

# **Campi Elettromagnetici**

**Corso di Laurea in Ingegneria Informatica,  
Biomedica e delle Telecomunicazioni**

**a.a. 2021-2022 - Laurea “Triennale” – Secondo semestre - Secondo anno**

**Università degli Studi di Napoli “Parthenope”**

**Stefano Perna**

# Color legend

New formulas, important considerations,  
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

# Plane Waves

Time domain (TD)

Spectral domains

Phasor Domain (PD)

Fourier Domain (FD)

Dispersive media: attenuation, distortion, phase velocity and group velocity

General expression of plane waves (PD)

Incidence

# Plane Waves

## Time domain

# Plane Waves (TD)

## Time domain - Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) + \vec{\mathbf{j}}_0(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) + \rho_0(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{cases}$$

## Time domain - Differential form

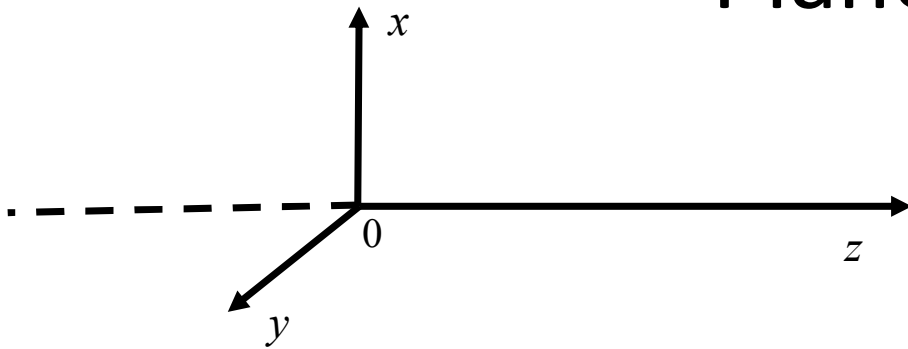
$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\mu \frac{\partial \vec{\mathbf{h}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \varepsilon \frac{\partial \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}_0(\vec{\mathbf{r}}, t) \\ \varepsilon \nabla \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \rho_0(\vec{\mathbf{r}}, t) \\ \mu \nabla \cdot \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = 0 \end{cases}$$

## Medium

- Linear
- Local (TND & SND)
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\begin{cases} \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \varepsilon \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \\ \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = \mu \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) \\ \sigma = 0 \end{cases}$$

# Plane Waves (TD)

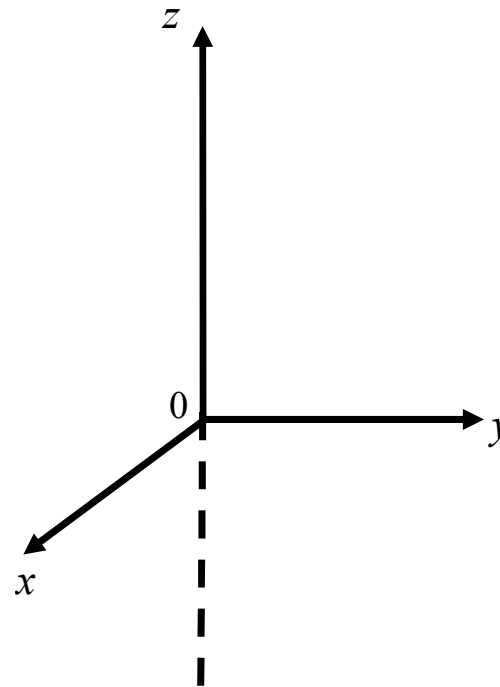


## Medium

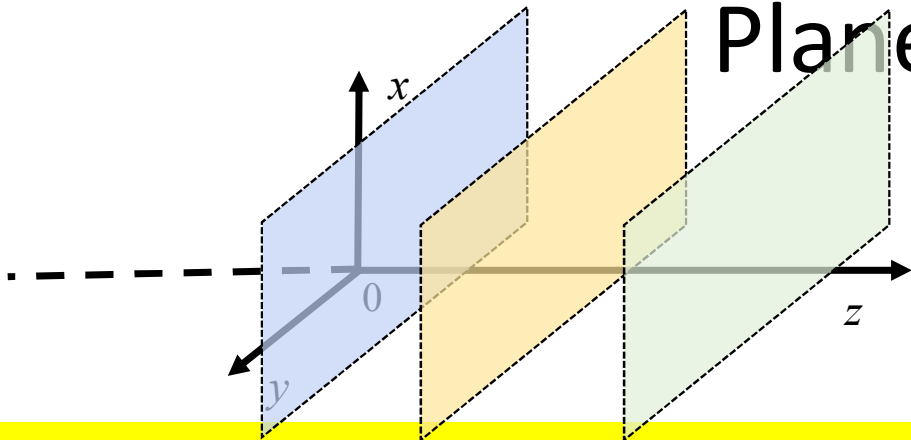
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$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\mu \frac{\partial \vec{\mathbf{h}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \varepsilon \frac{\partial \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}_0(\vec{\mathbf{r}}, t) \\ \varepsilon \nabla \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \rho_0(\vec{\mathbf{r}}, t) \\ \mu \nabla \cdot \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = 0 \end{cases}$$



# Plane Waves (TD)



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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = e_x(\vec{\mathbf{r}}, t)\hat{i}_x + e_y(\vec{\mathbf{r}}, t)\hat{i}_y + e_z(\vec{\mathbf{r}}, t)\hat{i}_z = e_x(\cancel{x}, \cancel{y}, z, t)\hat{i}_x + e_y(\cancel{x}, \cancel{y}, z, t)\hat{i}_y + e_z(\cancel{x}, \cancel{y}, z, t)\hat{i}_z$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = h_x(\vec{\mathbf{r}}, t)\hat{i}_x + h_y(\vec{\mathbf{r}}, t)\hat{i}_y + h_z(\vec{\mathbf{r}}, t)\hat{i}_z = h_x(\cancel{x}, \cancel{y}, z, t)\hat{i}_x + h_y(\cancel{x}, \cancel{y}, z, t)\hat{i}_y + h_z(\cancel{x}, \cancel{y}, z, t)\hat{i}_z$$

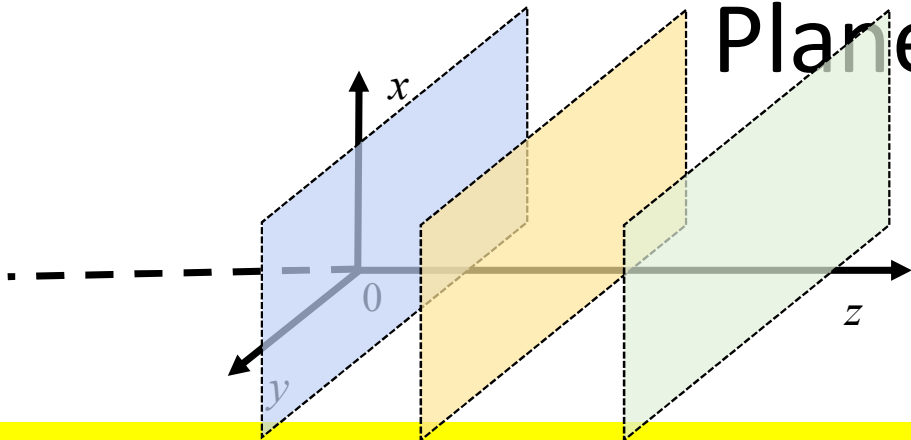
## Time domain - Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\mu \frac{\partial \vec{\mathbf{h}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \varepsilon \frac{\partial \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}_0(\vec{\mathbf{r}}, t) \\ \varepsilon \nabla \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \rho_0(\vec{\mathbf{r}}, t) \\ \mu \nabla \cdot \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = 0 \end{cases}$$

$$\nabla \times \vec{\mathbf{e}} = \left( \frac{\cancel{\partial e_z}}{\cancel{\partial y}} - \frac{\partial e_y}{\partial z} \right) \hat{i}_x + \left( \frac{\partial e_x}{\partial z} - \frac{\cancel{\partial e_z}}{\cancel{\partial x}} \right) \hat{i}_y + \left( \frac{\cancel{\partial e_y}}{\cancel{\partial x}} - \frac{\cancel{\partial e_x}}{\cancel{\partial y}} \right) \hat{i}_z$$

$$\nabla \times \vec{\mathbf{h}} = \left( \frac{\cancel{\partial h_z}}{\cancel{\partial y}} - \frac{\partial h_y}{\partial z} \right) \hat{i}_x + \left( \frac{\partial h_x}{\partial z} - \frac{\cancel{\partial h_z}}{\cancel{\partial x}} \right) \hat{i}_y + \left( \frac{\cancel{\partial h_y}}{\cancel{\partial x}} - \frac{\cancel{\partial h_x}}{\cancel{\partial y}} \right) \hat{i}_z$$

# Plane Waves (TD)



**Source-free**

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$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{e}}(z, t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = e_x(\vec{\mathbf{r}}, t)\hat{i}_x + e_y(\vec{\mathbf{r}}, t)\hat{i}_y + e_z(\vec{\mathbf{r}}, t)\hat{i}_z = e_x(z, t)\hat{i}_x + e_y(z, t)\hat{i}_y + e_z(z, t)\hat{i}_z$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = h_x(\vec{\mathbf{r}}, t)\hat{i}_x + h_y(\vec{\mathbf{r}}, t)\hat{i}_y + h_z(\vec{\mathbf{r}}, t)\hat{i}_z = h_x(z, t)\hat{i}_x + h_y(z, t)\hat{i}_y + h_z(z, t)\hat{i}_z$$

## Time domain - Differential form

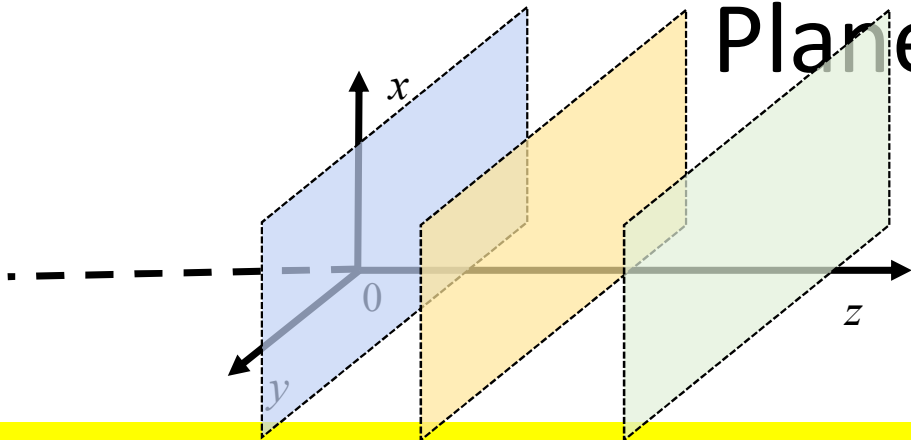
$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\mu \frac{\partial \vec{\mathbf{h}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \varepsilon \frac{\partial \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}_0(\vec{\mathbf{r}}, t) \\ \varepsilon \nabla \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = \rho_0(\vec{\mathbf{r}}, t) \\ \mu \nabla \cdot \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$$\nabla \times \vec{\mathbf{e}} = \left( -\frac{\partial e_y}{\partial z} \right) \hat{i}_x + \left( \frac{\partial e_x}{\partial z} \right) \hat{i}_y$$

$$\nabla \times \vec{\mathbf{h}} = \left( -\frac{\partial h_y}{\partial z} \right) \hat{i}_x + \left( \frac{\partial h_x}{\partial z} \right) \hat{i}_y$$



# Plane Waves (TD)



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$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \vec{\mathbf{h}}(z, t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = e_x(\vec{\mathbf{r}}, t)\hat{i}_x + e_y(\vec{\mathbf{r}}, t)\hat{i}_y + e_z(\vec{\mathbf{r}}, t)\hat{i}_z = e_x(z, t)\hat{i}_x + e_y(z, t)\hat{i}_y + e_z(z, t)\hat{i}_z$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = h_x(\vec{\mathbf{r}}, t)\hat{i}_x + h_y(\vec{\mathbf{r}}, t)\hat{i}_y + h_z(\vec{\mathbf{r}}, t)\hat{i}_z = h_x(z, t)\hat{i}_x + h_y(z, t)\hat{i}_y + h_z(z, t)\hat{i}_z$$

## Time domain - Differential form

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(z, t) = -\mu \frac{\partial \vec{\mathbf{h}}(z, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(z, t) = \varepsilon \frac{\partial \vec{\mathbf{e}}(z, t)}{\partial t} \end{array} \right.$$

$$\varepsilon \nabla \cdot \vec{\mathbf{e}}(z, t) = 0$$

$$\mu \nabla \cdot \vec{\mathbf{h}}(z, t) = 0$$

$$\nabla \times \vec{\mathbf{e}} = \left( -\frac{\partial e_y}{\partial z} \right) \hat{i}_x + \left( \frac{\partial e_x}{\partial z} \right) \hat{i}_y$$

$$\nabla \times \vec{\mathbf{h}} = \left( -\frac{\partial h_y}{\partial z} \right) \hat{i}_x + \left( \frac{\partial h_x}{\partial z} \right) \hat{i}_y$$

# Plane Waves (TD)

$$\begin{cases} \nabla \times \vec{e}(z,t) = -\mu \frac{\partial \vec{h}(z,t)}{\partial t} \\ \nabla \times \vec{h}(z,t) = \varepsilon \frac{\partial \vec{e}(z,t)}{\partial t} \end{cases}$$

$$\vec{e}(\vec{r},t) = e_x(z,t)\hat{i}_x + e_y(z,t)\hat{i}_y + \cancel{e_z(z,t)\hat{i}_z}$$

$$\vec{h}(\vec{r},t) = h_x(z,t)\hat{i}_x + h_y(z,t)\hat{i}_y + \cancel{h_z(z,t)\hat{i}_z}$$

$$\nabla \times \vec{e} = \left(-\frac{\partial e_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial e_x}{\partial z}\right)\hat{i}_y$$

$$-\mu \frac{\partial \vec{h}}{\partial t} = -\mu \frac{\partial h_x}{\partial t}\hat{i}_x - \mu \frac{\partial h_y}{\partial t}\hat{i}_y - \mu \frac{\partial h_z}{\partial t}\hat{i}_z$$

$$\nabla \times \vec{h} = \left(-\frac{\partial h_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial h_x}{\partial z}\right)\hat{i}_y$$

$$\varepsilon \frac{\partial \vec{e}(\vec{r},t)}{\partial t} = \varepsilon \frac{\partial e_x}{\partial t}\hat{i}_x + \varepsilon \frac{\partial e_y}{\partial t}\hat{i}_y + \varepsilon \frac{\partial e_z}{\partial t}\hat{i}_z$$

$$\begin{aligned} \frac{\partial e_z}{\partial t} &= 0 \\ \frac{\partial h_z}{\partial t} &= 0 \end{aligned}$$



$$e_z(z,t) = \text{const}$$



$$e_z(z,t) = 0$$



**TEM fields**

$$h_z(z,t) = \text{const}$$

$$h_z(z,t) = 0$$

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$$\vec{e}(\vec{r},t) = \vec{e}(z,t)$$

$$\vec{h}(\vec{r},t) = \vec{h}(z,t)$$



$$e_z(z,t) = h_z(z,t) = 0$$

# Plane Waves (TD)

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(z,t) = -\mu \frac{\partial \vec{\mathbf{h}}(z,t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(z,t) = \varepsilon \frac{\partial \vec{\mathbf{e}}(z,t)}{\partial t} \end{cases}$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = e_x(z,t)\hat{\mathbf{i}}_x + e_y(z,t)\hat{\mathbf{i}}_y$$

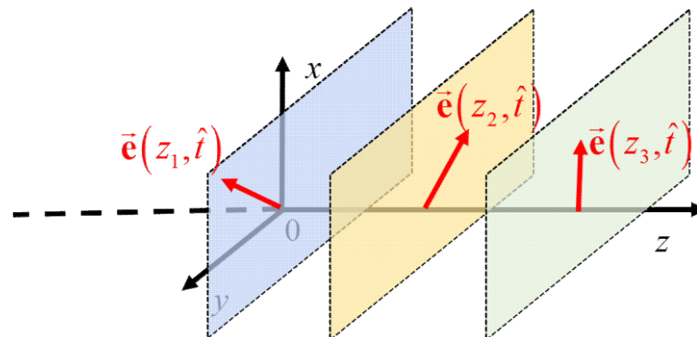
$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = h_x(z,t)\hat{\mathbf{i}}_x + h_y(z,t)\hat{\mathbf{i}}_y$$

$$\nabla \times \vec{\mathbf{e}} = \left(-\frac{\partial e_y}{\partial z}\right)\hat{\mathbf{i}}_x + \left(\frac{\partial e_x}{\partial z}\right)\hat{\mathbf{i}}_y$$

$$-\mu \frac{\partial \vec{\mathbf{h}}}{\partial t} = -\mu \frac{\partial h_x}{\partial t}\hat{\mathbf{i}}_x - \mu \frac{\partial h_y}{\partial t}\hat{\mathbf{i}}_y - \mu \frac{\partial h_z}{\partial t}\hat{\mathbf{i}}_z$$

$$\nabla \times \vec{\mathbf{h}} = \left(-\frac{\partial h_y}{\partial z}\right)\hat{\mathbf{i}}_x + \left(\frac{\partial h_x}{\partial z}\right)\hat{\mathbf{i}}_y$$

$$\varepsilon \frac{\partial \vec{\mathbf{e}}(\vec{\mathbf{r}},t)}{\partial t} = \varepsilon \frac{\partial e_x}{\partial t}\hat{\mathbf{i}}_x + \varepsilon \frac{\partial e_y}{\partial t}\hat{\mathbf{i}}_y + \varepsilon \frac{\partial e_z}{\partial t}\hat{\mathbf{i}}_z$$



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$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(z,t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \vec{\mathbf{h}}(z,t)$$



$$e_z(z,t) = h_z(z,t) = 0$$

# Plane Waves (TD)

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(z,t) = -\mu \frac{\partial \vec{\mathbf{h}}(z,t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(z,t) = \varepsilon \frac{\partial \vec{\mathbf{e}}(z,t)}{\partial t} \end{cases}$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = e_x(z,t)\hat{i}_x + e_y(z,t)\hat{i}_y$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = h_x(z,t)\hat{i}_x + h_y(z,t)\hat{i}_y$$

$$\nabla \times \vec{\mathbf{e}} = \left(-\frac{\partial e_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial e_x}{\partial z}\right)\hat{i}_y$$

$$-\mu \frac{\partial \vec{\mathbf{h}}}{\partial t} = -\mu \frac{\partial h_x}{\partial t}\hat{i}_x - \mu \frac{\partial h_y}{\partial t}\hat{i}_y - \mu \frac{\partial h_z}{\partial t}\hat{i}_z$$

$$\nabla \times \vec{\mathbf{h}} = \left(-\frac{\partial h_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial h_x}{\partial z}\right)\hat{i}_y$$

$$\varepsilon \frac{\partial \vec{\mathbf{e}}(\vec{\mathbf{r}},t)}{\partial t} = \varepsilon \frac{\partial e_x}{\partial t}\hat{i}_x + \varepsilon \frac{\partial e_y}{\partial t}\hat{i}_y + \varepsilon \frac{\partial e_z}{\partial t}\hat{i}_z$$

$$\frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t}$$

$$\frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t}$$

$$\frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t}$$

$$\frac{\partial h_x}{\partial z} = \varepsilon \frac{\partial e_y}{\partial t}$$

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$$e_z(z,t) = h_z(z,t) = 0$$

$\{e_y, h_x\}$   
 $\{e_x, h_y\}$  Independent each other

# Plane Waves (TD)

$$\begin{cases} \nabla \times \vec{e}(z,t) = -\mu \frac{\partial \vec{h}(z,t)}{\partial t} \\ \nabla \times \vec{h}(z,t) = \varepsilon \frac{\partial \vec{e}(z,t)}{\partial t} \end{cases}$$

$$\vec{e}(\vec{r},t) = e_x(z,t)\hat{i}_x + e_y(z,t)\hat{i}_y$$

$$\vec{h}(\vec{r},t) = h_x(z,t)\hat{i}_x + h_y(z,t)\hat{i}_y$$

$$\begin{cases} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t} \end{cases}$$

$$\begin{cases} \frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t} \\ \frac{\partial h_x}{\partial z} = \varepsilon \frac{\partial e_y}{\partial t} \end{cases}$$

$$\begin{cases} \frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t} \\ \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \end{cases}$$

$$\begin{cases} \frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t} \\ \frac{\partial h_x}{\partial z} = \varepsilon \frac{\partial e_y}{\partial t} \end{cases}$$

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$$\vec{e}(\vec{r},t) = \vec{e}(z,t)$$

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# Plane Waves (TD)

$$\vec{e}(\vec{r}, t) = e_x(z, t)\hat{i}_x + e_y(z, t)\hat{i}_y$$

$$\vec{h}(\vec{r}, t) = h_x(z, t)\hat{i}_x + h_y(z, t)\hat{i}_y$$

$$\left\{ \begin{array}{l} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t} \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t} \\ \frac{\partial h_x}{\partial z} = \varepsilon \frac{\partial e_y}{\partial t} \end{array} \right.$$

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$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

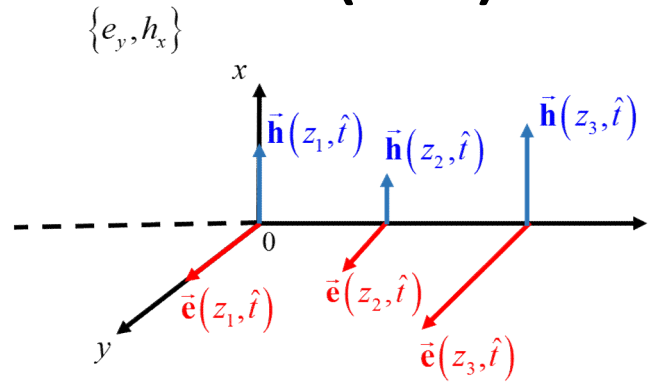
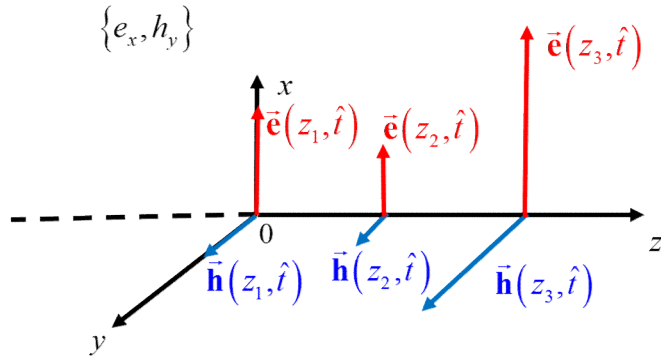
$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$e_z(z, t) = h_z(z, t) = 0$$

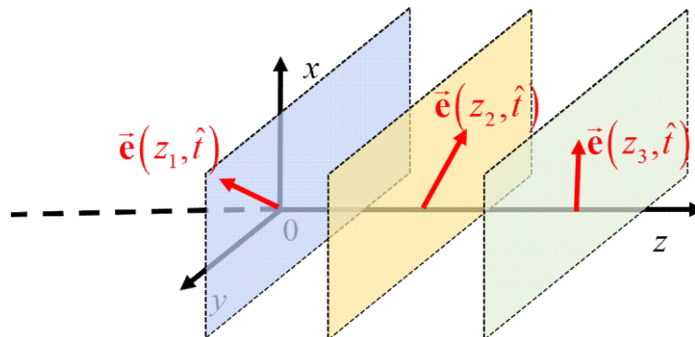
$\{e_y, h_x\}$   
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# Plane Waves (TD)



$$\begin{cases} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\epsilon \frac{\partial e_x}{\partial t} \end{cases}$$

$$\begin{cases} \frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t} \\ \frac{\partial h_x}{\partial z} = \epsilon \frac{\partial e_y}{\partial t} \end{cases}$$



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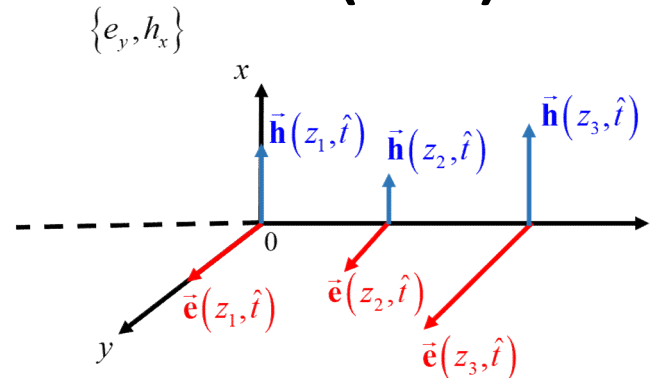
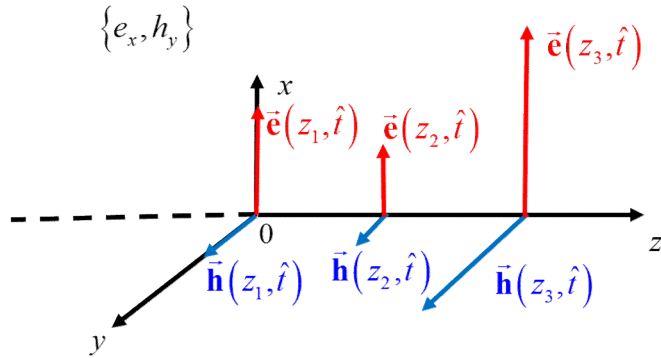


$$e_z(z, t) = h_z(z, t) = 0$$

$$\begin{cases} \{e_y, h_x\} \\ \{e_x, h_y\} \end{cases}$$

Independent  
each other

# Plane Waves (TD)



$$\begin{cases} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\epsilon \frac{\partial e_x}{\partial t} \end{cases} \Rightarrow \begin{cases} \frac{\partial^2 e_x}{\partial z^2} = -\mu \frac{\partial}{\partial z} \frac{\partial h_y}{\partial t} = -\mu \frac{\partial}{\partial t} \frac{\partial h_y}{\partial z} = \mu \epsilon \frac{\partial^2 e_x}{\partial t^2} \\ -\frac{\partial}{\partial t} \frac{\partial h_y}{\partial z} = \epsilon \frac{\partial^2 e_x}{\partial t^2} \end{cases} \Rightarrow \frac{\partial^2 e_x}{\partial z^2} = \mu \epsilon \frac{\partial^2 e_x}{\partial t^2}$$

$$\begin{cases} \frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t} \\ \frac{\partial h_x}{\partial z} = \epsilon \frac{\partial e_y}{\partial t} \end{cases} \Rightarrow \begin{cases} \frac{\partial^2 e_y}{\partial z^2} = \mu \frac{\partial}{\partial z} \frac{\partial h_x}{\partial t} = \mu \frac{\partial}{\partial t} \frac{\partial h_x}{\partial z} = \mu \epsilon \frac{\partial^2 e_y}{\partial t^2} \\ \frac{\partial}{\partial t} \frac{\partial h_x}{\partial z} = \epsilon \frac{\partial^2 e_y}{\partial t^2} \end{cases} \Rightarrow \frac{\partial^2 e_y}{\partial z^2} = \mu \epsilon \frac{\partial^2 e_y}{\partial t^2}$$

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$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

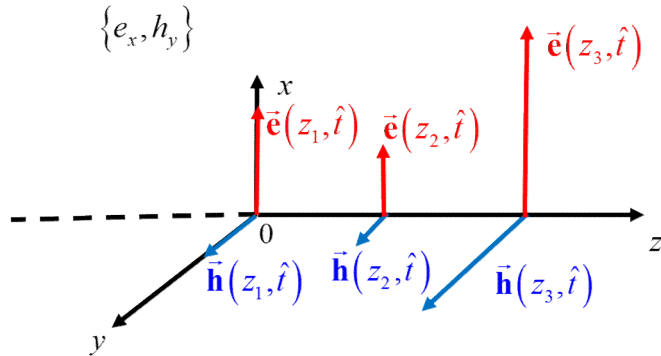
$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$

$$e_z(z, t) = h_z(z, t) = 0$$

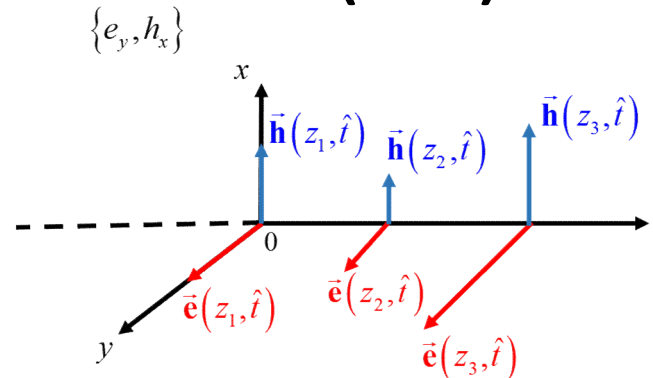
$\{e_y, h_x\}$   
 $\{e_x, h_y\}$  Independent each other



# Plane Waves (TD)



$$c = \frac{1}{\sqrt{\mu\epsilon}}$$



$$\begin{cases} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\epsilon \frac{\partial e_x}{\partial t} \end{cases} \quad \frac{\partial^2 e_x}{\partial z^2} = \mu\epsilon \frac{\partial^2 e_x}{\partial t^2} \quad \frac{\partial^2 e_x(z,t)}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e_x(z,t)}{\partial t^2} = 0$$

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Source-free

Medium

- Linear
- Local (TND & SND)
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- Lossless

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Independent each other

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$$f(z,t) = e^+(z-ct)$$

$$\alpha = z - ct$$

$$\frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

$$\frac{\partial e^+}{\partial z} = \frac{\partial e^+}{\partial \alpha} \frac{\partial \alpha}{\partial z} = \frac{\partial e^+}{\partial \alpha}$$



$$\frac{\partial e^+}{\partial \alpha} = \frac{\partial e^+}{\partial z}$$



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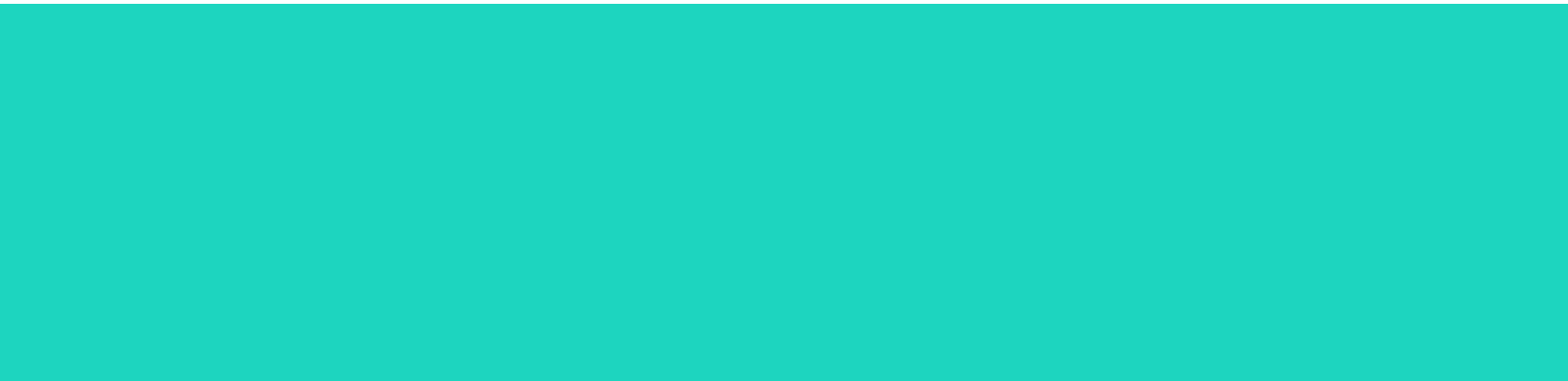
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$$f(z,t) = e^-(z+ct) \quad \beta = z+ct \quad \frac{\partial e^-}{\partial z} = \frac{1}{c} \frac{\partial e^-}{\partial t}$$

$$\begin{aligned} \frac{\partial e^-}{\partial z} &= \frac{\partial e^-}{\partial \beta} \frac{\partial \beta}{\partial z} = \frac{\partial e^-}{\partial \beta} & \longrightarrow & \frac{\partial e^-}{\partial \beta} = \frac{\partial e^-}{\partial z} \\ \frac{\partial e^-}{\partial t} &= \frac{\partial e^-}{\partial \beta} \frac{\partial \beta}{\partial t} = c \frac{\partial e^-}{\partial \beta} & \longrightarrow & \frac{\partial e^-}{\partial \beta} = \frac{1}{c} \frac{\partial e^-}{\partial t} \end{aligned}$$

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$$\begin{cases} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t} \end{cases} \quad \frac{\partial^2 e_x}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e_x}{\partial t^2} = 0$$

$\{e_x, h_y\}$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\begin{cases} \frac{\partial e_y}{\partial z} = \mu \frac{\partial h_x}{\partial t} \\ \frac{\partial h_x}{\partial z} = \varepsilon \frac{\partial e_y}{\partial t} \end{cases} \quad \frac{\partial^2 e_y}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e_y}{\partial t^2} = 0$$

$\{e_y, h_x\}$

$$f(z,t) = e^+(z-ct) \quad \frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e^+}{\partial t^2} = 0 \quad \frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

$$f(z,t) = e^-(z+ct) \quad \frac{\partial^2 e^-}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e^-}{\partial t^2} = 0 \quad \frac{\partial e^-}{\partial z} = \frac{1}{c} \frac{\partial e^-}{\partial t}$$

Source-free

Medium

- Linear
- Local (TND & SND)
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$e_z(z,t) = h_z(z,t) = 0$$

$$\begin{cases} \{e_y, h_x\} \\ \{e_x, h_y\} \end{cases}$$

Independent each other

# Plane Waves (TD)

$$\begin{cases} \frac{\partial e_x}{\partial z} = -\mu \frac{\partial h_y}{\partial t} \\ \frac{\partial h_y}{\partial z} = -\varepsilon \frac{\partial e_x}{\partial t} \end{cases} \quad \{e_x, h_y\}$$

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$$f(z,t) = e^+(z-ct)$$

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$$f(z,t) = e^-(z+ct)$$

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$$\frac{\partial e^-}{\partial z} = \frac{1}{c} \frac{\partial e^-}{\partial t}$$

$$e_x(z,t) = e_x^+(z-ct) + e_x^-(z+ct)$$

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$$f(z,t) = e^+(z-ct) \quad \frac{\partial^2 e^+}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 e^+}{\partial t^2} = 0$$

$$\frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

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$$\frac{\partial e^-}{\partial z} = \frac{1}{c} \frac{\partial e^-}{\partial t}$$

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$$e_x(z,t) = e_x^+(z-ct) + e_x^-(z+ct)$$

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$$e_x(z, t) = e_x^+(z - ct) + e_x^-(z + ct)$$

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$$\frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

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# Plane Waves (TD)

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$$e_x(z, t) = e_x^+(z - ct) + e_x^-(z + ct)$$

$\{e_x, h_y\}$

$$c = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\zeta = \sqrt{\frac{\mu}{\varepsilon}}$$

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$$e_y(z, t) = e_y^+(z - ct) + e_y^-(z + ct)$$

$\{e_y, h_x\}$

$$\mu c = \frac{\mu}{\sqrt{\varepsilon\mu}} = \sqrt{\frac{\mu}{\varepsilon}} = \zeta$$

$$\frac{\partial e^+}{\partial z} = -\frac{1}{c} \frac{\partial e^+}{\partial t}$$

$$\frac{\partial e^-}{\partial z} = \frac{1}{c} \frac{\partial e^-}{\partial t}$$

$$-\mu \frac{\partial h_y}{\partial t} = \frac{\partial e_x}{\partial z} = \frac{\partial e_x^+}{\partial z} + \frac{\partial e_x^-}{\partial z} = -\frac{1}{c} \frac{\partial e_x^+}{\partial t} + \frac{1}{c} \frac{\partial e_x^-}{\partial t}$$

$$\Rightarrow \mu c \frac{\partial h_y}{\partial t} = \frac{\partial}{\partial t} (e_x^+ - e_x^-)$$

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$$\mu c \frac{\partial h_y}{\partial t} = \frac{\partial}{\partial t} (e_x^+ - e_x^-) \implies \frac{\partial}{\partial t} [\zeta h_y - (e_x^+ - e_x^-)] = 0$$

$$\zeta h_y = (e_x^+ - e_x^-)$$

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$$\mu \frac{\partial h_x}{\partial t} = \frac{\partial e_y}{\partial z} = \frac{\partial e_y^+}{\partial z} + \frac{\partial e_y^-}{\partial z} = -\frac{1}{c} \frac{\partial e_y^+}{\partial t} + \frac{1}{c} \frac{\partial e_y^-}{\partial t}$$

$$\Rightarrow \mu c \frac{\partial h_x}{\partial t} = -\frac{\partial}{\partial t} (e_y^+ - e_y^-) \Rightarrow \frac{\partial}{\partial t} [\zeta h_x + (e_y^+ - e_y^-)] = 0$$

$$-\zeta h_x = (e_y^+ - e_y^-)$$

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$[\vec{e}] : \frac{\text{Volt}}{m}$

$[\vec{h}] : \frac{\text{Ampere}}{m}$

$$[\zeta] \frac{\text{Ampere}}{m} = \frac{\text{Volt}}{m} \quad \longrightarrow \quad [\zeta] = \frac{\text{Volt}}{\text{Ampere}} = \Omega$$

$\zeta$  : intrinsic resistance of the medium

in freespace

$$\zeta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\Omega$$

$\mu_0 = 4\pi \times 10^{-7} \text{ Henry / m}$

$\varepsilon_0 = 8.8 \times 10^{-12} \text{ Farad / m}$

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Independent each other

$$[\vec{e}]: \frac{\text{Volt}}{m}$$

$$[\vec{h}]: \frac{\text{Ampere}}{m}$$

$$\frac{\text{Volt}}{m} \frac{1}{m^2} = \frac{1}{[c]^2} \frac{\text{Volt}}{m} \frac{1}{s^2}$$

$$\Rightarrow [c]^2 = \left(\frac{m}{s}\right)^2 \Rightarrow [c] = \frac{m}{s}$$

$c$  is a speed

in free space

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3 \times 10^8 \text{ m/s}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ Henry / m}$$

$$\varepsilon_0 = 8.8 \times 10^{-12} \text{ Farad / m}$$

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- Local (TND & SND)
- Isotropic
- Homogeneous (TI – SI)
- Lossless

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

$$\vec{e}(\vec{r}, t) = \vec{e}(z, t)$$

$$\vec{h}(\vec{r}, t) = \vec{h}(z, t)$$



$$e_z(z, t) = h_z(z, t) = 0$$

$\{e_y, h_x\}$

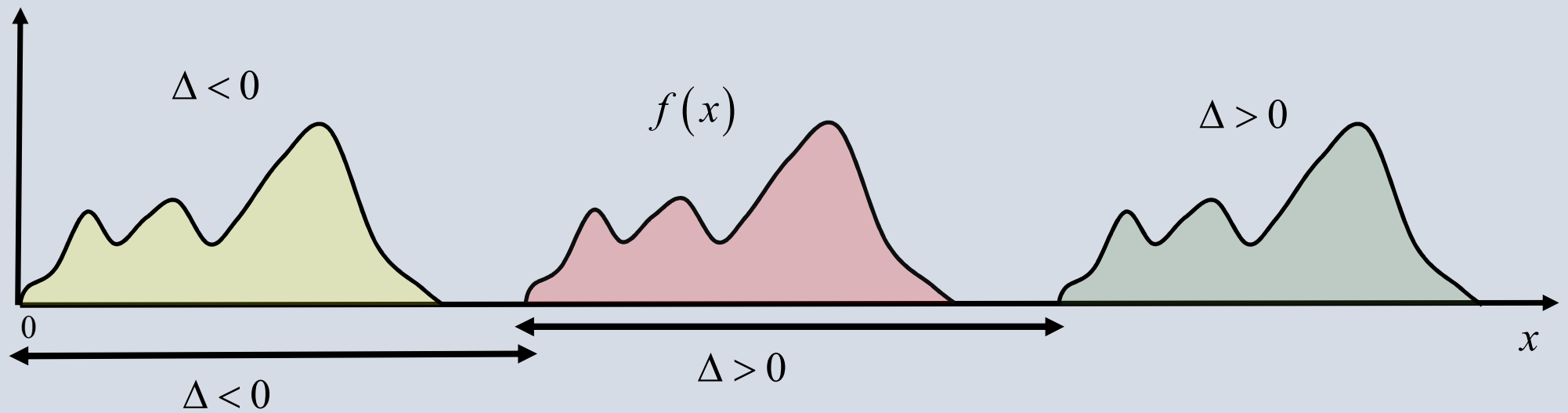
$\{e_x, h_y\}$

**Independent  
each other**

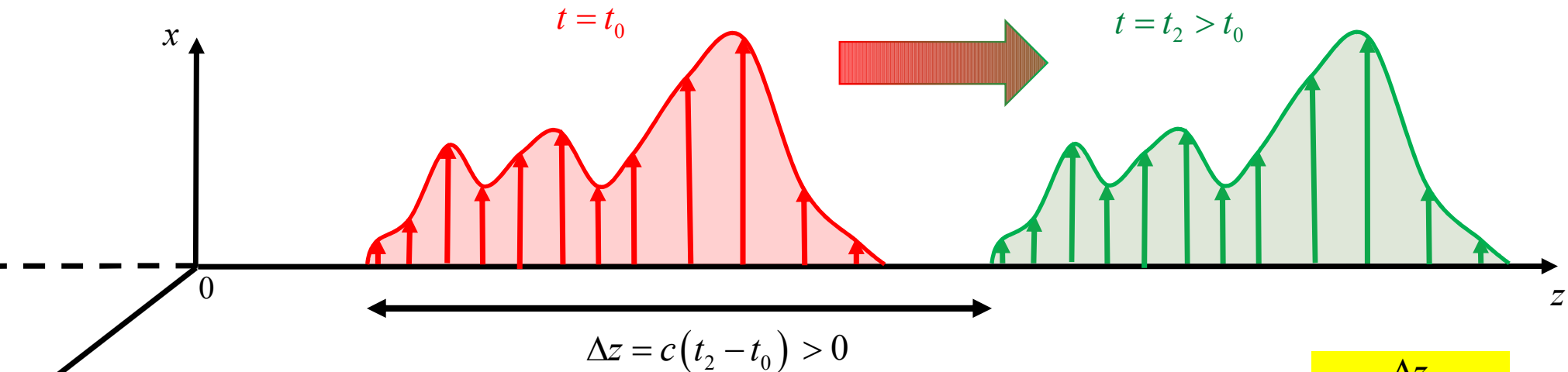


# MEMO

$$f(x - \Delta)$$



# Plane Waves (TD)



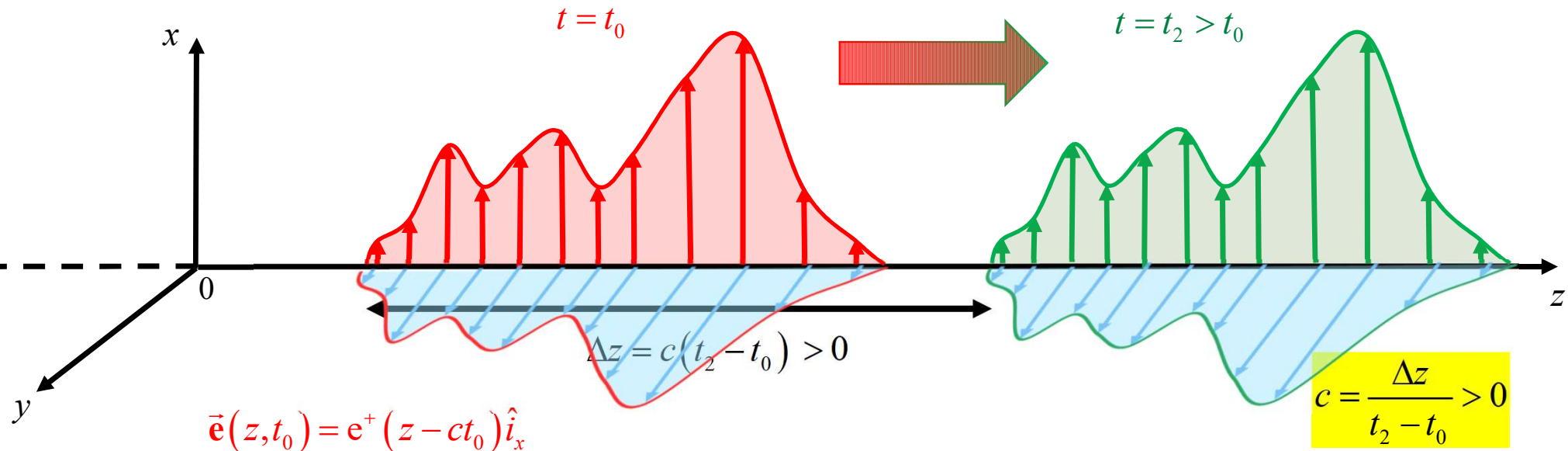
$$\vec{e}(z, t_0) = e^+(z - ct_0) \hat{i}_x$$

$$\vec{e}(z, t_2) = e^+(z - ct_2) \hat{i}_x = e^+(z - ct_0 + ct_0 - ct_2) \hat{i}_x = e^+(z - ct_0 - \underbrace{c[t_2 - t_0]}_{\Delta z}) \hat{i}_x$$

$$c = \frac{\Delta z}{t_2 - t_0} > 0$$

The electromagnetic perturbation **propagates** without deformation and with constant speed **c** along the positive sense of the z-axis

# Plane Waves (TD)



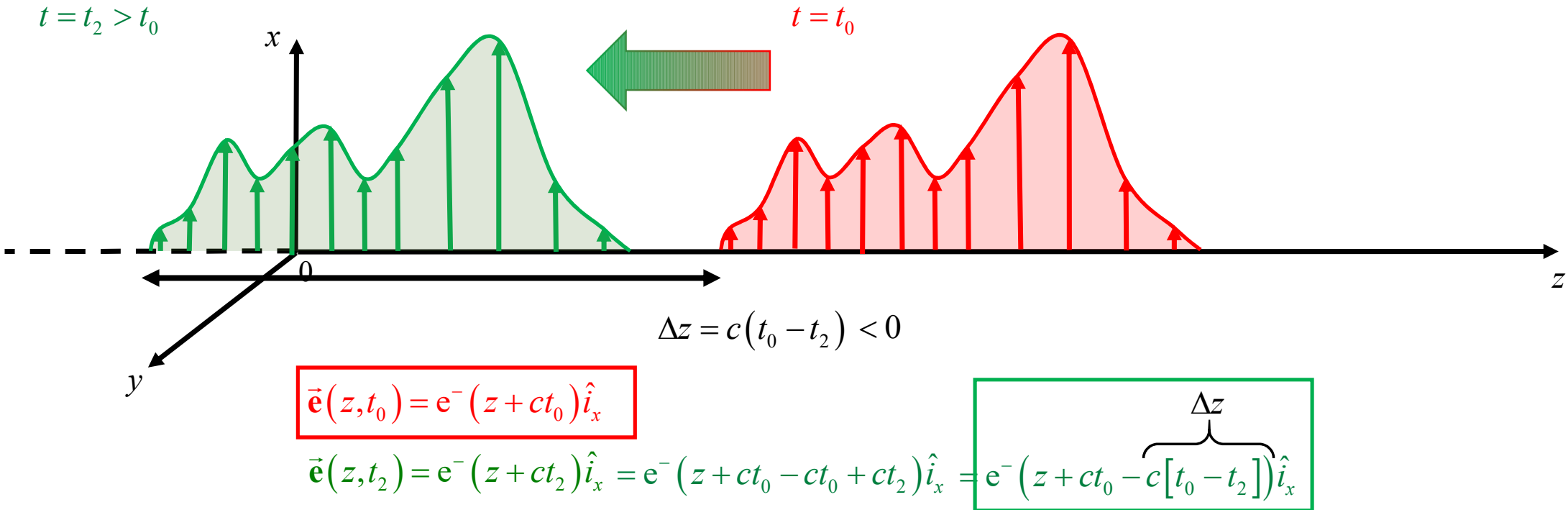
$$\vec{e}(z, t_0) = e^+(z - ct_0) \hat{i}_x$$

$$\vec{e}(z, t_2) = e^+(z - ct_2) \hat{i}_x = e^+(z - ct_0 + ct_0 - ct_2) \hat{i}_x = e^+(z - ct_0 - c[t_2 - t_0]) \hat{i}_x$$

The electromagnetic perturbation **propagates** without deformation and with constant speed  $c$  along the positive sense of the  $z$ -axis

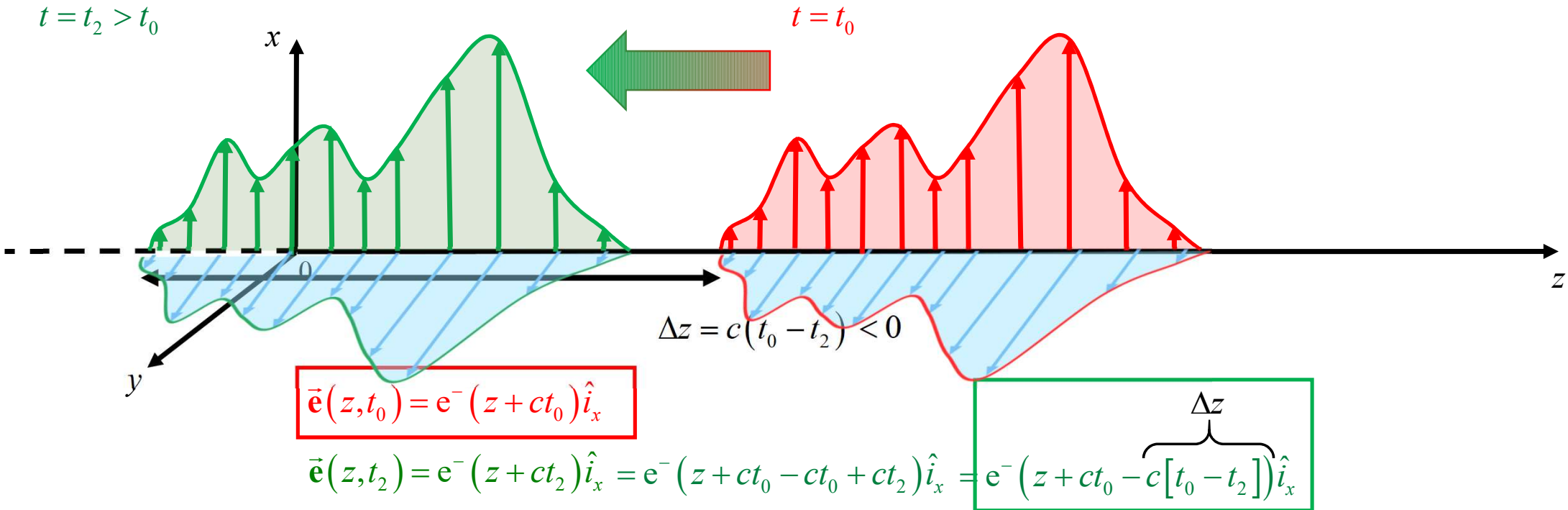
$\begin{cases} e^+(z - ct) \\ h^+(z - ct) \end{cases}$  is referred to as electromagnetic **progressive plane wave**

# Plane Waves (TD)



The electromagnetic perturbation **propagates** without deformation and with constant speed **c** along the negative sense of the z-axis

# Plane Waves (TD)



The electromagnetic perturbation **propagates** without deformation and with constant speed **c** along the negative sense of the z-axis

$\begin{cases} e^{- (z + ct)} \\ h^{- (z + ct)} \end{cases}$  is referred to as electromagnetic **regressive plane wave**