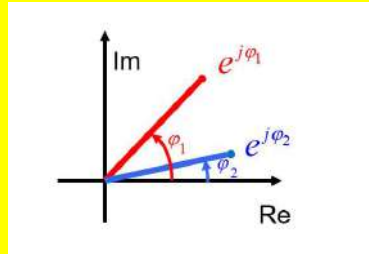
$$\frac{1}{\pi} \int_0^{\infty} E^+(\omega) e^{-j\beta(\omega)z} e^{j\omega t} d\omega \approx \frac{1}{\pi} \int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} E^+(\omega) e^{-j\beta(\omega)z} e^{j\omega t} d\omega$$

$$\beta(\omega) \approx \beta(\omega_0) + \beta'(\omega_0)(\omega - \omega_0) + \cancel{\frac{1}{2}\beta''(\omega_0)(\omega - \omega_0)^2} + \dots$$

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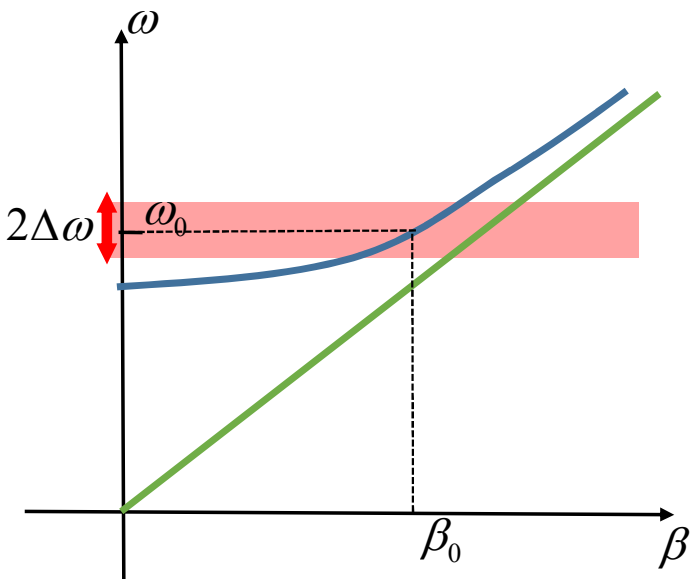
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