# Campi Elettromagnetici

## Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2021-2022 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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# Color legend

🟪 New formulas, important considerations, important formulas, important concepts

🟨 Very important for the discussion

⬜ Memo

🟩 Mathematical tools to be exploited

🟦 Mathematics

# Poynting theorem (TD)

$\vec{\mathbf{s}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right)\times\vec{\mathbf{h}}\left(\vec{\mathbf{r}},t\right)$ **Poynting vector** $\left[\vec{\mathbf{s}}\right]:\dfrac{Watt}{m^2}$

# Poynting theorem (TD)

$$\vec{\mathbf{s}}(\vec{\mathbf{r}},t)=\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

**Poynting vector** $[\vec{\mathbf{s}}]:\frac{Watt}{m^2}$

$$\nabla\cdot\vec{\mathbf{s}}(\vec{\mathbf{r}},t)+\frac{\partial}{\partial t}w(\vec{\mathbf{r}},t)+p_j(\vec{\mathbf{r}},t)=p_0(\vec{\mathbf{r}},t)$$

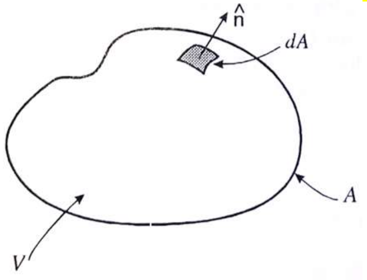
$$\boxed{\oint\!\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}+\frac{d}{dt}\iiint_V dV\,w(\vec{\mathbf{r}},t)\;+\iiint_V dV p_j(\vec{\mathbf{r}},t)\;=\iiint_V dV\,p_0(\vec{\mathbf{r}},t)$$

**Electromagnetic power flux**

$$P_S(t)+\frac{d}{dt}W(t)+P_j(t)=P_0(t)$$

**Hypotheses on the medium**

- **Linear**
- **Local (TND & SND)**
- **Isotropic**
- **Time-invariant**



$w(\vec{\mathbf{r}},t)=\frac{1}{2}\mu|\vec{\mathbf{h}}|^2+\frac{1}{2}\varepsilon|\vec{\mathbf{e}}|^2$ **Energy density of the e.m. field** ➡ $\iiint_V dV\,w(\vec{\mathbf{r}},t)=W(t)$ **Energy of the e.m. field**

$p_j(\vec{\mathbf{r}},t)=\sigma|\vec{\mathbf{e}}|^2$ **Power density dissipated in the conducting medium** ➡ $\iiint_V dV p_j(\vec{\mathbf{r}},t)=P_j(t)$ **Power dissipated in the conducting medium**

$p_0(\vec{\mathbf{r}},t)=-\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}$ **Power density delivered by the sources to the field** ➡ $\iiint_V dV\,p_0(\vec{\mathbf{r}},t)=P(t)$ **Power delivered by the sources to the field**

# THEOREMS

**Poynting**

Time domain – Phasor domain

**Uniqueness** (Interior problem – Exterior problem)

Time domain – Phasor domain

**Equivalence**

Phasor domain

**Image Theory**

**Reciprocity**

Phasor domain

# Mathematical tools that we will exploit today

$$\nabla \cdot \left[ \vec{\mathbf{A}}(\vec{\mathbf{r}}) \times \vec{\mathbf{B}}(\vec{\mathbf{r}}) \right] = \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot \left[ \nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}}) \right] - \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \left[ \nabla \times \vec{\mathbf{B}}(\vec{\mathbf{r}}) \right]$$

$$\iiint_V dV \, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS \, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

# Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}})=\frac{1}{2}\left[\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^{*}(\vec{\mathbf{r}})\right]$$

**Poynting vector**

$$\left[\vec{\mathbf{S}}\right]:\frac{Watt}{m^{2}}$$

$$\nabla\cdot\vec{\mathbf{S}}(\vec{\mathbf{r}})=\frac{1}{2}\nabla\cdot\left[\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^{*}(\vec{\mathbf{r}})\right]=\frac{1}{2}\vec{\mathbf{H}}^{*}(\vec{\mathbf{r}})\cdot\left[\nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}})\right]-\frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\left[\nabla\times\vec{\mathbf{H}}^{*}(\vec{\mathbf{r}})\right]=$$

$$=\frac{1}{2}\vec{\mathbf{H}}^{*}(\vec{\mathbf{r}})\cdot\left[\nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}})\right]-\frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\left[\nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}})\right]^{*}=$$

$$=\frac{1}{2}\vec{\mathbf{H}}^{*}(\vec{\mathbf{r}})\cdot\left[-j\omega_{0}\vec{\mathbf{B}}(\vec{\mathbf{r}})\right]-\frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\left[j\omega_{0}\vec{\mathbf{D}}(\vec{\mathbf{r}})+\vec{\mathbf{J}}(\vec{\mathbf{r}})+\vec{\mathbf{J}}_{0}(\vec{\mathbf{r}})\right]^{*}$$

$$=-\frac{1}{2}j\omega_{0}\vec{\mathbf{H}}^{*}(\vec{\mathbf{r}})\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}})+\frac{1}{2}j\omega_{0}\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{D}}^{*}(\vec{\mathbf{r}})-\frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}^{*}(\vec{\mathbf{r}})-\frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_{0}^{*}(\vec{\mathbf{r}})$$

$$\nabla\cdot\left[\vec{\mathbf{A}}(\vec{\mathbf{r}})\times\vec{\mathbf{B}}(\vec{\mathbf{r}})\right]=\vec{\mathbf{B}}(\vec{\mathbf{r}})\cdot\left[\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right]-\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\left[\nabla\times\vec{\mathbf{B}}(\vec{\mathbf{r}})\right]$$

Phasor domain

$$\nabla\times\vec{\mathbf{E}}(\vec{\mathbf{r}})=-j\omega_{0}\vec{\mathbf{B}}(\vec{\mathbf{r}})$$

$$\nabla\times\vec{\mathbf{H}}(\vec{\mathbf{r}})=j\omega_{0}\vec{\mathbf{D}}(\vec{\mathbf{r}})+\vec{\mathbf{J}}(\vec{\mathbf{r}})+\vec{\mathbf{J}}_{0}(\vec{\mathbf{r}})$$

$$\nabla\cdot\vec{\mathbf{D}}(\vec{\mathbf{r}})=\rho(\vec{\mathbf{r}})+\rho_{0}(\vec{\mathbf{r}})$$

$$\nabla\cdot\vec{\mathbf{B}}(\vec{\mathbf{r}})=0$$

# Poynting theorem (PD)

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**Poynting vector**

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- **Linear**
- **Isotropic**
- **Time-invariant**
- **Local (TND & SND)**

$$\begin{cases} \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{e}}(\vec{\mathbf{r}},t) \\ \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \mu(\vec{\mathbf{r}})\vec{\mathbf{h}}(\vec{\mathbf{r}},t) \\ \vec{\mathbf{j}}(\vec{\mathbf{r}},t) = \sigma\vec{\mathbf{e}}(\vec{\mathbf{r}},t) \end{cases}$$

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$$\begin{cases} \varepsilon : real \\ \mu : real \\ \sigma : real \end{cases}$$

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$$\nabla \cdot \vec{\mathbf{S}} = -\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^* \cdot \vec{\mathbf{B}} + \frac{1}{2} j\omega_0 \vec{\mathbf{E}} \cdot \vec{\mathbf{D}}^* - \frac{1}{2}\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}^* - \frac{1}{2}\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*$$

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$$\nabla \cdot \vec{\mathbf{S}} = \boxed{-\frac{1}{2} j\omega_0 \vec{\mathbf{H}}^{*} \cdot \vec{\mathbf{B}}} + \frac{1}{2} j\omega_0 \vec{\mathbf{E}} \cdot \vec{\mathbf{D}}^{*} - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}^{*} - \frac{1}{2} \vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^{*}$$

$$= -\frac{1}{2} j\omega_0 \mu_1 \left|\vec{\mathbf{H}}\right|^2 - \frac{1}{2} \omega_0 \mu_2 \left|\vec{\mathbf{H}}\right|^2$$

$$-j\omega_0 \vec{\mathbf{H}}^{*} \cdot \vec{\mathbf{B}} = -j\omega_0 \vec{\mathbf{H}}^{*} \cdot \left[\left(\mu_1 - j\mu_2\right)\vec{\mathbf{H}}\right] = -j\omega_0 \mu_1 \left|\vec{\mathbf{H}}\right|^2 - \omega_0 \mu_2 \left|\vec{\mathbf{H}}\right|^2$$

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$$\nabla\cdot\vec{\mathbf{S}} = -\frac{1}{2}j\omega_0\vec{\mathbf{H}}^*\cdot\vec{\mathbf{B}} \boxed{+\frac{1}{2}j\omega_0\vec{\mathbf{E}}\cdot\vec{\mathbf{D}}^*} -\frac{1}{2}\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}^* - \frac{1}{2}\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^*$$

$$= -\frac{1}{2}j\omega_0\mu_1\left|\vec{\mathbf{H}}\right|^2 - \frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}\right|^2 + \frac{1}{2}j\omega_0\varepsilon_1\left|\vec{\mathbf{E}}\right|^2 - \frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}\right|^2$$

$$j\omega_0\vec{\mathbf{E}}\cdot\vec{\mathbf{D}}^* = j\omega_0\vec{\mathbf{E}}\cdot\left[\left(\varepsilon_1 - j\varepsilon_2\right)\vec{\mathbf{E}}\right]^* = j\omega_0\vec{\mathbf{E}}\cdot\left[\left(\varepsilon_1 + j\varepsilon_2\right)\vec{\mathbf{E}}^*\right] = j\omega_0\varepsilon_1\left|\vec{\mathbf{E}}\right|^2 - \omega_0\varepsilon_2\left|\vec{\mathbf{E}}\right|^2$$

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**Poynting vector**

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$$\nabla\cdot\vec{\mathbf{S}} = -\frac{1}{2}j\omega_0\vec{\mathbf{H}}^*\cdot\vec{\mathbf{B}} + \frac{1}{2}j\omega_0\vec{\mathbf{E}}\cdot\vec{\mathbf{D}}^* \boxed{-\frac{1}{2}\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}^*} - \frac{1}{2}\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^*$$

$$= -\frac{1}{2}j\omega_0\mu_1\left|\vec{\mathbf{H}}\right|^2 - \frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}\right|^2 + \frac{1}{2}j\omega_0\varepsilon_1\left|\vec{\mathbf{E}}\right|^2 - \frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}\right|^2 - \frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2 - \frac{1}{2}\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^*$$

$$-\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}^* = -\vec{\mathbf{E}}\cdot\sigma\vec{\mathbf{E}}^* = -\sigma\left|\vec{\mathbf{E}}\right|^2$$

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**Poynting vector**

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$$\boxed{\nabla\cdot\vec{\mathbf{S}}}=-\frac{1}{2}j\omega_0\vec{\mathbf{H}}^{*}\cdot\vec{\mathbf{B}}+\frac{1}{2}j\omega_0\vec{\mathbf{E}}\cdot\vec{\mathbf{D}}^{*}-\frac{1}{2}\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}^{*}-\frac{1}{2}\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^{*}$$

$$=\boxed{-\frac{1}{2}j\omega_0\mu_1\left|\vec{\mathbf{H}}\right|^2}\boxed{-\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}\right|^2}\boxed{+\frac{1}{2}j\omega_0\varepsilon_1\left|\vec{\mathbf{E}}\right|^2}\boxed{-\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}\right|^2-\frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2}\boxed{-\frac{1}{2}\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^{*}}$$

$$\nabla\cdot\vec{\mathbf{S}}(\vec{\mathbf{r}})=\nabla\cdot\vec{\mathbf{S}}_1(\vec{\mathbf{r}})+j\nabla\cdot\vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\nabla\cdot\vec{\mathbf{S}}_1=-\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}\right|^2-\frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}\right|^2-\frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^{*}\right\}$$

$$\nabla\cdot\vec{\mathbf{S}}_2=-\frac{1}{2}\omega_0\mu_1\left|\vec{\mathbf{H}}\right|^2+\frac{1}{2}\omega_0\varepsilon_1\left|\vec{\mathbf{E}}\right|^2-\frac{1}{2}\mathrm{Im}\left\{\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^{*}\right\}$$

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$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j\vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\nabla \cdot \vec{\mathbf{S}}_1 + \frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}\right|^2 + \frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}\right|^2 + \frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2 = -\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*\right\}$$

$$\nabla \cdot \vec{\mathbf{S}}_2 + \frac{1}{2}\omega_0\mu_1\left|\vec{\mathbf{H}}\right|^2 - \frac{1}{2}\omega_0\varepsilon_1\left|\vec{\mathbf{E}}\right|^2 = -\frac{1}{2}\mathrm{Im}\left\{\vec{\mathbf{E}} \cdot \vec{\mathbf{J}}_0^*\right\}$$

$$\nabla \cdot \vec{\mathbf{S}}(\vec{\mathbf{r}}) = \nabla \cdot \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j\nabla \cdot \vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

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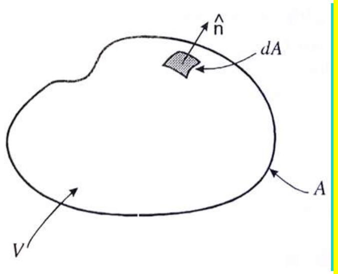
$$\nabla\cdot\vec{\mathbf{S}}_1 + \frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}\right|^2 + \frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}\right|^2 + \frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2 = -\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^{*}\right\}$$

$$\nabla\cdot\vec{\mathbf{S}}_2 + \frac{1}{2}\omega_0\mu_1\left|\vec{\mathbf{H}}\right|^2 - \frac{1}{2}\omega_0\varepsilon_1\left|\vec{\mathbf{E}}\right|^2 = -\frac{1}{2}\mathrm{Im}\left\{\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^{*}\right\}$$



### Hypotheses on the medium (PD)

- Linear
- Isotropic
- Time-invariant
- Time-Dispersive
- Space-Nondispersive

$$\begin{cases}\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})\\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}})\vec{\mathbf{H}}(\vec{\mathbf{r}})\\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma\vec{\mathbf{E}}(\vec{\mathbf{r}})\end{cases}$$

$$\begin{cases}\varepsilon = \varepsilon_1 - j\varepsilon_2\\ \mu = \mu_1 - j\mu_2\\ \sigma : \ real\end{cases}$$

$$\iiint_V dV\,\nabla\cdot\vec{\mathbf{S}}_1 + \iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}\right|^2 + \frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}\right|^2 + \frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2\right] = \iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^{*}\right\}\right]$$

$$\oiint_A dA\,\vec{\mathbf{S}}_1\cdot\hat{\mathbf{n}} + \iiint_V dV\left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}\right|^2 + \frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}\right|^2 + \frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2\right] = \iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^{*}\right\}\right]$$

$$\iiint_V dV\,\nabla\cdot\vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{\mathbf{n}}$$

# Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2}\left[\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^{*}(\vec{\mathbf{r}})\right]$$

**Poynting vector**

$$\left[\vec{\mathbf{S}}\right] : \frac{Watt}{m^{2}}$$

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \vec{\mathbf{S}}_{1}(\vec{\mathbf{r}}) + j\vec{\mathbf{S}}_{2}(\vec{\mathbf{r}})$$

$$\nabla\cdot\vec{\mathbf{S}}_{1} + \frac{1}{2}\omega_{0}\mu_{2}\left|\vec{\mathbf{H}}\right|^{2} + \frac{1}{2}\omega_{0}\varepsilon_{2}\left|\vec{\mathbf{E}}\right|^{2} + \frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^{2} = -\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_{0}^{*}\right\}$$

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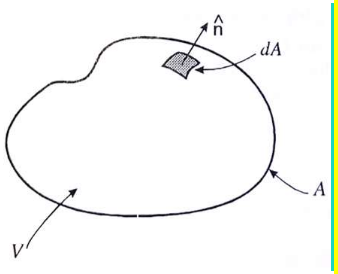

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$$\vec{\mathbf{f}}_1(\vec{\mathbf{r}},t) \longrightarrow \vec{\mathbf{F}}_1(\vec{\mathbf{r}})$$

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$$\left\langle \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t)\right\rangle = \frac{1}{T}\int_0^T \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t)\,dt \quad = \frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{F}}_1(\vec{\mathbf{r}})\cdot\vec{\mathbf{F}}_2^*(\vec{\mathbf{r}})\right\}$$

$$\left\langle \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\times\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t)\right\rangle = \frac{1}{T}\int_0^T \vec{\mathbf{f}}_1(\vec{\mathbf{r}},t)\times\vec{\mathbf{f}}_2(\vec{\mathbf{r}},t)\,dt \quad = \frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{F}}_1(\vec{\mathbf{r}})\times\vec{\mathbf{F}}_2^*(\vec{\mathbf{r}})\right\}$$

$$\vec{\mathbf{S}}_1(\vec{\mathbf{r}}) = \mathrm{Re}\left\{\vec{\mathbf{S}}(\vec{\mathbf{r}})\right\} = \mathrm{Re}\left\{\frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right\} = \frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}(\vec{\mathbf{r}})\times\vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right\}$$

$$= \left\langle \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right\rangle = \left\langle \vec{\mathbf{s}}(\vec{\mathbf{r}},t)\right\rangle$$

$$\boxed{\oiint_A dA\,\vec{\mathbf{S}}_1\cdot\hat{\mathbf{n}}} + \iiint_V dV\,\frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2 = \iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^*\right\}\right]$$

**Time averaged power flux associated to the e.m. field**

$$\begin{cases}\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})\\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}})\vec{\mathbf{H}}(\vec{\mathbf{r}})\\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma\vec{\mathbf{E}}(\vec{\mathbf{r}})\end{cases}$$

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$$\mathrm{Re}\left\{-\frac{1}{2}\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{J}}_0^*(\vec{\mathbf{r}})\right\} = \left\langle -\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right\rangle$$

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Time averaged power flux associated to the e.m. field

Time averaged power delivered by the sources to the field

**TD**

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Power flux associated to the e.m. field

Time derivative of the energy of the e.m. field

Power dissipated in the conducting medium

Power delivered by the sources to the field

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$$\frac{1}{2}\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2 = \frac{1}{2}\mathrm{Re}\left\{\sigma\left|\vec{\mathbf{E}}(\vec{\mathbf{r}})\right|^2\right\} \quad = \frac{1}{2}\mathrm{Re}\left\{\sigma\vec{\mathbf{E}}(\vec{\mathbf{r}})\cdot\vec{\mathbf{E}}^*(\vec{\mathbf{r}})\right\} \quad = \left\langle \sigma\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right\rangle$$

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**Where is the energy?**

**Remember that the energy is a state function**

$$\int_{t_1}^{t_2} dt \frac{\partial}{\partial t} w(t) = w(t_2) - w(t_1)$$

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$$\left[\vec{\mathbf{S}}\right] : \frac{Watt}{m^{2}}$$

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j\vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\boxed{\oint\!\!\!\oint_A dA\,\vec{\mathbf{S}}_1\cdot\hat{\mathbf{n}}} + \boxed{\iiint_V dV\,\frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^{2}} = \boxed{\iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^{*}\right\}\right]}$$

**Time averaged power flux associated to the e.m. field**

**Time averaged power dissipated in the conducting medium**

**Time averaged power delivered by the sources to the field**

**TD**

$$\underbrace{\oint\!\!\!\oint_A dA\,\vec{\mathbf{s}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}}_{\text{Power flux associated to the e.m. field}} + \underbrace{\frac{d}{dt}\iiint_V dV\left[\frac{1}{2}\mu\left|\vec{\mathbf{h}}\right|^{2} + \frac{1}{2}\varepsilon\left|\vec{\mathbf{e}}\right|^{2}\right]}_{\text{Time derivative of the energy of the e.m. field}} + \underbrace{\iiint_V dV\,\sigma\left|\vec{\mathbf{e}}\right|^{2}}_{\text{Power dissipated in the conducting medium}} = \underbrace{-\iiint_V dV\,\vec{\mathbf{j}}_0\cdot\vec{\mathbf{e}}}_{\text{Power delivered by the sources to the field}}$$

**Hypotheses on the medium (PD)**

- **Linear**
- **Isotropic**
- **Time-invariant**
- **-Time-Dispersive**
- **-Space-Nondispersive**

$$\begin{cases}\vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}})\\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}})\vec{\mathbf{H}}(\vec{\mathbf{r}})\\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma\vec{\mathbf{E}}(\vec{\mathbf{r}})\end{cases}$$

$$\begin{cases}\varepsilon = \varepsilon_1 - j\varepsilon_2\\ \mu = \mu_1 - j\mu_2\\ \sigma : real\end{cases}$$

$$\begin{cases}\varepsilon_2 = 0\\ \mu_2 = 0\end{cases}$$

# Poynting theorem (PD)

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \frac{1}{2}\left[\vec{\mathbf{E}}(\vec{\mathbf{r}}) \times \vec{\mathbf{H}}^*(\vec{\mathbf{r}})\right]$$

**Poynting vector**

$$\left[\vec{\mathbf{S}}\right] : \frac{Watt}{m^2}$$

$$\vec{\mathbf{S}}(\vec{\mathbf{r}}) = \vec{\mathbf{S}}_1(\vec{\mathbf{r}}) + j\vec{\mathbf{S}}_2(\vec{\mathbf{r}})$$

$$\oint\!\!\!\oint_A dA\,\vec{\mathbf{S}}_1 \cdot \hat{\mathbf{n}} + \iiint_V dV \left[\frac{1}{2}\omega_0\mu_2\left|\vec{\mathbf{H}}\right|^2 + \frac{1}{2}\omega_0\varepsilon_2\left|\vec{\mathbf{E}}\right|^2 + \frac{1}{2}\sigma\left|\vec{\mathbf{E}}\right|^2\right] = \iiint_V dV\left[-\frac{1}{2}\mathrm{Re}\left\{\vec{\mathbf{E}}\cdot\vec{\mathbf{J}}_0^*\right\}\right]$$

**Time averaged power flux associated to the e.m. field**

**LOSSES**

$(\varepsilon_2)$ electric losses

$(\mu_2)$ magnetic losses

**Time averaged power dissipated in the conducting medium**

**Time averaged power delivered by the sources to the field**

$$\varepsilon_2 > 0;\ \mu_2 > 0;\ \sigma > 0$$

**Dispersion and losses are related each other: a (time) dispersive medium presents losses**

**Hypotheses on the medium (PD)**

- **Linear**
- **Isotropic**
- **Time-invariant**
- **Time- Dispersive**
- **Space-Nondispersive**

$$\begin{cases} \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \varepsilon(\vec{\mathbf{r}})\vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{B}}(\vec{\mathbf{r}}) = \mu(\vec{\mathbf{r}})\vec{\mathbf{H}}(\vec{\mathbf{r}}) \\ \vec{\mathbf{J}}(\vec{\mathbf{r}}) = \sigma\vec{\mathbf{E}}(\vec{\mathbf{r}}) \end{cases}$$

$$\begin{cases} \varepsilon = \varepsilon_1 - j\varepsilon_2 \\ \mu = \mu_1 - j\mu_2 \\ \sigma : \ real \end{cases}$$