

A large satellite dish antenna is mounted on a tall metal tower. The dish is dark and has a grid-like structure. The background is a sunset sky with soft, colorful clouds in shades of orange, yellow, and blue. The overall scene is slightly blurred, giving it a cinematic feel.

Campi Elettromagnetici

**Corso di Laurea in Ingegneria Informatica,
Biomedica e delle Telecomunicazioni**

a.a. 2021-2022 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

Phasor Domain (PD)

Fourier Domain (FD)

Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

Plane Waves (Spectral Domains)

$$\begin{cases} \frac{dE_x}{dz} = -j\omega_0\mu H_y \\ \frac{dH_y}{dz} = -j\omega_0\varepsilon E_x \end{cases} \quad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$\begin{cases} k = \omega\sqrt{\mu\varepsilon} \\ k = \beta - j\alpha \end{cases}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain

$$\begin{cases} \frac{dE_x}{dz} = -j\omega\mu H_y \\ \frac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \quad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z, \omega) = E_x^+(\omega) e^{-jkz} + E_x^-(\omega) e^{jkz}$$

$$\zeta H_y(z, \omega) = E_x^+(\omega) e^{-jkz} - E_x^-(\omega) e^{jkz}$$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \end{cases}$$

$$k(\omega) = \omega \sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

Fourier Domain

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- ~~- Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

Independent

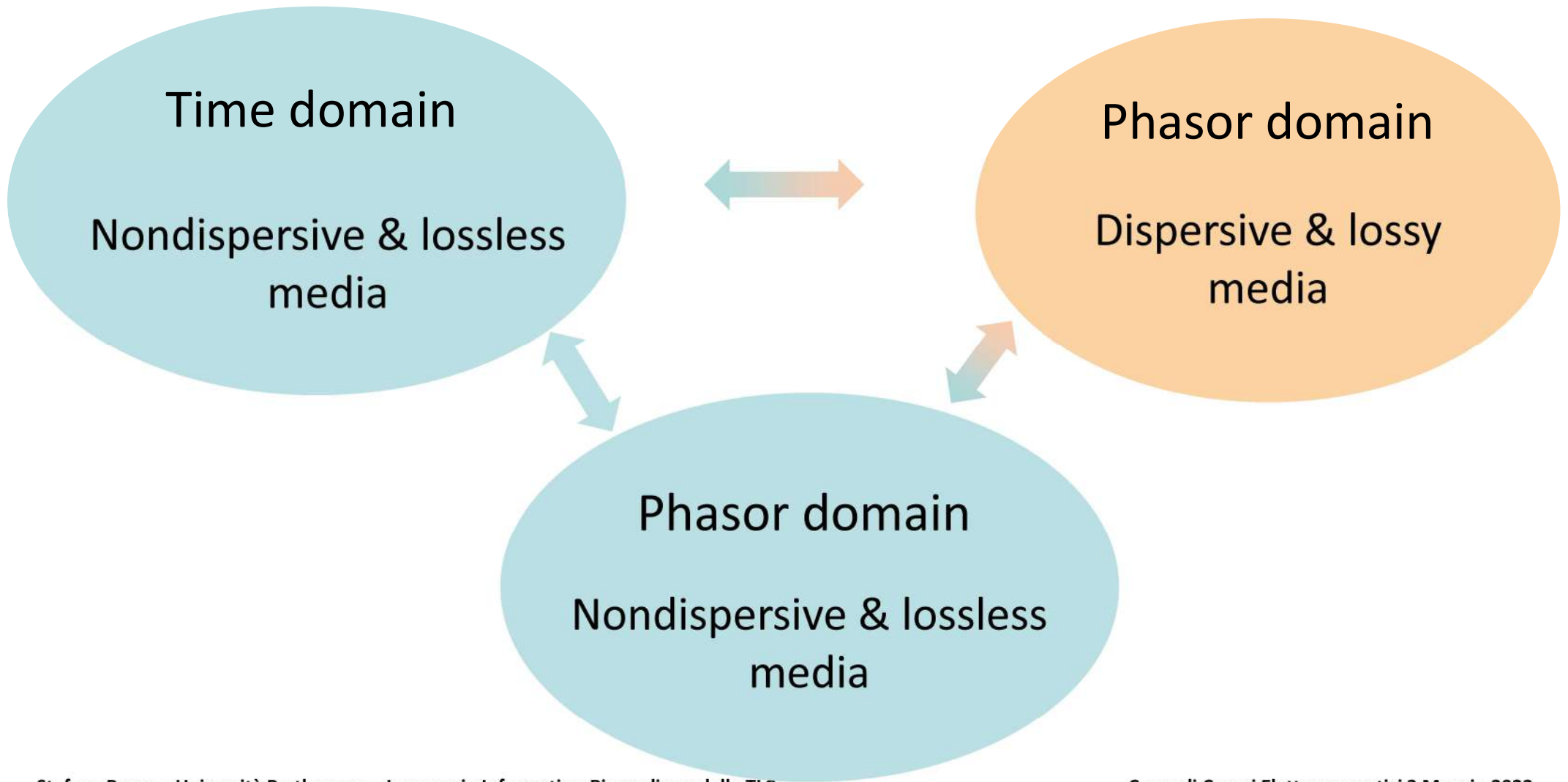
$$\{E_x, H_y\}$$

each other

Plane Waves

Phasor domain

Razionale



Plane Waves (**Phasor Domain**)

$$\begin{cases} \frac{dE_x}{dz} = -j\omega_0\mu H_y \\ \frac{dH_y}{dz} = -j\omega_0\varepsilon E_x \end{cases} \quad \{E_x, H_y\}$$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain

- Source-free**
- Medium**
- Linear
 - **Time dispersive**
 - Space non-dispersive
 - Isotropic
 - Homogeneous (TI - SI)
 - ~~- Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

↓

$$E_z = H_z = 0$$

$\{E_y, H_x\}$ Independent each other

$\{E_x, H_y\}$ Independent each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

Source-free

Medium

- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- Lossless

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma \neq 0 \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

Independent
each other

$$\{E_x, H_y\}$$

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\epsilon} = \beta(\omega_0)$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$E_x^-(z) = E^- e^{j\beta z}$$

$$\zeta H_y^-(z) = -E^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\epsilon}}$$

Source-free

Medium

- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- Lossless

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \phi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$E^- e^{j\beta z} \rightarrow e_x^-(z, t) = |E^-| \cos(\omega_0 t + \beta z + \phi^-) = e_x^-(z + v_p t)$$

Regressive plane wave

Time nondispersive & lossless

$$\begin{cases} \epsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\epsilon}}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\epsilon} = \beta(\omega_0)$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
 $\{E_x, H_y\}$ Independent each other

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}} = c$$

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0 \sqrt{\mu\epsilon} = \beta(\omega_0)$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$E_x^-(z) = E^- e^{j\beta z}$$

$$\zeta H_y^-(z) = -E^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\epsilon}}$$

Source-free

Medium

- Linear
- **Time nondispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossless**

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \phi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$E^- e^{j\beta z} \rightarrow e_x^-(z, t) = |E^-| \cos(\omega_0 t + \beta z + \phi^-) = e_x^-(z + v_p t)$$

Regressive plane wave

Time nondispersive & lossless

$$\begin{cases} \epsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$$\vec{S}(\vec{r}) = \frac{1}{2} \vec{E}(\vec{r}) \times \vec{H}^*(\vec{r})$$

$$\vec{S}^+(\vec{r}) = \frac{1}{2} \vec{E}^+(\vec{r}) \times \vec{H}^{+*}(\vec{r}) = \frac{1}{2} E_x^+(z) \hat{i}_x \times H_y^{+*}(z) \hat{i}_y = \frac{1}{2} (E^+ e^{-j\beta z}) \frac{(E^+ e^{-j\beta z})^*}{\zeta} \hat{i}_z$$

$$= \frac{|E^+|^2}{2\zeta} \hat{i}_z = \frac{|E_x^+(z)|^2}{2\zeta} \hat{i}_z = \zeta \frac{|H_y^+(z)|^2}{2} \hat{i}_z$$

$$\hat{i}_z = \hat{i}_x \times \hat{i}_y$$

$$\hat{i}_y = \hat{i}_z \times \hat{i}_x$$

$$\hat{i}_x = \hat{i}_y \times \hat{i}_z$$

$\{E_y, H_x\}$
 $\{E_x, H_y\}$ Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0 \sqrt{\mu\epsilon} = \beta(\omega_0)$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$E_x^-(z) = E^- e^{j\beta z}$$

$$\zeta H_y^-(z) = -E^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\epsilon}}$$

Source-free

Medium

- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- Lossless

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \phi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$E^- e^{j\beta z} \rightarrow e_x^-(z, t) = |E^-| \cos(\omega_0 t + \beta z + \phi^-) = e_x^-(z + v_p t)$$

Regressive plane wave

Time nondispersive & lossless

$$\begin{cases} \epsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$$\vec{S}(\vec{r}) = \frac{1}{2} \vec{E}(\vec{r}) \times \vec{H}^*(\vec{r})$$

$$\vec{S}^-(\vec{r}) = \frac{1}{2} \vec{E}^-(\vec{r}) \times \vec{H}^{-*}(\vec{r}) = \frac{1}{2} E_x^-(z) \hat{i}_x \times H_y^{-*}(z) \hat{i}_y = -\frac{|E_x^-(z)|^2}{2\zeta} \hat{i}_z$$

$$= -\zeta \frac{|H_y^-(z)|^2}{2} \hat{i}_z$$

$$\hat{i}_z = \hat{i}_x \times \hat{i}_y$$

$$\hat{i}_y = \hat{i}_z \times \hat{i}_x$$

$$\hat{i}_x = \hat{i}_y \times \hat{i}_z$$

$\{E_y, H_x\}$
 $\{E_x, H_y\}$ Independent each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$E_x^-(z) = E^- e^{j\beta z}$$

$$\zeta H_y^-(z) = -E^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Source-free

Medium

- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- Lossless

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$E^- e^{j\beta z} \rightarrow e_x^-(z, t) = |E^-| \cos(\omega_0 t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

Regressive plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

↓

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
 $\{E_x, H_y\}$ Independent each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\epsilon} = \beta(\omega_0)$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\epsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Time nondispersive & lossless

$$\begin{cases} \epsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\epsilon}}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\epsilon} = \beta(\omega_0)$$

Source-free

Medium

- Linear
- **Time nondispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

Independent

$$\{E_x, H_y\}$$

each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_0 = 2\pi f_0$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Time nondispersive & lossless

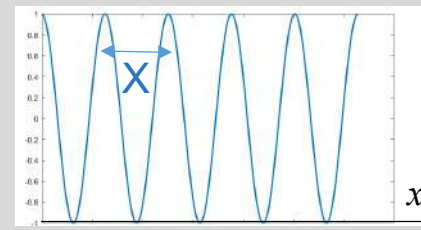
$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Memo



$$\cos(2\pi \nu x) = \cos\left(\frac{2\pi}{X} x\right)$$

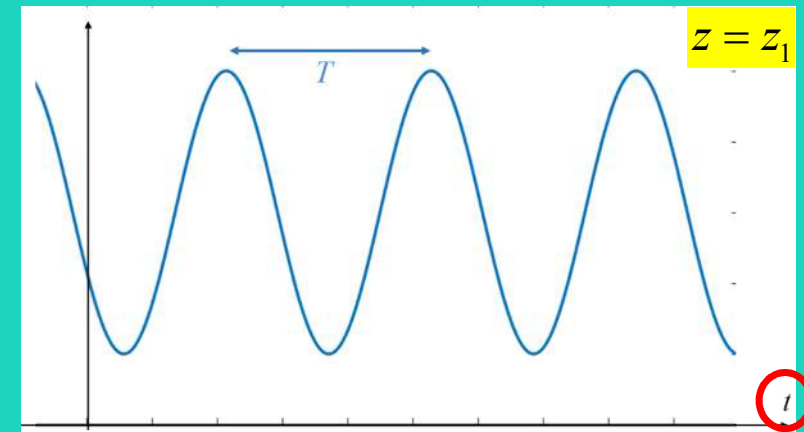
ν : frequency

$X = \frac{1}{\nu}$: period

$$e_x^+(z = z_1, t) = |E^+| \cos(\omega_0 t - \beta z_1 + \varphi^+) = |E^+| \cos\left(2\pi \left[\frac{\omega_0}{2\pi}\right] t - \beta z_1 + \varphi^+\right)$$

$$\text{frequency: } f_0 = \frac{\omega_0}{2\pi}$$

$$\text{period: } T = \frac{1}{f_0} = \frac{2\pi}{\omega_0}$$



Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_0 = 2\pi f_0$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

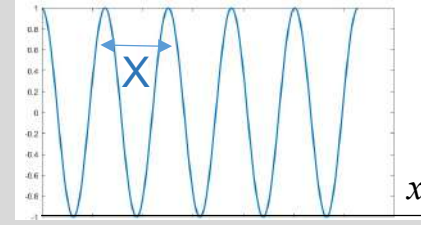
$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Memo



$$\cos(2\pi \nu x) = \cos\left(\frac{2\pi}{X} x\right)$$

ν : frequency

$X = \frac{1}{\nu}$: period

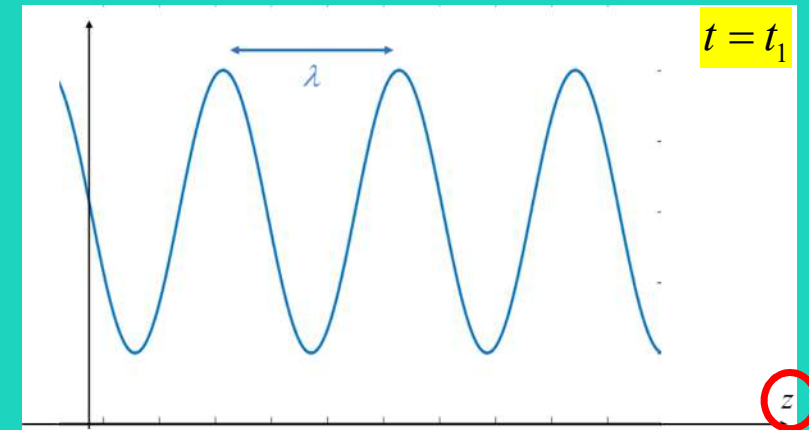
$$e_x^+(z, t = t_1) = |E^+| \cos(\omega_0 t_1 - \beta z + \varphi^+) = |E^+| \cos(-\omega_0 t_1 + \beta z - \varphi^+)$$

$$= |E^+| \cos\left(2\pi \left[\frac{\beta}{2\pi}\right] z - \omega_0 t_1 - \varphi^+\right)$$

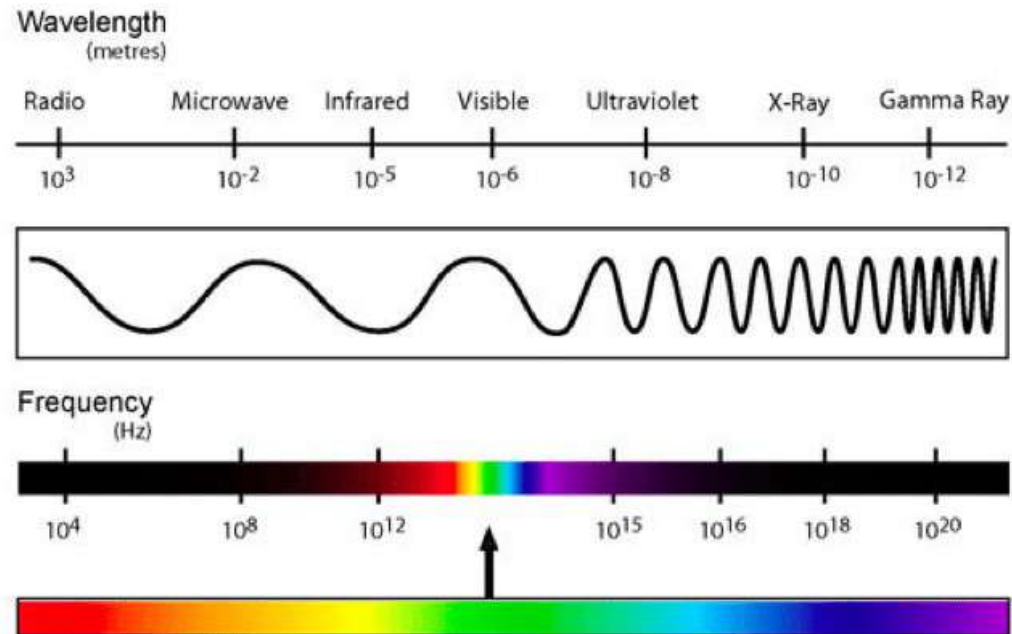
λ : wavelength

$$\text{frequency: } \nu = \frac{\beta}{2\pi}$$

$$\text{period: } \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{c}{f_0}$$



Electromagnetic spectrum



Free space (Linear, isotropic, local, homogeneous)

$$\lambda = \frac{c}{f}$$

f : frequency
 λ : wavelength
 c : light speed

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_0 = 2\pi f_0$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

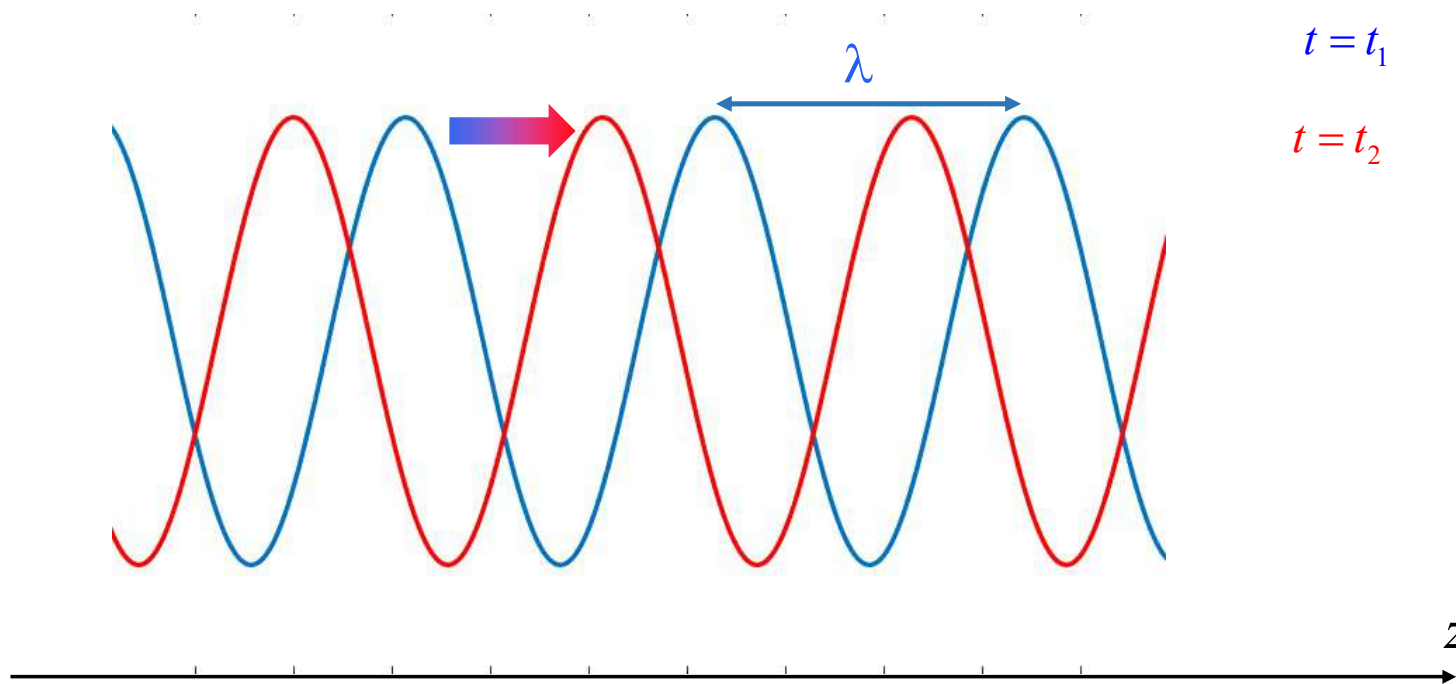
Progressive plane wave

- The term $e^{-j\beta z}$ is related to the propagation along the (positive sense of the) z-axis

Plane Waves (Phasor Domain)

$$e_x^+(z,t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}} = c$$



Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_0 = 2\pi f_0$$

Time nondispersive & lossless

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

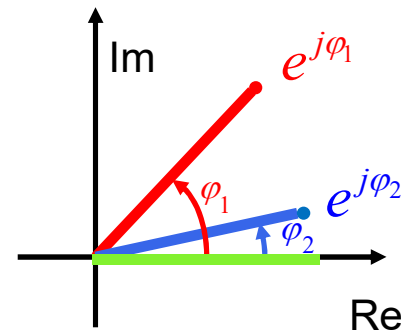
$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$e^{-j\beta z} = e^{-j\frac{2\pi}{\lambda} z}$$

$$z \ll \lambda \Rightarrow \frac{2\pi}{\lambda} z \ll 2\pi \Rightarrow e^{-j\frac{2\pi}{\lambda} z} \approx 1$$



Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_0 = 2\pi f_0$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

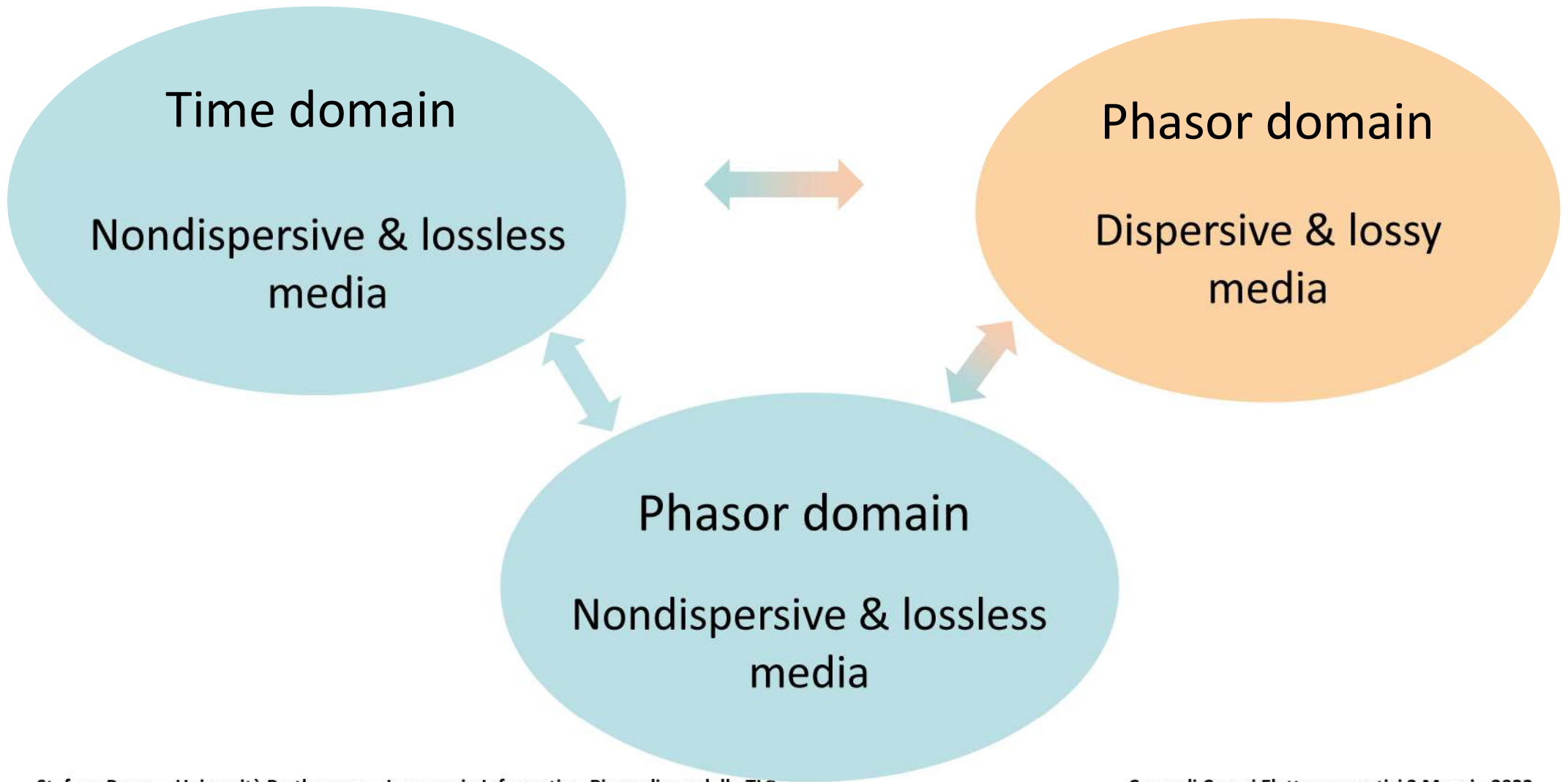
$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

- The term $e^{-j\beta z}$ is related to the propagation along the (positive sense of the) z-axis
- When z is small with respect to λ , the propagation effects become negligible

Razionale



Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_0 = 2\pi f_0$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\omega_0 = 2\pi f_0$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\omega_0 = 2\pi f_0$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\omega_0 = 2\pi f_0$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Source-free

Medium

- Linear
- **Time nondispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\omega_0 = 2\pi f_0$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Source-free

Medium

- Linear
- **Time nondispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu \epsilon} = \beta(\omega_0)$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu \epsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\omega_0 = 2\pi f_0$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \phi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

Time nondispersive & lossless

$$\begin{cases} \epsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu \epsilon}}$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

~~$$k(\omega_0) = \omega_0 \sqrt{\mu\epsilon} = \beta(\omega_0)$$~~

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\omega_0 = 2\pi f_0$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \phi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Time dispersive (lossy)

$$\begin{cases} \epsilon(\omega_0) = \epsilon_1(\omega_0) - j\epsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma: \text{real} \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \epsilon: \text{real} \\ \mu: \text{real} \\ \sigma = 0 \end{cases} \quad c = \frac{1}{\sqrt{\mu\epsilon}}$$

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f_0}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\omega_0 = 2\pi f_0$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma: real \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon: real \\ \mu: real \\ \sigma = 0 \end{cases} \quad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

~~$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$~~

$$\omega_0 = 2\pi f_0$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma: \text{real} \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon: \text{real} \\ \mu: \text{real} \\ \sigma = 0 \end{cases} \quad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\omega_0 = 2\pi f_0$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma: \text{real} \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon: \text{real} \\ \mu: \text{real} \\ \sigma = 0 \end{cases} \quad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

~~$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$~~

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\omega_0 = 2\pi f_0$$

~~$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$~~

Progressive plane wave

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma: real \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon: real \\ \mu: real \\ \sigma = 0 \end{cases} \quad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x^+(z) = E^+ e^{-jkz}$$

$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\omega_0 = 2\pi f_0$$

$$E^+ e^{-jkz} \rightarrow$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma: real \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon: real \\ \mu: real \\ \sigma = 0 \end{cases} \quad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Source-free

Medium

- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- Lossy

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$\lambda = \frac{2\pi}{\beta}$$

$$E_x^+(z) = E^+ e^{-jkz}$$

$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\omega_0 = 2\pi f_0$$

$$E^+ e^{-jkz} \rightarrow e^+(z, t) = e^{-\alpha z} |E^+| \cos(\omega_0 t - \beta z + \varphi^+)$$

$$E^+ e^{-jkz} = E^+ e^{-j(\beta - j\alpha)z} = E^+ e^{(-j\beta z - \alpha z)} = E^+ e^{-j\beta z} e^{-\alpha z}$$

$$E^+ = |E^+| e^{j\varphi^+} \Rightarrow E^+ e^{-jkz} = |E^+| e^{j\varphi^+} e^{-j\beta z} e^{-\alpha z}$$

$$e_x^+(z, t) = \text{Re} \left\{ |E^+| e^{j\varphi^+} e^{-j\beta z} e^{-\alpha z} e^{j\omega_0 t} \right\} = e^{-\alpha z} |E^+| \cos(\omega_0 t - \beta z + \varphi^+)$$

$$= e^{-\alpha z} |E^+| \cos \left(-\beta \left[z - \frac{\omega_0}{\beta} t \right] + \varphi^+ \right) = e^{-\alpha z} |E^+| \cos \left(-\beta [z - v_p t] + \varphi^+ \right)$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma: \text{real} \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon: \text{real} \\ \mu: \text{real} \\ \sigma = 0 \end{cases} \quad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$$\begin{cases} \{E_y, H_x\} \\ \{E_x, H_y\} \end{cases} \quad \text{Independent each other}$$

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\epsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$\lambda = \frac{2\pi}{\beta}$$

$$E_x^+(z) = E^+ e^{-jkz}$$

$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\epsilon(\omega_0)}}$$

$$\omega_0 = 2\pi f_0$$

$$E^+ e^{-jkz} \rightarrow e^+(z,t) = e^{-\alpha z} |E^+| \cos(\omega_0 t - \beta z + \phi^+)$$

Progressive plane wave

Time dispersive (lossy)

$$\begin{cases} \epsilon(\omega_0) = \epsilon_1(\omega_0) - j\epsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma: real \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \epsilon: real \\ \mu: real \\ \sigma = 0 \end{cases} \quad c = \frac{1}{\sqrt{\mu\epsilon}}$$

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f_0}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\omega_0 = 2\pi f_0$$

$$E^+ e^{-jkz} \rightarrow e^+(z,t) = e^{-\alpha z} |E^+| \cos(\omega_0 t - \beta z + \varphi^+)$$

Progressive plane wave

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega_0} v_p = \frac{v_p}{f_0}$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma: \text{real} \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon: \text{real} \\ \mu: \text{real} \\ \sigma = 0 \end{cases} \quad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f_0}$$

$$E_x^+(z) = E^+ e^{-jkz}$$

$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\omega_0 = 2\pi f_0$$

$$E^+ e^{-jkz} \rightarrow e^+(z,t) = e^{-\alpha z} |E^+| \cos(\omega_0 t - \beta z + \varphi^+)$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma: \text{real} \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon: \text{real} \\ \mu: \text{real} \\ \sigma = 0 \end{cases} \quad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent
each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f_0}$$

$$E_x^+(z) = E^+ e^{-jkz}$$

$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\omega_0 = 2\pi f_0$$

$$E^+ e^{-jkz} \rightarrow e^+(z, t) = e^{-\alpha z} |E^+| \cos(\omega_0 t - \beta z + \varphi^+)$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \text{real} \end{cases}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu \varepsilon} = \beta(\omega_0)$$

$$\omega_0 = 2\pi f_0$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f_0}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu \varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

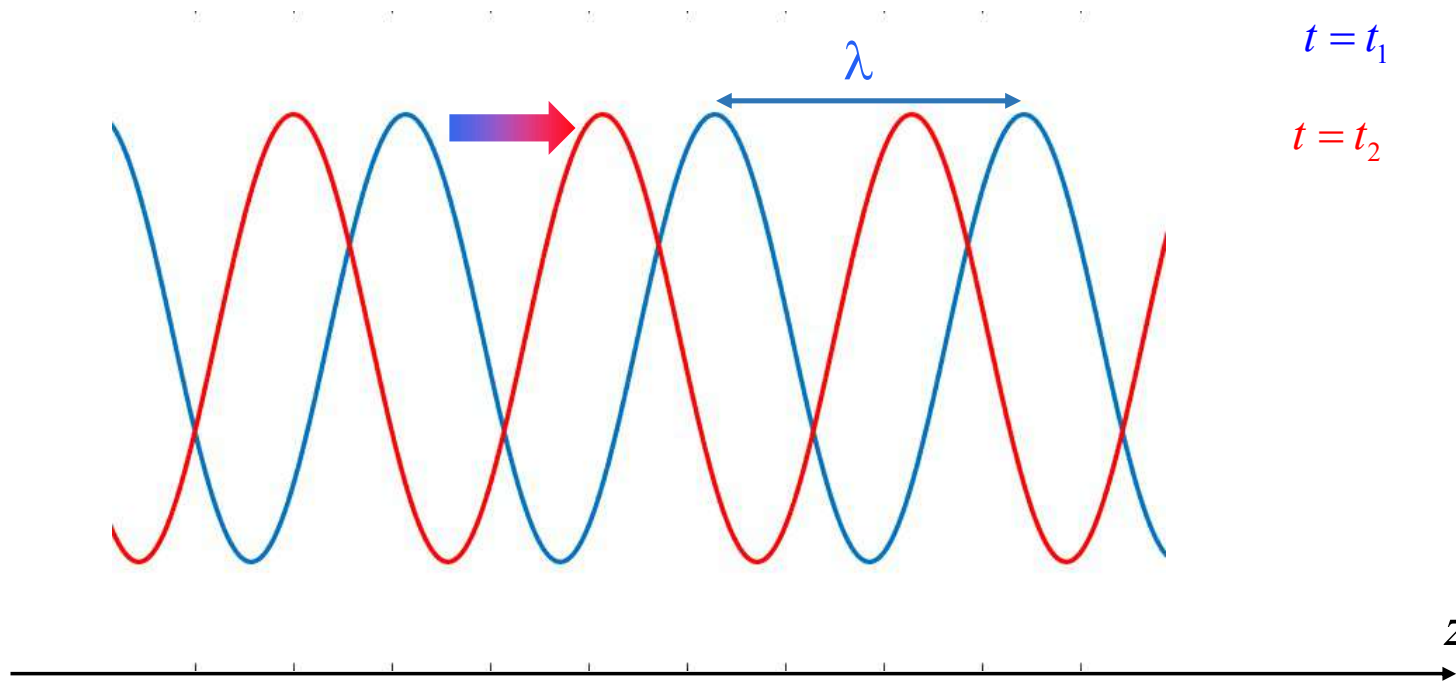
Independent each other

Plane Waves (Phasor Domain)

Time nondispersive & lossless medium

$$e_x^+(z,t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

$$v_p = \frac{\omega_0}{\beta}$$

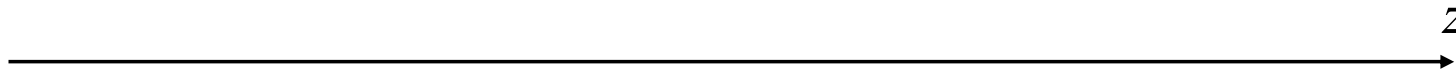


Plane Waves (Phasor Domain)

Time dispersive & lossy medium

$$e_x^+(z,t) = |E^+| e^{-\alpha z} \cos(\omega_0 t - \beta z + \phi^+) = e^{-\alpha z} e_x^+(z - v_p t)$$

$$v_p = \frac{\omega_0}{\beta}$$

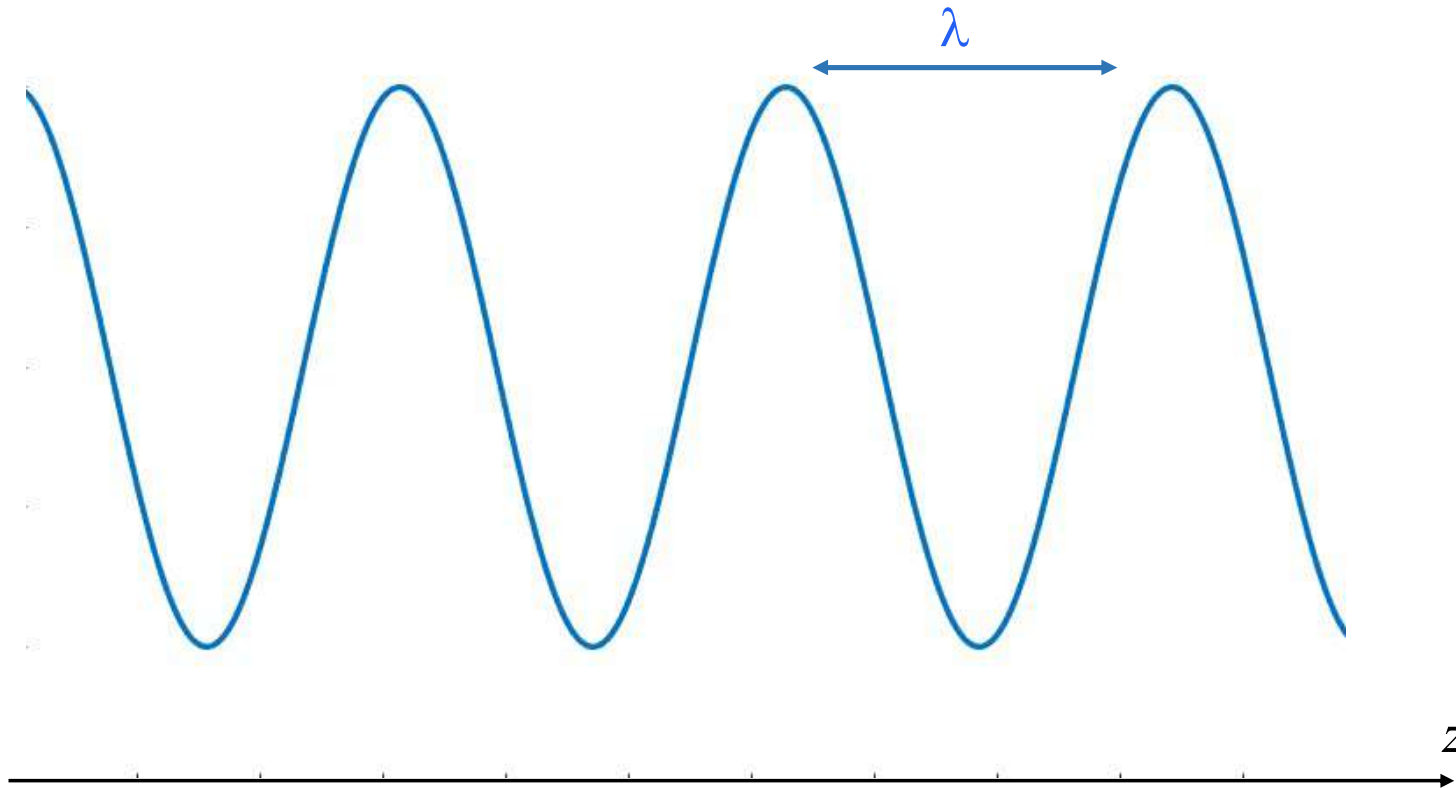


Plane Waves (Phasor Domain)

Time dispersive & lossy medium

$$e_x^+(z,t) = |E^+| e^{-\alpha z} \cos(\omega_0 t - \beta z + \varphi^+) = e^{-\alpha z} e_x^+(z - v_p t)$$

$$v_p = \frac{\omega_0}{\beta}$$



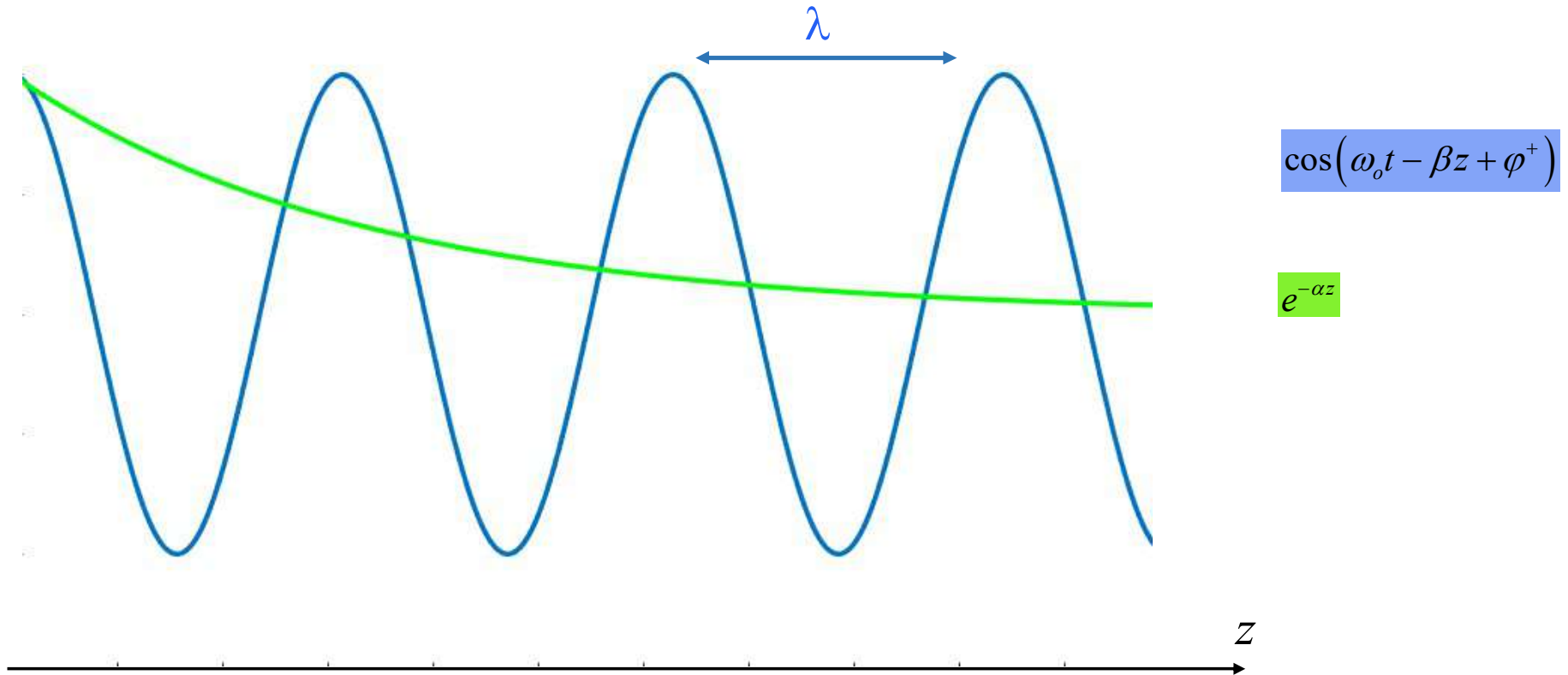
$$\cos(\omega_0 t - \beta z + \varphi^+)$$

Plane Waves (Phasor Domain)

Time dispersive & lossy medium

$$e_x^+(z,t) = |E^+| e^{-\alpha z} \cos(\omega_0 t - \beta z + \varphi^+) = e^{-\alpha z} e_x^+(z - v_p t)$$

$$v_p = \frac{\omega_0}{\beta}$$

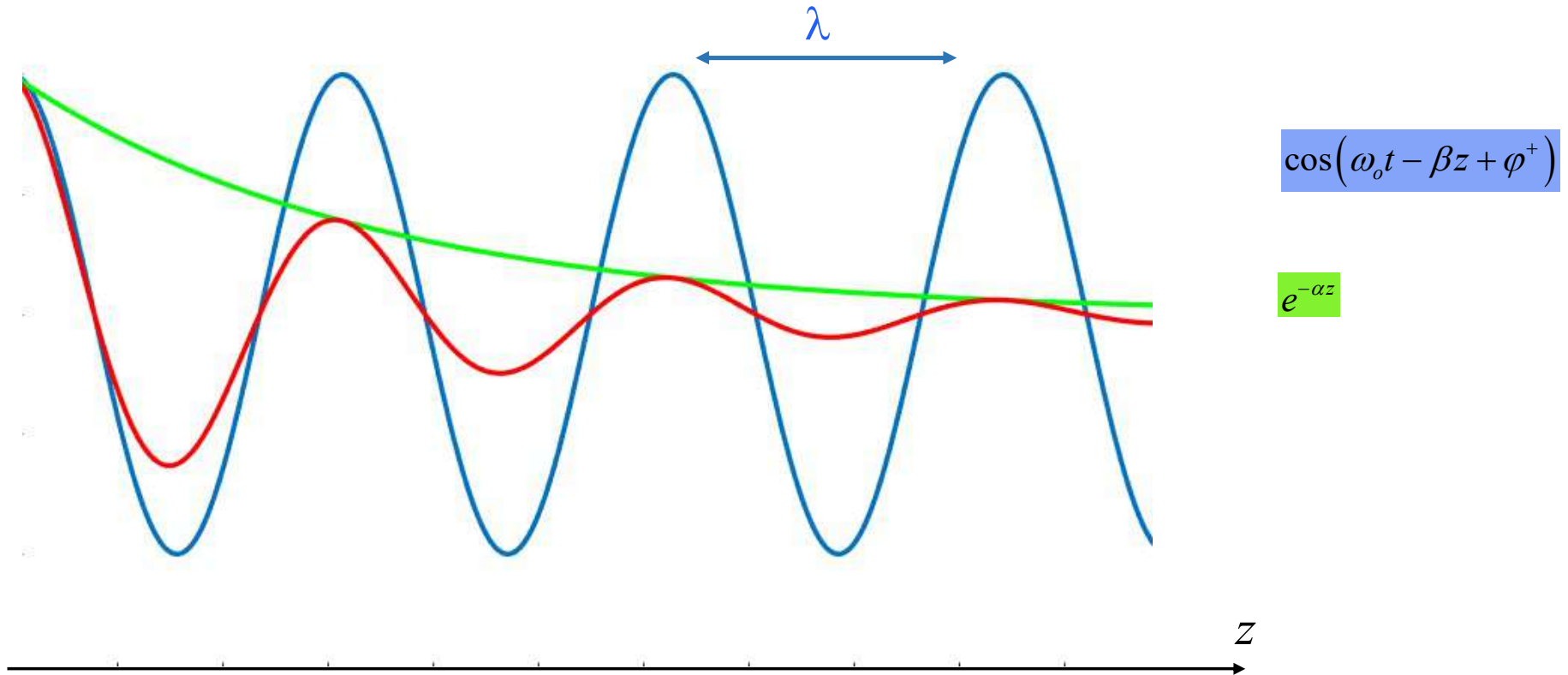


Plane Waves (Phasor Domain)

Time dispersive & lossy medium

$$e_x^+(z,t) = |E^+| e^{-\alpha z} \cos(\omega_0 t - \beta z + \varphi^+) = e^{-\alpha z} e_x^+(z - v_p t)$$

$$v_p = \frac{\omega_0}{\beta}$$

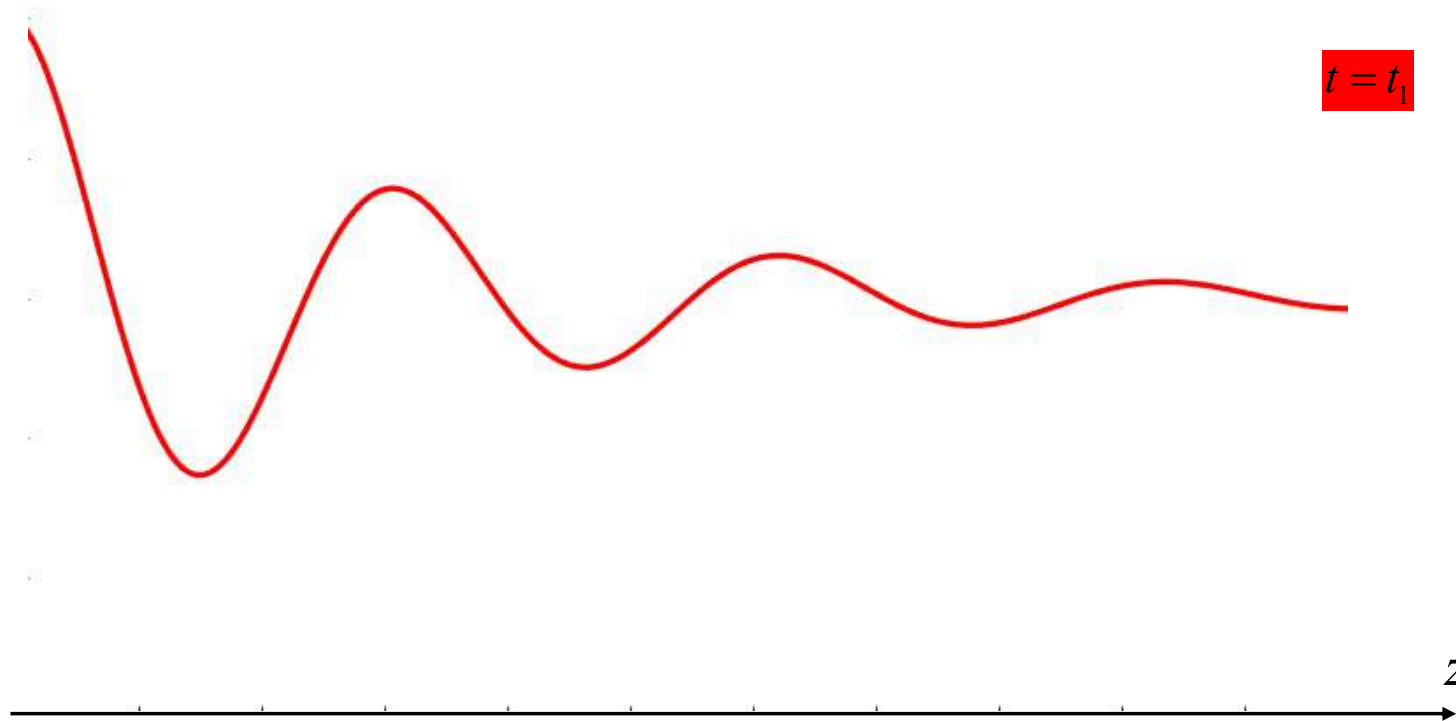


Plane Waves (Phasor Domain)

Time dispersive & lossy medium

$$e_x^+(z,t) = |E^+| e^{-\alpha z} \cos(\omega_0 t - \beta z + \varphi^+) = e^{-\alpha z} e_x^+(z - v_p t)$$

$$v_p = \frac{\omega_0}{\beta}$$

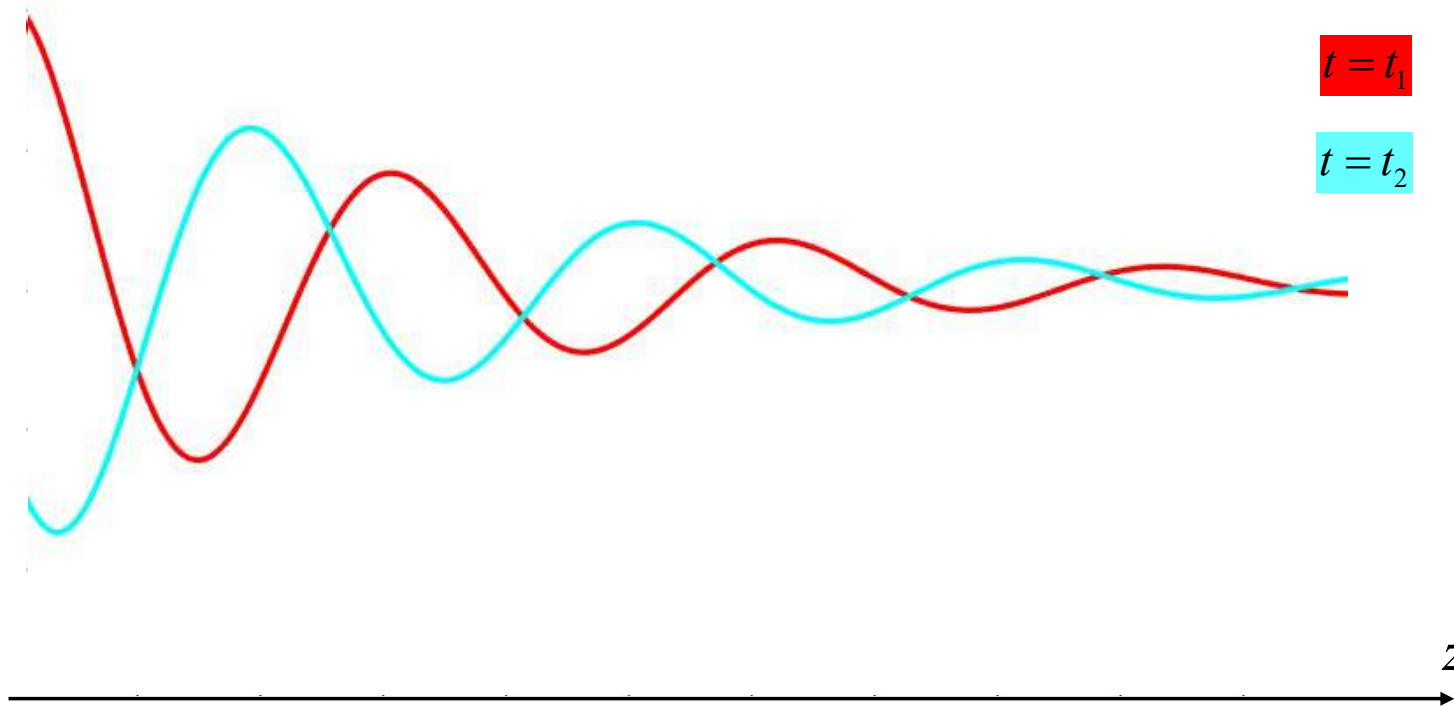


Plane Waves (Phasor Domain)

Time dispersive & lossy medium

$$e_x^+(z,t) = |E^+| e^{-\alpha z} \cos(\omega_0 t - \beta z + \varphi^+) = e^{-\alpha z} e_x^+(z - v_p t)$$

$$v_p = \frac{\omega_0}{\beta}$$



Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f_0}$$

$$E_x^+(z) = E^+ e^{-jkz}$$

$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

$$\omega_0 = 2\pi f_0$$

$$E^+ e^{-jkz} \rightarrow e^+(z, t) = e^{-\alpha z} |E^+| \cos(\omega_0 t - \beta z + \varphi^+)$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \text{real} \end{cases}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu \varepsilon} = \beta(\omega_0)$$

$$\omega_0 = 2\pi f_0$$

$$v_p = \frac{\omega_0}{\beta}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{v_p}{f_0}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$c = \frac{1}{\sqrt{\mu \varepsilon}}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x^+(z) = E^+ e^{-jkz}$$

$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$v_p = \frac{\omega_0}{\beta}$$

$$E^+ e^{-jkz} \rightarrow e^+(z, t) = e^{-\alpha z} |E^+| \cos(\omega_0 t - \beta z + \phi^+)$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \text{real} \end{cases}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu \varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta}$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \phi^+) = e_x^+(z - v_p t)$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases} \quad c = \frac{1}{\sqrt{\mu \varepsilon}}$$

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$$\begin{cases} \{E_y, H_x\} \\ \{E_x, H_y\} \end{cases} \quad \text{Independent each other}$$

Plane Waves (Phasor Domain)

$$\{E_x, H_y\}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x^+(z) = E^+ e^{-jkz}$$

$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\operatorname{Re}\{\omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)}\}} = \frac{1}{\operatorname{Re}\{\sqrt{\mu(\omega_0) \varepsilon(\omega_0)}\}} = v_p(\omega_0)$$

$$E^+ e^{-jkz} \rightarrow e^+(z, t) = e^{-\alpha z} |E^+| \cos(\omega_0 t - \beta z + \varphi^+)$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \text{real} \end{cases}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu \varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu \varepsilon}} = c$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases} \quad c = \frac{1}{\sqrt{\mu \varepsilon}}$$

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

$$\begin{cases} \{E_y, H_x\} \\ \{E_x, H_y\} \end{cases} \quad \text{Independent each other}$$

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x^+(z) = E^+ e^{-jkz}$$

$$\zeta H_y^+(z) = E^+ e^{-jkz}$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\operatorname{Re}\{\omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)}\}} = \frac{1}{\operatorname{Re}\{\sqrt{\mu(\omega_0) \varepsilon(\omega_0)}\}} = v_p(\omega_0)$$

$$E^+ e^{-jkz} \rightarrow e^+(z, t) = e^{-\alpha z} |E^+| \cos(\omega_0 t - \beta z + \phi^+)$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \text{real} \end{cases}$$

$$E_x^+(z) = E^+ e^{-j\beta z}$$

$$\zeta H_y^+(z) = E^+ e^{-j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu \varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0 \sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu \varepsilon}} = c$$

$$E^+ e^{-j\beta z} \rightarrow e_x^+(z, t) = |E^+| \cos(\omega_0 t - \beta z + \phi^+) = e_x^+(z - v_p t)$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases} \quad c = \frac{1}{\sqrt{\mu \varepsilon}}$$

Source-free

Medium

- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI - SI)
- **Lossy**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$E_z = H_z = 0$$

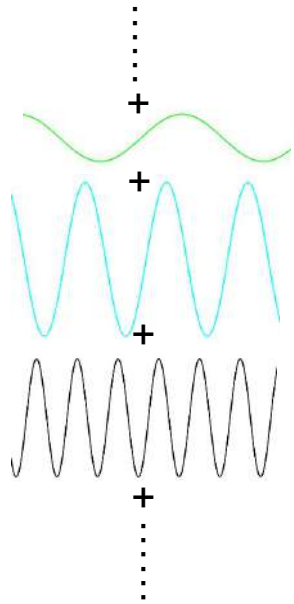
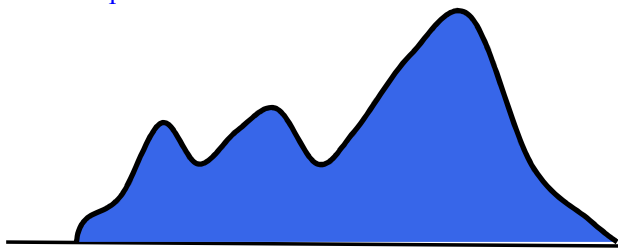
$\{E_y, H_x\}$
 $\{E_x, H_y\}$ **Independent each other**

Plane Waves

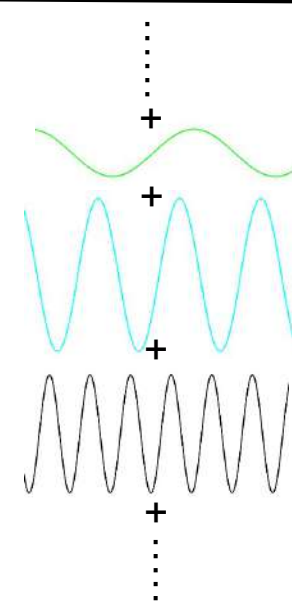
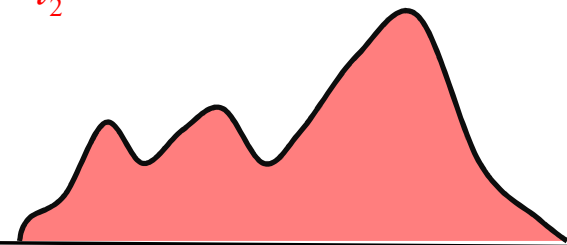
Time nondispersive & lossless medium

$$v_p = c$$

$t = t_1$



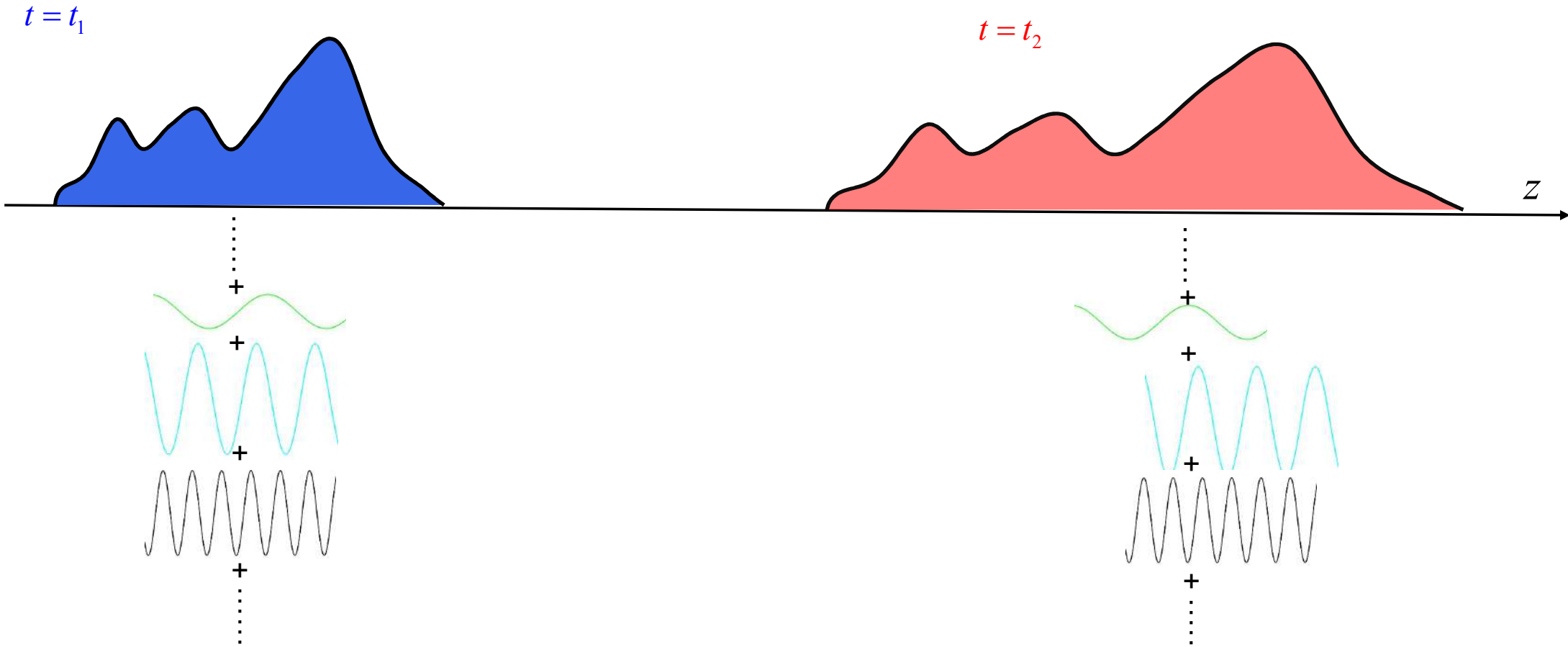
$t = t_2$



Plane Waves

Time dispersive medium

$$v_p = v_p(\omega_0)$$



Plane Waves (Phasor Domain)

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu \varepsilon} = \beta(\omega_0)$$

$$c = \frac{1}{\sqrt{\mu \varepsilon}}$$

$$v_p = \frac{\omega_0}{\beta} = c$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \text{real} \end{cases}$$

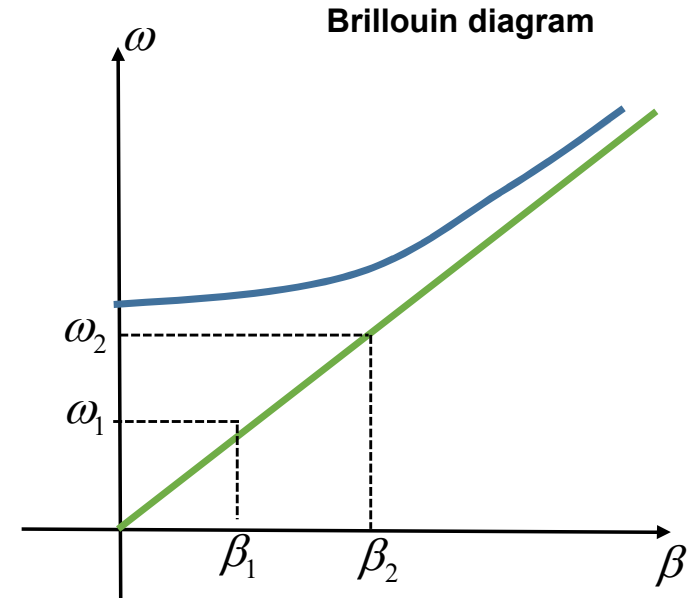
$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

Attenuation

$$\alpha \neq 0$$

Distortion

$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$



nondispersive

dispersive

$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = c$$

Plane Waves (Phasor Domain)

Time nondispersive & lossless

$$\begin{cases} \varepsilon : \text{real} \\ \mu : \text{real} \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu \varepsilon} = \beta(\omega_0)$$

$$c = \frac{1}{\sqrt{\mu \varepsilon}}$$

$$v_p = \frac{\omega_0}{\beta} = c$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma : \text{real} \end{cases}$$

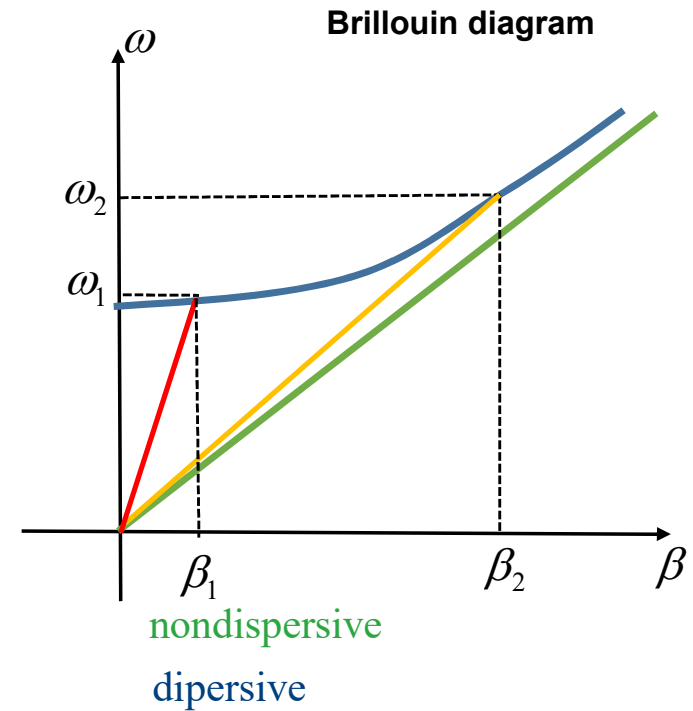
$$k(\omega_0) = \omega_0 \sqrt{\mu(\omega_0) \varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

Attenuation

$$\alpha \neq 0$$

Distortion

$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$



$$v_p = \frac{\omega_0}{\beta} = v_p(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = c$$