Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2021-2022 - Laurea "Triennale" – Secondo semestre - Secondo anno

Università degli Studi di Napoli "Parthenope"

Stefano Perna

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

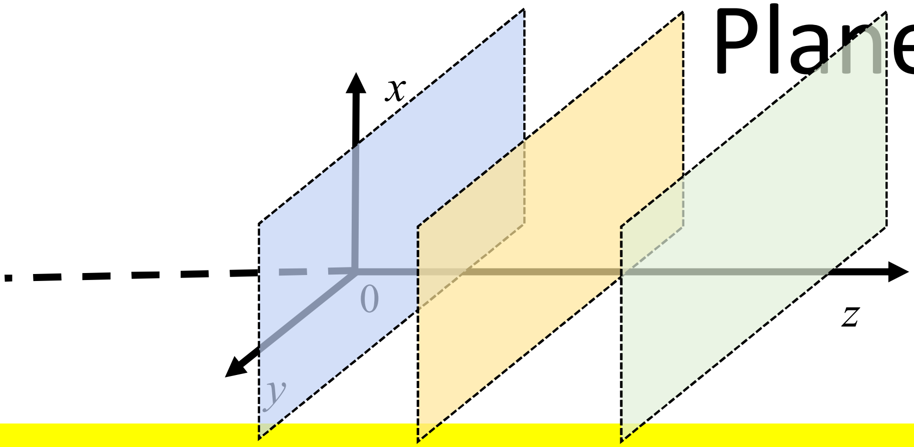
General expression of plane waves (PD)

Incidence

Plane Waves

Time domain

Plane Waves (TD)



Source-free

Medium
- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=e_x(\vec{\mathbf{r}},t)\hat{i}_x+e_y(\vec{\mathbf{r}},t)\hat{i}_y+e_z(\vec{\mathbf{r}},t)\hat{i}_z=e_x(z,t)\hat{i}_x+e_y(z,t)\hat{i}_y+e_z(z,t)\hat{i}_z$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=h_x(\vec{\mathbf{r}},t)\hat{i}_x+h_y(\vec{\mathbf{r}},t)\hat{i}_y+h_z(\vec{\mathbf{r}},t)\hat{i}_z=h_x(z,t)\hat{i}_x+h_y(z,t)\hat{i}_y+h_z(z,t)\hat{i}_z$$

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial y}=0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\vec{\mathbf{h}}(z,t)$$

Time domain - Differential form

$$\begin{cases}\nabla\times\vec{\mathbf{e}}(z,t)=-\mu\dfrac{\partial\vec{\mathbf{h}}(z,t)}{\partial t}\\ \nabla\times\vec{\mathbf{h}}(z,t)=\varepsilon\dfrac{\partial\vec{\mathbf{e}}(z,t)}{\partial t}\\ \varepsilon\nabla\cdot\vec{\mathbf{e}}(z,t)=0\\ \mu\nabla\cdot\vec{\mathbf{h}}(z,t)=0\end{cases}$$

$$\nabla\times\vec{\mathbf{e}}=\left(-\frac{\partial e_y}{\partial z}\right)\hat{i}_x+\left(\frac{\partial e_x}{\partial z}\right)\hat{i}_y$$

$$\nabla\times\vec{\mathbf{h}}=\left(-\frac{\partial h_y}{\partial z}\right)\hat{i}_x+\left(\frac{\partial h_x}{\partial z}\right)\hat{i}_y$$

Plane Waves (TD)

$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$e_x(z,t) = e_x^+(z-ct) + e_x^-(z+ct)$$

$$\zeta h_y(z,t) = e_x^+(z-ct) - e_x^-(z+ct)$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y, h_x\}$

$$\begin{cases} \dfrac{\partial e_y}{\partial z} = \mu \dfrac{\partial h_x}{\partial t} \\ \dfrac{\partial h_x}{\partial z} = \varepsilon \dfrac{\partial e_y}{\partial t} \end{cases} \qquad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2} = 0$$

$$e_y(z,t) = e_y^+(z-ct) + e_y^-(z+ct)$$

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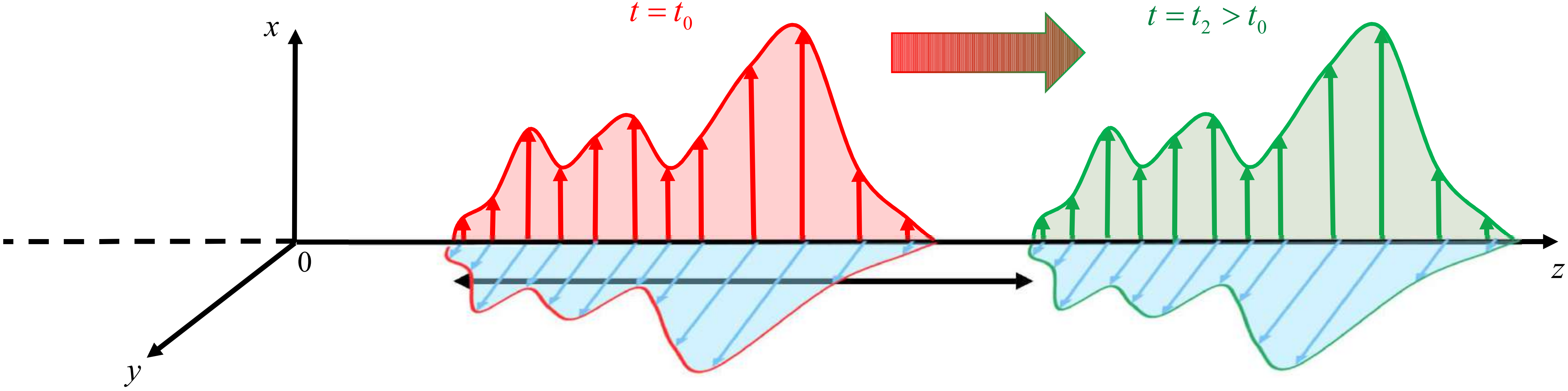
$\{e_y, h_x\}$, $\{e_x, h_y\}$ **Independent each other**

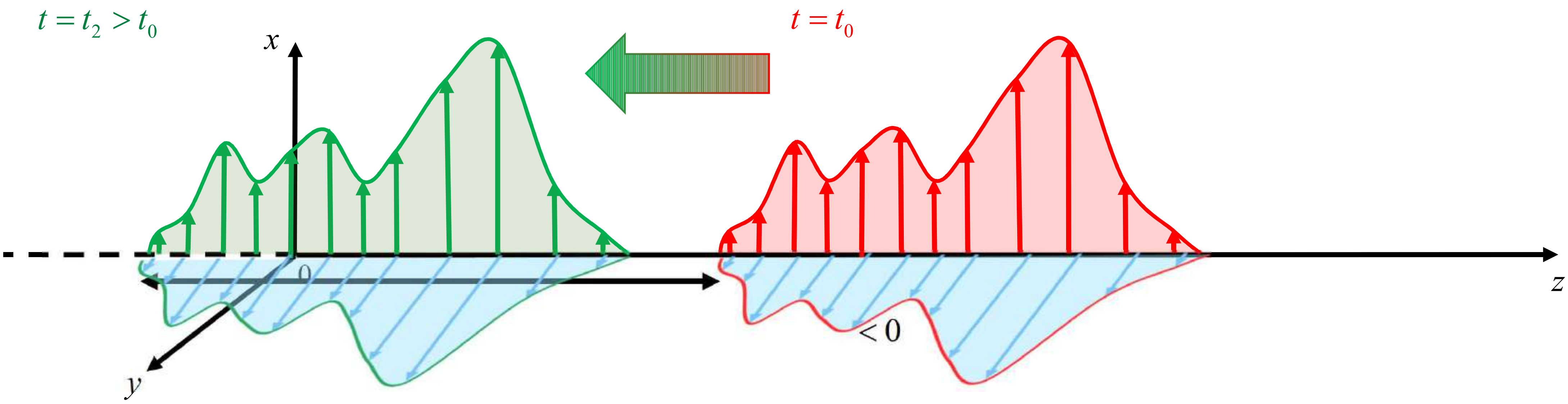
Plane Waves (TD)



The electromagnetic perturbation **propagates** without deformation and with constant speed *c* along the positive sense of the z-axis

$$\begin{cases} e^{+}\left(z-ct\right) \\ h^{+}\left(z-ct\right) \end{cases}$$ is referred to as electromagnetic **<u>progressive</u> plane wave**

Plane Waves (TD)



The electromagnetic perturbation **propagates** without deformation and with constant speed ***c*** along the negative sense of the z-axis

$$\begin{cases} e^{-}(z+ct) \\ h^{-}(z+ct) \end{cases}$$ is referred to as electromagnetic **<u>regressive</u> plane wave**

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$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(z,t)$$

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⇓

$$e_z(z,t)=h_z(z,t)=0$$

$\{e_y,h_x\}$ $\{e_x,h_y\}$ **Independent each other**

$\{e_x,h_y\}$

$$\begin{cases}\dfrac{\partial e_x}{\partial z}=-\mu\dfrac{\partial h_y}{\partial t}\\ \dfrac{\partial h_y}{\partial z}=-\varepsilon\dfrac{\partial e_x}{\partial t}\end{cases}\qquad \frac{\partial^2 e_x}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2}=0$$

$$\begin{cases}e_x^+(z,t)=e_x^+(z-ct)\\ \zeta h_y^+(z,t)=e_x^+(z-ct)\end{cases}\qquad \{e_x^+,h_y^+\}$$

$$\begin{cases}e_x^-(z,t)=e_x^-(z+ct)\\ \zeta h_y^-(z,t)=-e_x^-(z+ct)\end{cases}\qquad \{e_x^-,h_y^-\}$$

$$\vec{\mathbf{s}}^+=\frac{\left[e_x^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^-=-\frac{\left[e_x^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$$c=\frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta=\sqrt{\frac{\mu}{\varepsilon}}$$

$\{e_y,h_x\}$

$$\begin{cases}\dfrac{\partial e_y}{\partial z}=\mu\dfrac{\partial h_x}{\partial t}\\ \dfrac{\partial h_x}{\partial z}=\varepsilon\dfrac{\partial e_y}{\partial t}\end{cases}\qquad \frac{\partial^2 e_y}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 e_y}{\partial t^2}=0$$

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$$\vec{\mathbf{s}}^+=\frac{\left[e_y^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^-=-\frac{\left[e_y^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$\{e_x^+,h_y^+\}$ $\{e_x^-,h_y^-\}$ $\{e_y^+,h_x^+\}$ $\{e_y^-,h_x^-\}$

In all these 4 cases the Poynting vector is directed along the direction of propagation

Plane Waves (TD)

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0 \qquad \{e_x, h_y\}$$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

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$\{e_x^+, h_y^+\}$ $\{e_x^-, h_y^-\}$ $\{e_y^+, h_x^+\}$ $\{e_y^-, h_x^-\}$

In all these 4 cases the Poynting vector can be written as follows:

$$\vec{\mathbf{s}} = \frac{|\vec{\mathbf{e}}|^2}{\zeta}\hat{i}_p = \zeta|\vec{\mathbf{h}}|^2\hat{i}_p$$

where $\hat{i}_p$ points to the propagation direction

$$\vec{\mathbf{s}}^+ = \frac{\left[e_x^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_x^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$$\vec{\mathbf{s}}^+ = \frac{\left[e_y^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_y^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$$\vec{\mathbf{s}}^+ = \zeta\left[h_y^+(z-ct)\right]^2\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\zeta\left[h_y^-(z+ct)\right]^2\hat{i}_z$$

$$\vec{\mathbf{s}}^+ = \zeta\left[h_x^+(z-ct)\right]^2\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\zeta\left[h_x^-(z+ct)\right]^2\hat{i}_z$$

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$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$
$\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

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⬇

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$\{e_x, h_y\}$

$$\begin{cases} \dfrac{\partial e_x}{\partial z} = -\mu \dfrac{\partial h_y}{\partial t} \\ \dfrac{\partial h_y}{\partial z} = -\varepsilon \dfrac{\partial e_x}{\partial t} \end{cases} \qquad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 e_x}{\partial t^2} = 0$$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases} \qquad \{e_x^+, h_y^+\}$$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases} \qquad \{e_x^-, h_y^-\}$$

$$\vec{\mathbf{s}}^+ = \frac{\left[e_x^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_x^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

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$\{e_y, h_x\}$

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$$\vec{\mathbf{s}}^+ = \frac{\left[e_y^+(z-ct)\right]^2}{\zeta}\hat{i}_z \qquad \vec{\mathbf{s}}^- = -\frac{\left[e_y^-(z+ct)\right]^2}{\zeta}\hat{i}_z$$

$\{e_x^+, h_y^+\}$ $\{e_x^-, h_y^-\}$ $\{e_y^+, h_x^+\}$ $\{e_y^-, h_x^-\}$

In all these 4 cases the electric and magnetic fields are related each other through the following relation:

$$\zeta\vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$$

where $\hat{i}_p$ points to the propagation direction

$\{e_y, h_x\}$ $\{e_x, h_y\}$ **Independent each other**

Plane Waves (TD)

$\{e_x^+, h_y^+\}$

$$\begin{cases} e_x^+(z,t) = e_x^+(z-ct) \\ \zeta h_y^+(z,t) = e_x^+(z-ct) \end{cases}$$

$\{e_y^+, h_x^+\}$

$$\begin{cases} e_y^+(z,t) = e_y^+(z-ct) \\ \zeta h_x^+(z,t) = -e_y^+(z-ct) \end{cases}$$

the e.m. field propagates along $\hat{i}_p = \hat{i}_z$

$\{e_x^-, h_y^-\}$

$$\begin{cases} e_x^-(z,t) = e_x^-(z+ct) \\ \zeta h_y^-(z,t) = -e_x^-(z+ct) \end{cases}$$

$\{e_y^-, h_x^-\}$

$$\begin{cases} e_y^-(z,t) = e_y^-(z+ct) \\ \zeta h_x^-(z,t) = e_y^-(z+ct) \end{cases}$$

the e.m. field propagates along $\hat{i}_p = -\hat{i}_z$

- the e.m. field lies on the plane xy orthogonal to the propagation direction
- $|\vec{\mathbf{e}}|$ and $|\vec{\mathbf{h}}|$ are proportional through ζ
- $\zeta\vec{\mathbf{h}} = \hat{i}_p \times \vec{\mathbf{e}}$

Source-free

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⇓

$$e_z(z,t) = h_z(z,t) = 0$$

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Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

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Plane Waves (Spectral Domains)

Phasor domain (PD)

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0\vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0\vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla\cdot\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}}) \\ \nabla\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega\vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

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Plane Waves (Spectral Domains)

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Phasor domain (PD)

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0\vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0\vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla\cdot\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}}) \\ \nabla\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

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$$\vec{\mathbf{D}} = \varepsilon\vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu\vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma\vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

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Fourier domain (FD)

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$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

Plane Waves (Spectral Domains)

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$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

PD

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

FD

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

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Phasor domain (PD)

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0\vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0\vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla\cdot\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}}) \\ \nabla\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega\vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{D}} = \varepsilon\vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu\vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma\vec{\mathbf{E}} \qquad \begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time non-dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) + \rho_0(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}},\omega) + \vec{\mathbf{J}}_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}},\omega) = \rho(\vec{\mathbf{r}},\omega) + \rho_0(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \boldsymbol{\varepsilon} \neq real = \varepsilon_1 - j\varepsilon_2 \\ \boldsymbol{\mu} \neq real = \mu_1 - j\mu_2 \\ \boldsymbol{\sigma} \neq \mathbf{0} \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \mu \vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \nabla \cdot \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}}) = 0 \\ \nabla \cdot \mu \vec{\mathbf{H}}(\vec{\mathbf{r}}) = 0 \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = -j\omega \mu \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = j\omega \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) \\ \nabla \cdot \varepsilon \vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = 0 \\ \nabla \cdot \mu \vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = 0 \end{cases}$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

FD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\begin{cases} \vec{\mathbf{J}}_0 = \mathbf{0} \\ \rho_0 = 0 \end{cases}$$

$$\vec{\mathbf{D}} = \varepsilon \vec{\mathbf{E}} \qquad \vec{\mathbf{B}} = \mu \vec{\mathbf{H}} \qquad \vec{\mathbf{J}} = \sigma \vec{\mathbf{E}}$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma \neq 0 \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z) = -j\omega_0 \mu \vec{\mathbf{H}}(z) \\ \nabla \times \vec{\mathbf{H}}(z) = j\omega_0 \varepsilon \vec{\mathbf{E}}(z) \end{cases}$$

Fourier domain (FD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z,\omega) = -j\omega \mu \vec{\mathbf{H}}(z,\omega) \\ \nabla \times \vec{\mathbf{H}}(z,\omega) = j\omega \varepsilon \vec{\mathbf{E}}(z,\omega) \end{cases}$$

PD

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

FD

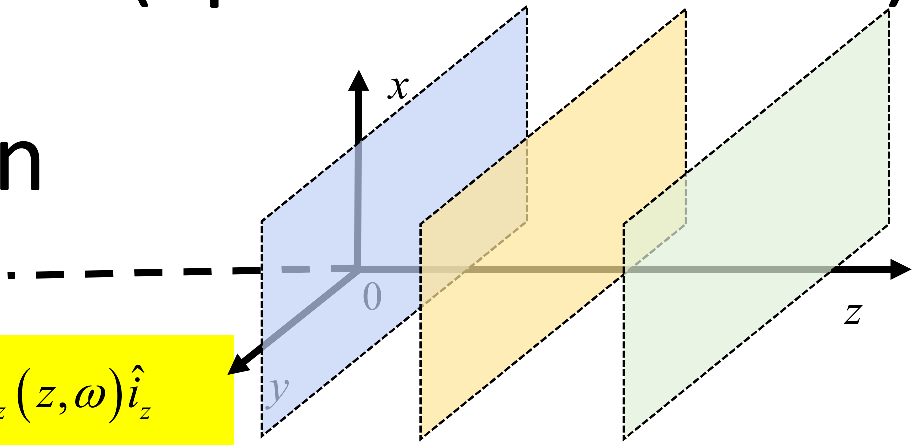
$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma \neq 0 \end{cases}$$

Plane Waves (Spectral Domains)

Fourier Domain



Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}},\omega) = E_x(z,\omega)\hat{i}_x + E_y(z,\omega)\hat{i}_y + E_z(z,\omega)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}},\omega) = H_x(z,\omega)\hat{i}_x + H_y(z,\omega)\hat{i}_y + H_z(z,\omega)\hat{i}_z$$

Fourier domain (FD)

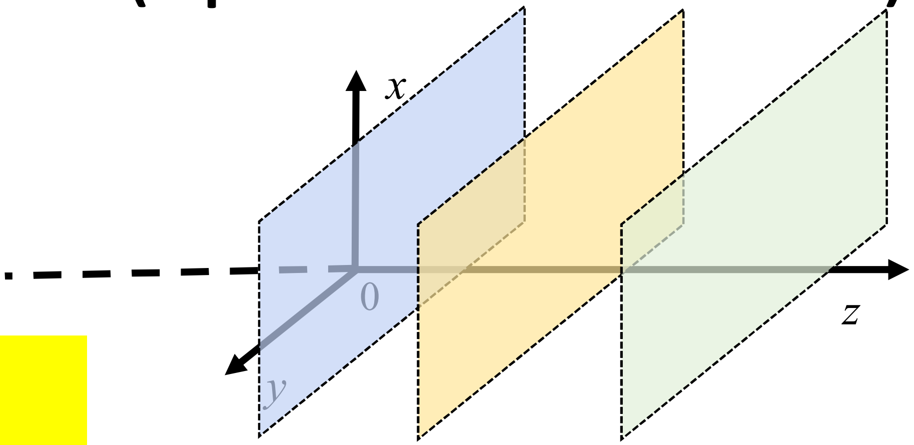
$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z,\omega) = -j\omega\mu\vec{\mathbf{H}}(z,\omega) \\ \nabla \times \vec{\mathbf{H}}(z,\omega) = j\omega\varepsilon\vec{\mathbf{E}}(z,\omega) \end{cases}$$

$$\nabla \times \vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$\nabla \times \vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

Plane Waves (Spectral Domains)

Phasor Domain



Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = E_x(z)\hat{i}_x + E_y(z)\hat{i}_y + E_z(z)\hat{i}_z$$

$$\vec{\mathbf{H}}(\vec{\mathbf{r}}) = H_x(z)\hat{i}_x + H_y(z)\hat{i}_y + H_z(z)\hat{i}_z$$

Phasor domain (PD)

$$\begin{cases} \nabla \times \vec{\mathbf{E}}(z) = -j\omega_0 \mu \vec{\mathbf{H}}(z) \\ \nabla \times \vec{\mathbf{H}}(z) = j\omega_0 \varepsilon \vec{\mathbf{E}}(z) \end{cases}$$

$$\nabla \times \vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$\nabla \times \vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

Plane Waves (Spectral Domains)

$$\begin{cases}\nabla\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}}\\ \nabla\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}}\end{cases}$$

$$\vec{\mathbf{E}} = E_x\hat{i}_x + E_y\hat{i}_y + \cancel{E_z\hat{i}_z}$$

$$\vec{\mathbf{H}} = H_x\hat{i}_x + H_y\hat{i}_y + \cancel{H_z\hat{i}_z}$$

$$\nabla\times\vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$-j\omega\mu\vec{\mathbf{H}} = -j\omega\mu H_x\hat{i}_x - j\omega\mu H_y\hat{i}_y - j\omega\mu H_z\hat{i}_z$$

$$\nabla\times\vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

$$j\omega\varepsilon\vec{\mathbf{E}} = j\omega\varepsilon E_x\hat{i}_x + j\omega\varepsilon E_y\hat{i}_y + j\omega\varepsilon E_z\hat{i}_z$$

$$E_z = 0 \qquad H_z = 0$$

⇒

$$e_z(z,t) = 0 \qquad h_z(z,t) = 0$$

⇒ **TEM fields**

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

⇓

$$E_z = H_z = 0$$

Plane Waves (Spectral Domains)

$$\begin{cases}\nabla\times\vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}}\\ \nabla\times\vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}}\end{cases}$$

$$\vec{\mathbf{E}} = E_x\hat{i}_x + E_y\hat{i}_y$$

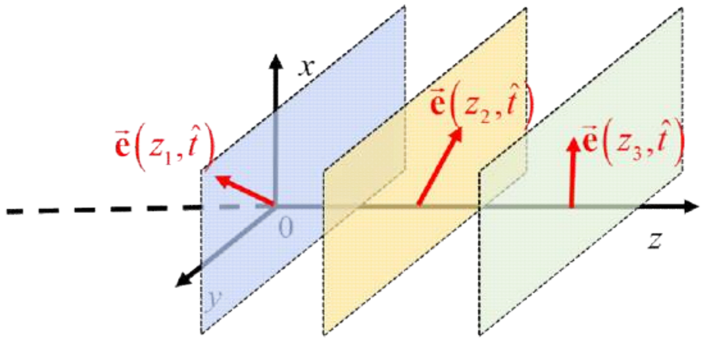
$$\vec{\mathbf{H}} = H_x\hat{i}_x + H_y\hat{i}_y$$

$$\nabla\times\vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$-j\omega\mu\vec{\mathbf{H}} = -j\omega\mu H_x\hat{i}_x - j\omega\mu H_y\hat{i}_y - j\omega\mu H_z\hat{i}_z$$

$$\nabla\times\vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

$$j\omega\varepsilon\vec{\mathbf{E}} = j\omega\varepsilon E_x\hat{i}_x + j\omega\varepsilon E_y\hat{i}_y + j\omega\varepsilon E_z\hat{i}_z$$



Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

Plane Waves (Spectral Domains)

$$\begin{cases} \nabla \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \end{cases}$$

$$\vec{\mathbf{E}} = E_x\hat{i}_x + E_y\hat{i}_y$$

$$\vec{\mathbf{H}} = H_x\hat{i}_x + H_y\hat{i}_y$$

$$\nabla \times \vec{\mathbf{E}} = \left(-\frac{dE_y}{dz}\right)\hat{i}_x + \left(\frac{dE_x}{dz}\right)\hat{i}_y$$

$$-j\omega\mu\vec{\mathbf{H}} = -j\omega\mu H_x\hat{i}_x - j\omega\mu H_y\hat{i}_y - j\omega\mu H_z\hat{i}_z$$

$$\nabla \times \vec{\mathbf{H}} = \left(-\frac{dH_y}{dz}\right)\hat{i}_x + \left(\frac{dH_x}{dz}\right)\hat{i}_y$$

$$j\omega\varepsilon\vec{\mathbf{E}} = j\omega\varepsilon E_x\hat{i}_x + j\omega\varepsilon E_y\hat{i}_y + j\omega\varepsilon E_z\hat{i}_z$$

$$\frac{dE_y}{dz} = j\omega\mu H_x$$

$$\frac{dE_x}{dz} = -j\omega\mu H_y$$

$$\frac{dH_y}{dz} = -j\omega\varepsilon E_x$$

$$\frac{dH_x}{dz} = j\omega\varepsilon E_y$$

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Spectral Domains)

$$\begin{cases} \nabla \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} \\ \nabla \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} \end{cases}$$

$$\vec{\mathbf{E}} = E_x \hat{i}_x + E_y \hat{i}_y$$

$$\vec{\mathbf{H}} = H_x \hat{i}_x + H_y \hat{i}_y$$

$$\frac{dE_x}{dz} = -j\omega\mu H_y$$

$$\frac{dH_y}{dz} = -j\omega\varepsilon E_x$$

$$\frac{dE_y}{dz} = j\omega\mu H_x$$

$$\frac{dH_x}{dz} = j\omega\varepsilon E_y$$

$$\frac{dE_y}{dz} = j\omega\mu H_x$$

$$\frac{dE_x}{dz} = -j\omega\mu H_y$$

$$\frac{dH_y}{dz} = -j\omega\varepsilon E_x$$

$$\frac{dH_x}{dz} = j\omega\varepsilon E_y$$

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\frac{dE_x}{dz} = -j\omega\mu H_y$$

$$\frac{dH_y}{dz} = -j\omega\varepsilon E_x$$

$\{E_y, H_x\}$

$$\frac{dE_y}{dz} = j\omega\mu H_x$$

$$\frac{dH_x}{dz} = j\omega\varepsilon E_y$$

Source-free

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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$\{E_y, H_x\}$

Source-free

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- **Linear**
- **Time dispersive**
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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases}$$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases}$$

Plane Waves (Spectral Domains)

Source-free

Medium
- Linear
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- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \Rightarrow \frac{d^2E_x}{dz^2} = -j\omega\mu\frac{dH_y}{dz} = -\omega^2\mu\varepsilon E_x \Rightarrow \frac{d^2E_x}{dz^2} + \omega^2\mu\varepsilon E_x = 0$$

$\{E_y, H_x\}$

$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \Rightarrow \frac{d^2E_y}{dz^2} = j\omega\mu\frac{dH_x}{dz} = -\omega^2\mu\varepsilon E_y \Rightarrow \frac{d^2E_y}{dz^2} + \omega^2\mu\varepsilon E_y = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~Lossless~~

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

k : (complex) propagation constant

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
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$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2E_x}{dz^2} + \omega^2\mu\varepsilon E_x = 0 \qquad \frac{d^2E_x}{dz^2} + k^2 E_x = 0$$

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$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2E_y}{dz^2} + \omega^2\mu\varepsilon E_y = 0 \qquad \frac{d^2E_y}{dz^2} + k^2 E_y = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2E_x}{dz^2} + k^2 E_x = 0$$

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Source-free

Medium

- **Linear**
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$\{E_y, H_x\}$
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Independent each other

Plane Waves (Spectral Domains)

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Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases}$$

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$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases}$$

$$\frac{d^2E_y}{dz^2} + k^2E_y = 0$$

$$E_y = E_y^+ e^{-jkz} + E_y^- e^{jkz}$$

$$\frac{d^2f(z)}{dz^2} + k^2 f(z) = 0$$

$$\xi^2 + k^2 = 0 \qquad \xi = \pm jk$$

$$f(z) = C_1e^{-jkz} + C_2e^{+jkz}$$

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$$\begin{cases} \dfrac{dE_y}{dz} = j\omega\mu H_x \\ \dfrac{dH_x}{dz} = j\omega\varepsilon E_y \end{cases} \qquad \frac{d^2 E_y}{dz^2} + k^2 E_y = 0$$

$$E_y = E_y^+ e^{-jkz} + E_y^- e^{jkz}$$

$$-j\omega\mu H_y = \frac{dE_x}{dz} = -jkE_x^+ e^{-jkz} + jkE_x^- e^{jkz}$$

$$\omega\mu H_y = kE_x^+ e^{-jkz} - kE_x^- e^{jkz} \qquad \frac{\omega\mu}{k} H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz} \qquad \zeta H_y = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

ζ : intrinsic impedance of the medium

$$\frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = \zeta$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

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$$-\omega\mu H_x = kE_y^+ e^{-jkz} - kE_y^- e^{jkz} \qquad -\frac{\omega\mu}{k} H_x = E_y^+ e^{-jkz} - E_y^- e^{jkz} \qquad -\zeta H_x = E_y^+ e^{-jkz} - E_y^- e^{jkz}$$

$$\frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = \zeta$$

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$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

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Plane Waves (Spectral Domains)

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Plane Waves (Spectral Domains)

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Plane Waves (Spectral Domains)

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Phasor Domain

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Fourier Domain

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

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Plane Waves (Spectral Domains)

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⬇

$$E_z = H_z = 0$$

$$\{E_y, H_x\}$$

$$\{E_x, H_y\}$$

Independent each other

Plane Waves (Spectral Domains)

$\{E_x, H_y\}$

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$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Spectral Domains)

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases}$$

$\{E_x, H_y\}$

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$$k = \omega\sqrt{\mu\varepsilon}$$

$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega \varepsilon E_x \end{cases}$$

$\{E_x, H_y\}$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z,\omega) = E_x^+(\omega) e^{-jkz} + E_x^-(\omega) e^{jkz}$$

$$\zeta H_y(z,\omega) = E_x^+(\omega) e^{-jkz} - E_x^-(\omega) e^{jkz}$$

Fourier Domain

Source-free

Medium

- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

Plane Waves (Spectral Domains)

Source-free

Medium
- Linear
- Time dispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

$$k = \omega\sqrt{\mu\varepsilon} \qquad k = \beta - j\alpha \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Phasor Domain

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$

$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Fourier Domain

$\{E_x, H_y\}$

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega\mu H_y \\ \dfrac{dH_y}{dz} = -j\omega\varepsilon E_x \end{cases} \qquad \frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$E_x(z,\omega) = E_x^+(\omega) e^{-jkz} + E_x^-(\omega) e^{jkz}$$

$$\zeta H_y(z,\omega) = E_x^+(\omega) e^{-jkz} - E_x^-(\omega) e^{jkz}$$

$$\begin{cases} \varepsilon(\omega) = \varepsilon_1(\omega) - j\varepsilon_2(\omega) \\ \mu(\omega) = \mu_1(\omega) - j\mu_2(\omega) \end{cases}$$

$$k(\omega) = \omega\sqrt{\mu(\omega)\varepsilon(\omega)}$$

$$k(\omega) = \beta(\omega) - j\alpha(\omega)$$

Plane Waves

Time domain (TD)

Spectral domains

- Phasor Domain (PD)
- Fourier Domain (FD)
- Dispersive media: attenuation, distortion, phase velocity and group velocity

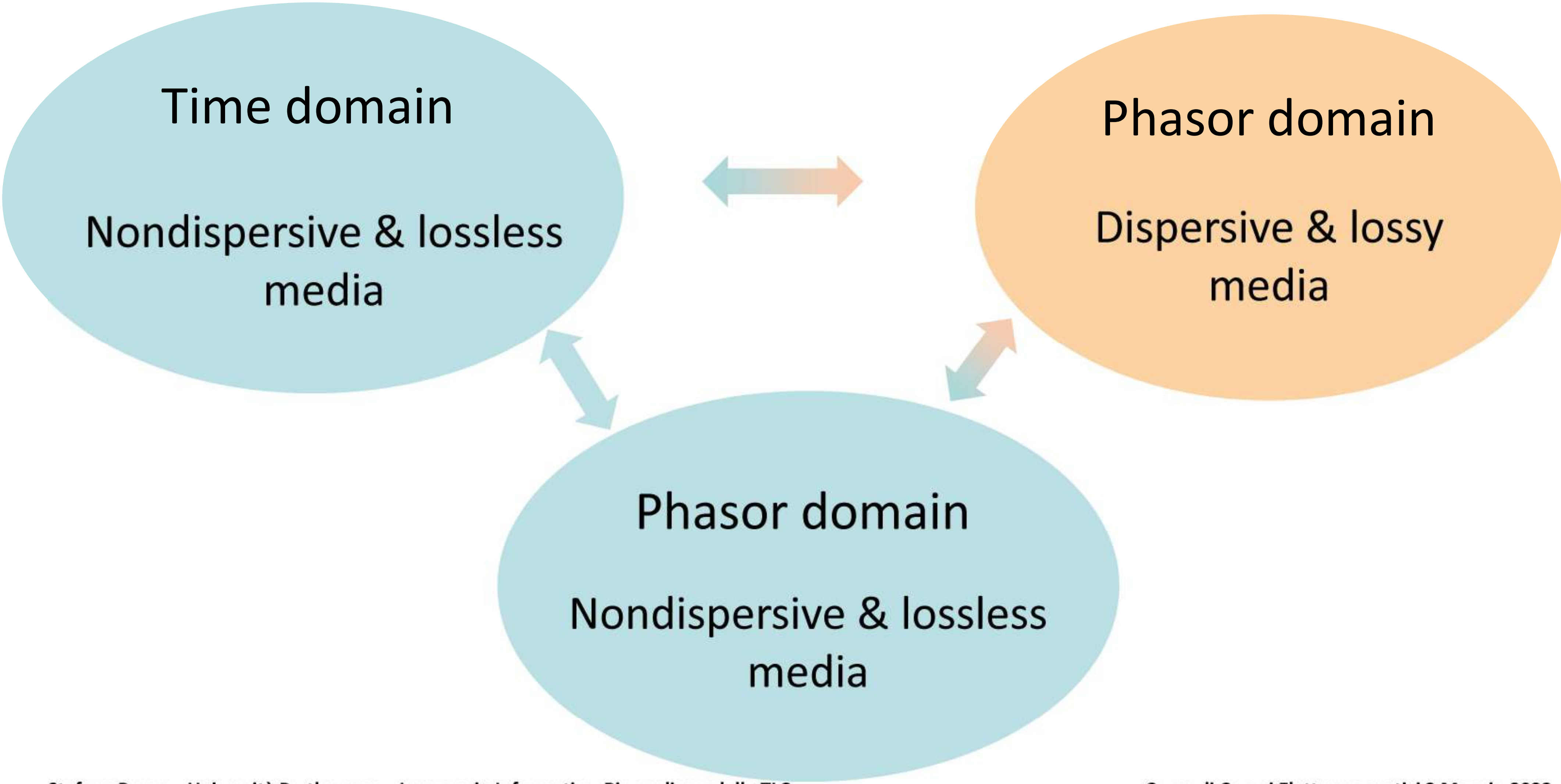
General expression of plane waves (PD)

Incidence

Plane Waves

Phasor domain

Rationale


Time domain
Nondispersive & lossless media
Phasor domain
Dispersive & lossy media
Phasor domain
Nondispersive & lossless media

Plane Waves (Phasor Domain)

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases}$$

$\{E_x, H_y\}$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

$$k = \omega\sqrt{\mu\varepsilon}$$
$$k = \beta - j\alpha$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

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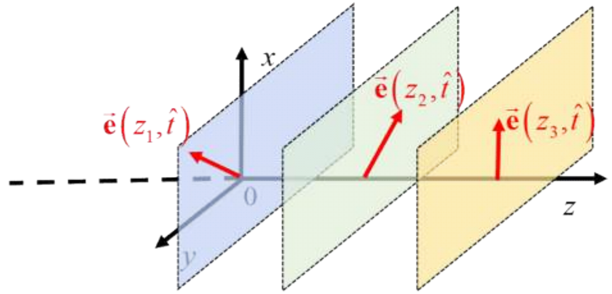
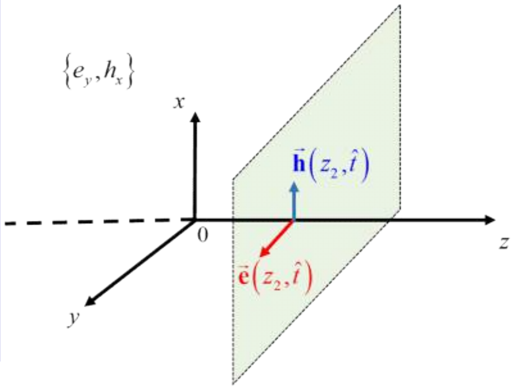
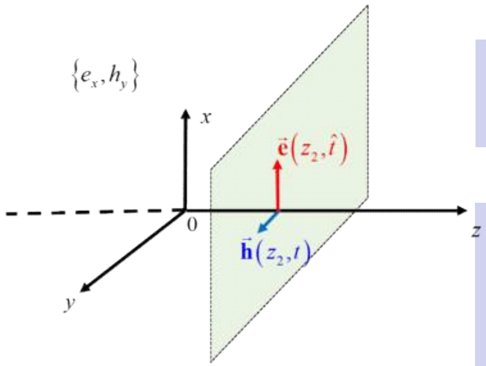
$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$
$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain







Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

$$\begin{cases} \dfrac{dE_x}{dz} = -j\omega_0 \mu H_y \\ \dfrac{dH_y}{dz} = -j\omega_0 \varepsilon E_x \end{cases}$$

$\{E_x, H_y\}$

$$\frac{d^2 E_x}{dz^2} + k^2 E_x = 0$$

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$$k = \beta - j\alpha$$

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$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \end{cases}$$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)}$$
$$k(\omega_0) = \beta(\omega_0) - j\alpha(\omega_0)$$

Phasor Domain

Source-free

Medium
- **Linear**
- **Time dispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- ~~**Lossless**~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma \neq 0 \end{cases}$$

Source-free

Medium
- Linear
- **Time dispersive**
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- ~~Lossless~~

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

⬇

$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu(\omega_0)\varepsilon(\omega_0)} = \beta(\omega_0) - j\alpha(\omega_0)$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$\zeta(\omega_0) = \sqrt{\frac{\mu(\omega_0)}{\varepsilon(\omega_0)}}$$

Time dispersive (lossy)

$$\begin{cases} \varepsilon(\omega_0) = \varepsilon_1(\omega_0) - j\varepsilon_2(\omega_0) \\ \mu(\omega_0) = \mu_1(\omega_0) - j\mu_2(\omega_0) \\ \sigma \neq 0 \end{cases}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Source-free

Medium
- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Phasor Domain)

$\left\{E_x, H_y\right\}$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$

$$\zeta H_y(z) = E_x^+ e^{-jkz} - E_x^- e^{jkz}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu(\cancel{\omega_0})\varepsilon(\cancel{\omega_0})} = \beta(\omega_0) \cancel{- j\alpha(\omega_0)}$$

$$\zeta(\cancel{\omega_0}) = \sqrt{\frac{\mu(\cancel{\omega_0})}{\varepsilon(\cancel{\omega_0})}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Source-free

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- **Linear**
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$$E_z = H_z = 0$$

$\left\{E_y, H_x\right\}$

$\left\{E_x, H_y\right\}$

Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$E_x(z) = E_x^+ e^{-jkz} + E_x^- e^{jkz}$$
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$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

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$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$
$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases}$$

$$k(\omega_0) = \omega_0 \sqrt{\mu\varepsilon} = \beta(\omega_0)$$

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$$E_z = H_z = 0$$

$\{E_y, H_x\}$
$\{E_x, H_y\}$
Independent each other

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

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$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

$$E_x^+ = |E_x^+| e^{j\varphi^+}$$

$$E_x^+ e^{-j\beta z} = |E_x^+| e^{j\varphi^+} e^{-j\beta z}$$

$$e_x^+(z,t) = \mathrm{Re}\left\{|E_x^+| e^{j\varphi^+} e^{-j\beta z} e^{j\omega_0 t}\right\} = |E_x^+|\cos(\omega_o t - \beta z + \varphi^+)$$

$$= |E_x^+|\cos\left(-\beta\left[z - \frac{\omega_0}{\beta}t\right] + \varphi^+\right) = |E_x^+|\cos\left(-\beta\left[z - v_p t\right] + \varphi^+\right)$$

$$= e_x^+(z - v_p t) = e_x^+(z - ct)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Source-free

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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E_x^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

$$E_x^+ e^{-j\beta z} = |E_x^+| e^{j\varphi^+} e^{-j\beta z}$$

$$e_x^+(z,t) = \mathrm{Re}\left\{|E_x^+| e^{j\varphi^+} e^{-j\beta z} e^{j\omega_0 t}\right\} = |E_x^+|\cos(\omega_o t - \beta z + \varphi^+)$$

$$= |E_x^+|\cos\left(-\beta\left[z - \frac{\omega_0}{\beta}t\right] + \varphi^+\right) = |E_x^+|\cos\left(-\beta\left[z - v_p t\right] + \varphi^+\right)$$

$$= e_x^+(z - v_p t) = e_x^+(z - ct)$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$
$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E_x^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Source-free

Medium
- Linear
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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$
$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$, $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$
$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E_x^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$E_x^- = |E_x^-| e^{j\varphi^-}$$
$$E_x^- e^{j\beta z} = |E_x^-| e^{j\varphi^-} e^{j\beta z}$$
$$e_x^-(z,t) = \mathrm{Re}\left\{|E_x^-| e^{j\varphi^-} e^{j\beta z} e^{j\omega_0 t}\right\} = |E_x^-|\cos(\omega_o t + \beta z + \varphi^-)$$
$$= e_x^-(z + v_p t) = e_x^-(z + ct)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

Source-free

Medium
- **Linear**
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$$E_z = H_z = 0$$

$\{E_y, H_x\}$ $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Source-free

Medium

- Linear
- Time nondispersive
- Space non-dispersive
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E_x^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

$$E_x^- e^{j\beta z} \rightarrow e_x^-(z,t) = |E_x^-|\cos(\omega_o t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

$$E_x^- e^{j\beta z} = |E_x^-| e^{j\varphi^-} e^{j\beta z}$$

$$e_x^-(z,t) = \mathrm{Re}\left\{|E_x^-| e^{j\varphi^-} e^{j\beta z} e^{j\omega_0 t}\right\} = |E_x^-|\cos(\omega_o t + \beta z + \varphi^-)$$

$$= e_x^-(z + v_p t) = e_x^-(z + ct)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$ $\{E_x, H_y\}$ **Independent each other**

Plane Waves (Phasor Domain)

$\{E_x, H_y\}$

$$E_x(z) = E_x^+ e^{-j\beta z} + E_x^- e^{j\beta z}$$

$$\zeta H_y(z) = E_x^+ e^{-j\beta z} - E_x^- e^{j\beta z}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

Source-free

Medium

- **Linear**
- **Time nondispersive**
- **Space non-dispersive**
- **Isotropic**
- **Homogeneous (TI – SI)**
- **Lossless**

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$

$\{E_x, H_y\}$

Independent each other

$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = |E_x^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

Progressive plane wave

$$E_x^- e^{j\beta z} \rightarrow e_x^-(z,t) = |E_x^-|\cos(\omega_o t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

Regressive plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$v_p = \frac{\omega_0}{\beta} = \frac{\omega_0}{\omega_0\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}} = c$$

Plane Waves (Phasor Domain)

Source-free

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- **Space non-dispersive**
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$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$

$$E_z = H_z = 0$$

$\{E_y, H_x\}$ $\{E_x, H_y\}$ **Independent each other**

$\{E_x, H_y\}$

$$k(\omega_0) = \omega_0\sqrt{\mu\varepsilon} = \beta(\omega_0)$$

$$E_x^+(z) = E_x^+ e^{-j\beta z}$$
$$\zeta H_y^+(z) = E_x^+ e^{-j\beta z}$$

$$E_x^-(z) = E_x^- e^{j\beta z}$$
$$\zeta H_y^-(z) = -E_x^- e^{j\beta z}$$

$$v_p = \frac{\omega_0}{\beta} \qquad \zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

$$E_x^+ e^{-j\beta z} \rightarrow e_x^+(z,t) = \left|E_x^+\right|\cos\left(\omega_o t - \beta z + \varphi^+\right) = e_x^+(z - v_p t)$$

Progressive plane wave

$$E_x^- e^{j\beta z} \rightarrow e_x^-(z,t) = \left|E_x^-\right|\cos\left(\omega_o t + \beta z + \varphi^-\right) = e_x^-(z + v_p t)$$

Regressive plane wave

Time nondispersive & lossless

$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma = 0 \end{cases} \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

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<u>Progressive</u> plane wave

$$E^- e^{j\beta z} \rightarrow e_x^-(z,t) = |E^-|\cos(\omega_o t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

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$$E^+ e^{-j\beta z} \quad \rightarrow e_x^+(z,t) = |E^+|\cos(\omega_o t - \beta z + \varphi^+) = e_x^+(z - v_p t)$$

<u>Progressive</u> plane wave

$$E^- e^{j\beta z} \quad \rightarrow e_x^-(z,t) = |E^-|\cos(\omega_o t + \beta z + \varphi^-) = e_x^-(z + v_p t)$$

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