

Campi Elettromagnetici

**Corso di Laurea in Ingegneria Informatica,
Biomedica e delle Telecomunicazioni**

a.a. 2021-2022 - Laurea “Triennale” – Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

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Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Mathematical tools that we will exploit today

$$\vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot [\vec{\mathbf{B}}(\vec{\mathbf{r}}) \times \vec{\mathbf{C}}(\vec{\mathbf{r}})] = \vec{\mathbf{C}}(\vec{\mathbf{r}}) \cdot [\vec{\mathbf{A}}(\vec{\mathbf{r}}) \times \vec{\mathbf{B}}(\vec{\mathbf{r}})] = \vec{\mathbf{B}}(\vec{\mathbf{r}}) \cdot [\vec{\mathbf{C}}(\vec{\mathbf{r}}) \times \vec{\mathbf{A}}(\vec{\mathbf{r}})]$$

Fields at boundaries

Let us consider nonhomogeneous media, so that constitutive relations may change as a function of space.

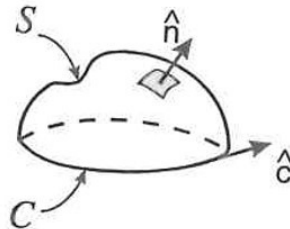
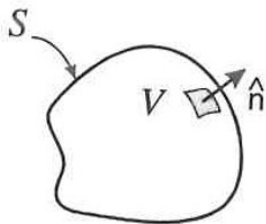
Let us suppose that this change is abrupt across space, that is, we have two media of different characteristics separated by a smooth surface.

The solutions of Maxwell equations can be obtained in both the two regions: matching conditions are needed on the boundaries.

Fields at boundaries

Integral form

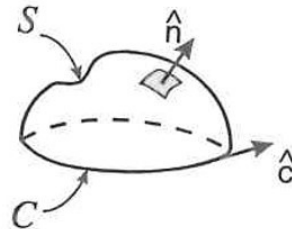
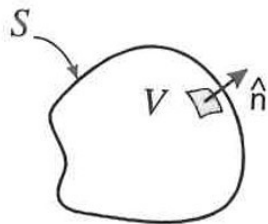
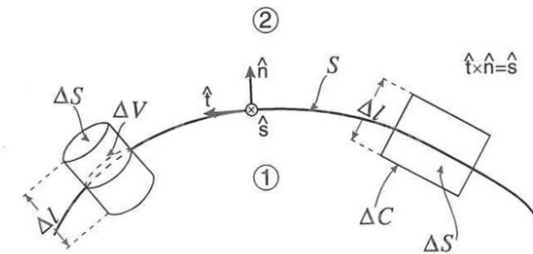
$$\left\{ \begin{array}{l} \oint_C d\mathbf{c} \, \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = -\frac{d}{dt} \iint_S dS \, \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oint_C d\mathbf{c} \, \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = \frac{d}{dt} \iint_S dS \, \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} + \iint_S dS \, \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oiint_S dS \, \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = \iiint_V dV \, \rho(\vec{\mathbf{r}}, t) \\ \oiint_S dS \, \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = 0 \end{array} \right.$$



Fields at boundaries

Integral form

$$\left\{ \begin{array}{l} \oint_C d\mathbf{c} \, \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = -\frac{d}{dt} \iint_S dS \, \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oint_C d\mathbf{c} \, \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = \frac{d}{dt} \iint_S dS \, \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} + \iint_S dS \, \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oiint_S dS \, \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = \iiint_V dV \, \rho(\vec{\mathbf{r}}, t) \\ \oiint_S dS \, \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = 0 \end{array} \right.$$

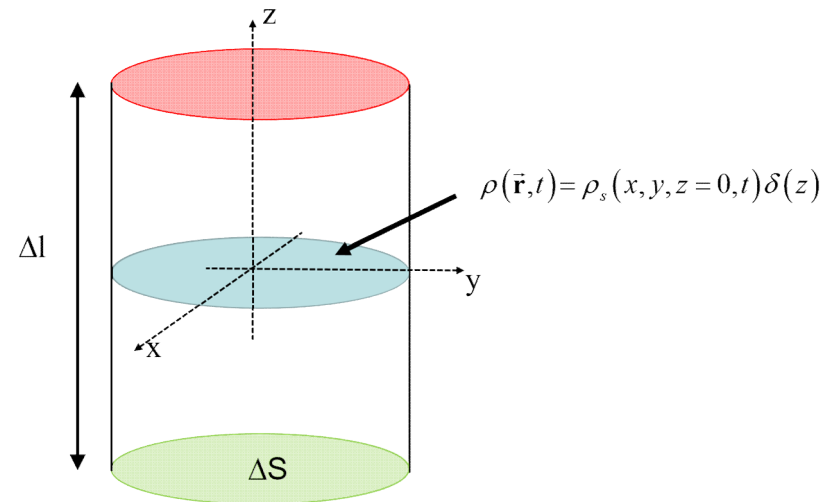


Fields at boundaries

$$\rho(\vec{r}, t) = \rho_s(x, y, z=0, t) \delta(z)$$

$$\iiint_V dV \rho(x, y, z, t) = \iint_{\Delta S} dS \int_{\Delta l} dz \rho(x, y, z, t)$$

$$= \iint_{\Delta S} dS \int_{\Delta l} dz \rho_s(x, y, z=0, t) \delta(z) = \iint_{\Delta S} dS \rho_s(x, y, z=0, t)$$



$$\oiint_S dS \vec{d}(\vec{r}, t) \cdot \hat{n} = \iiint_V dV \rho(\vec{r}, t)$$

$$\Delta l \rightarrow 0$$

When a localized charge distribution is present over the boundary, so that ρ is infinite there

$$\iint_{\Delta S} dS \vec{d}_2(\vec{r}, t) \cdot \hat{n} - \iint_{\Delta S} dS \vec{d}_1(\vec{r}, t) \cdot \hat{n} = \iiint_V dV \rho(\vec{r}, t)$$

$$\iint_{\Delta S} dS [\vec{d}_2(\vec{r}, t) - \vec{d}_1(\vec{r}, t)] \cdot \hat{n} = \iint_{\Delta S} dS \rho_s(\vec{r}, t)$$

$$(\vec{d}_2 - \vec{d}_1) \cdot \hat{n} = \rho_s$$

$$[\rho]: \frac{\text{Coulomb}}{m^3}$$

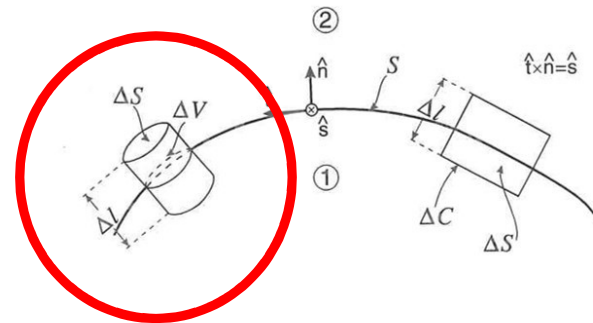
$$[\delta(z)]: \frac{1}{m}$$

$$[\rho_s]: \frac{\text{Coulomb}}{m^2}$$

Fields at boundaries

Integral form

$$\left\{ \begin{array}{l} \oint_C d\mathbf{c} \, \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = -\frac{d}{dt} \iint_S dS \, \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oint_C d\mathbf{c} \, \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = \frac{d}{dt} \iint_S dS \, \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} + \iint_S dS \, \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oiint_S dS \, \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = \iiint_V dV \, \rho(\vec{\mathbf{r}}, t) \\ \oiint_S dS \, \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = 0 \end{array} \right.$$



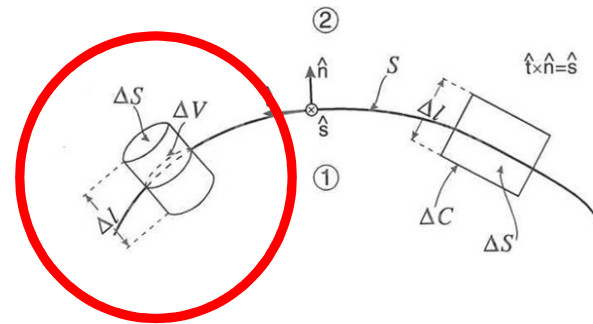
$$\oiint_S dS \, \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = \iiint_V dV \, \rho(\vec{\mathbf{r}}, t) \quad \longrightarrow \quad (\vec{\mathbf{d}}_2 - \vec{\mathbf{d}}_1) \cdot \hat{\mathbf{n}} = \rho_s$$

$$\oiint_S dS \, \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = 0 \quad \longrightarrow \quad (\vec{\mathbf{b}}_2 - \vec{\mathbf{b}}_1) \cdot \hat{\mathbf{n}} = 0$$

Fields at boundaries

Integral form

$$\left\{ \begin{array}{l} \oint_C d\mathbf{c} \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = -\frac{d}{dt} \iint_S dS \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oint_C d\mathbf{c} \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = \frac{d}{dt} \iint_S dS \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} + \iint_S dS \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oiint_S dS \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = \iiint_V dV \rho(\vec{\mathbf{r}}, t) \\ \oiint_S dS \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = 0 \end{array} \right.$$



$$(\vec{\mathbf{d}}_2 - \vec{\mathbf{d}}_1) \cdot \hat{\mathbf{n}} = \rho_s$$

$$(\vec{\mathbf{b}}_2 - \vec{\mathbf{b}}_1) \cdot \hat{\mathbf{n}} = 0$$

Fields at boundaries

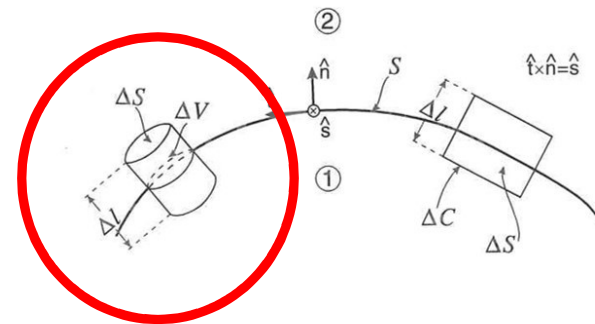
Integral form

$$\left\{ \begin{array}{l} \oint_C d\mathbf{c} \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = -\frac{d}{dt} \iint_S dS \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oint_C d\mathbf{c} \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = \frac{d}{dt} \iint_S dS \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} + \iint_S dS \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oiint_S dS \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = \iiint_V dV \rho(\vec{\mathbf{r}}, t) \\ \oiint_S dS \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = 0 \end{array} \right.$$

+

Current density equation

$$\oiint_S dS \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} + \frac{dq(t)}{dt} = 0$$



$$(\vec{\mathbf{d}}_2 - \vec{\mathbf{d}}_1) \cdot \hat{\mathbf{n}} = \rho_s$$

$$(\vec{\mathbf{b}}_2 - \vec{\mathbf{b}}_1) \cdot \hat{\mathbf{n}} = 0$$

Fields at boundaries

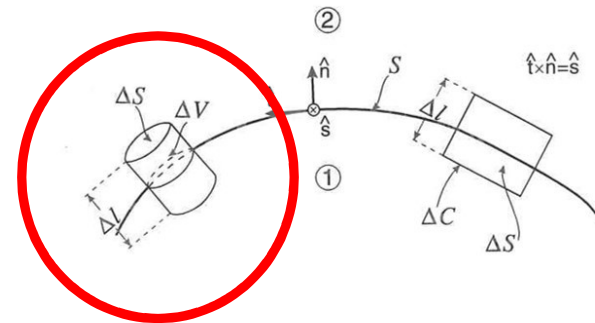
Integral form

$$\left\{ \begin{aligned} \oint_C d\mathbf{c} \, \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} &= -\frac{d}{dt} \iint_S dS \, \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oint_C d\mathbf{c} \, \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} &= \frac{d}{dt} \iint_S dS \, \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} + \iint_S dS \, \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oiint_S dS \, \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} &= \iiint_V dV \, \rho(\vec{\mathbf{r}}, t) \\ \oiint_S dS \, \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} &= 0 \end{aligned} \right.$$

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Current density equation

$$\oiint_S dS \, \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} + \frac{d}{dt} \iiint_V dV \, \rho(\vec{\mathbf{r}}, t) = 0$$



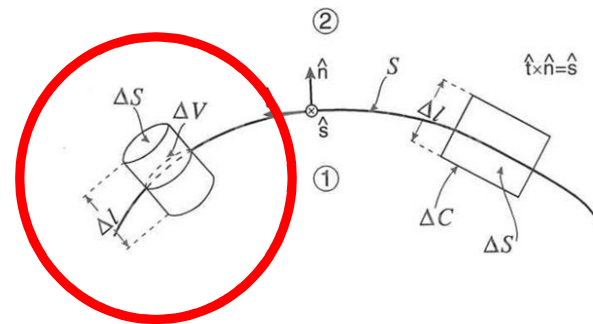
$$(\vec{\mathbf{d}}_2 - \vec{\mathbf{d}}_1) \cdot \hat{\mathbf{n}} = \rho_s$$

$$(\vec{\mathbf{b}}_2 - \vec{\mathbf{b}}_1) \cdot \hat{\mathbf{n}} = 0$$

Fields at boundaries

Integral form

$$\left\{ \begin{aligned} \oint_C d\mathbf{c} \, \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} &= -\frac{d}{dt} \iint_S dS \, \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oint_C d\mathbf{c} \, \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} &= \frac{d}{dt} \iint_S dS \, \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} + \iint_S dS \, \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oiint_S dS \, \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} &= \iiint_V dV \, \rho(\vec{\mathbf{r}}, t) \\ \oiint_S dS \, \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} &= 0 \end{aligned} \right.$$



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Current density equation

$$\oiint_S dS \, \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} + \frac{d}{dt} \iiint_V dV \, \rho(\vec{\mathbf{r}}, t) = 0$$



$$(\vec{\mathbf{j}}_2 - \vec{\mathbf{j}}_1) \cdot \hat{\mathbf{n}} = -\frac{\partial \rho_s}{\partial t}$$

$$(\vec{\mathbf{d}}_2 - \vec{\mathbf{d}}_1) \cdot \hat{\mathbf{n}} = \rho_s$$

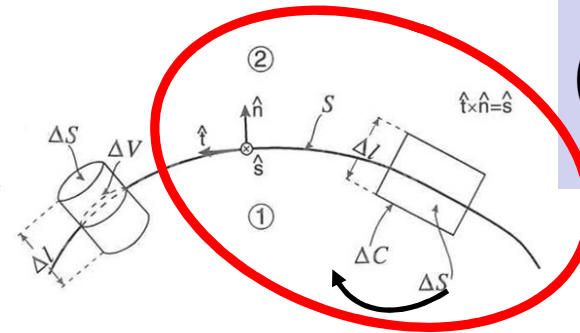
$$(\vec{\mathbf{b}}_2 - \vec{\mathbf{b}}_1) \cdot \hat{\mathbf{n}} = 0$$

Fields at boundaries

Integral form

$$\left\{ \begin{aligned} \oint_C dc \vec{e}(\vec{r}, t) \cdot \hat{c} &= -\frac{d}{dt} \iint_S dS \vec{b}(\vec{r}, t) \cdot \hat{n} \\ \oint_C dc \vec{h}(\vec{r}, t) \cdot \hat{c} &= \frac{d}{dt} \iint_S dS \vec{d}(\vec{r}, t) \cdot \hat{n} + \iint_S dS \vec{j}(\vec{r}, t) \cdot \hat{n} \\ \oiint_S dS \vec{d}(\vec{r}, t) \cdot \hat{n} &= \iiint_V dV \rho(\vec{r}, t) \\ \oiint_S dS \vec{b}(\vec{r}, t) \cdot \hat{n} &= 0 \end{aligned} \right.$$

$$\hat{t} = \hat{n} \times \hat{s}$$



$$(\vec{d}_2 - \vec{d}_1) \cdot \hat{n} = \rho_s$$

$$(\vec{b}_2 - \vec{b}_1) \cdot \hat{n} = 0$$

$$(\vec{j}_2 - \vec{j}_1) \cdot \hat{n} = -\frac{\partial \rho_s}{\partial t}$$

$$\Delta l \rightarrow 0$$

$$\vec{A} \cdot [\vec{B} \times \vec{C}] = \vec{C} \cdot [\vec{A} \times \vec{B}]$$

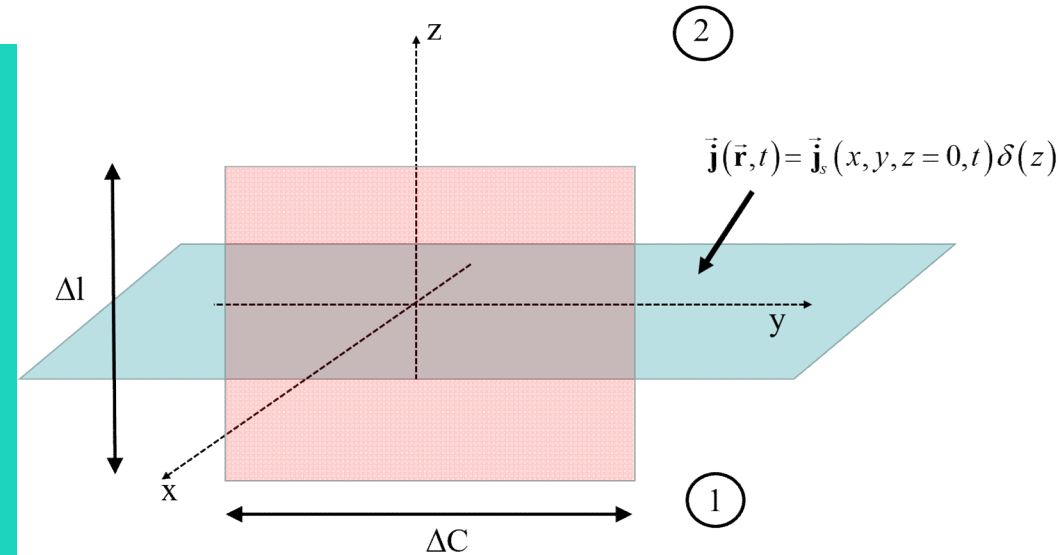
$$\begin{aligned} \oint_C dc \vec{h}(\vec{r}, t) \cdot \hat{c} &= \int_{\Delta C} dc \vec{h}_1(\vec{r}, t) \cdot \hat{t} - \int_{\Delta C} dc \vec{h}_2(\vec{r}, t) \cdot \hat{t} = \int_{\Delta C} dc [\vec{h}_1(\vec{r}, t) - \vec{h}_2(\vec{r}, t)] \cdot \hat{t} = \int_{\Delta C} dc [\vec{h}_1(\vec{r}, t) - \vec{h}_2(\vec{r}, t)] \cdot (\hat{n} \times \hat{s}) \\ &= \int_{\Delta C} dc \hat{s} \cdot ([\vec{h}_1(\vec{r}, t) - \vec{h}_2(\vec{r}, t)] \times \hat{n}) = \int_{\Delta C} dc \hat{s} \cdot (\hat{n} \times [\vec{h}_2(\vec{r}, t) - \vec{h}_1(\vec{r}, t)]) = \int_{\Delta C} dc (\hat{n} \times [\vec{h}_2(\vec{r}, t) - \vec{h}_1(\vec{r}, t)]) \cdot \hat{s} \end{aligned}$$

Fields at boundaries

$$\vec{j}(x, y, z, t) = \vec{j}_s(x, y, z=0, t) \delta(z)$$

$$\iint_{\Delta S} dS \vec{j}(x, y, z, t) \cdot \hat{i}_x = \int_{\Delta C} dy \int_{\Delta l} dz \vec{j}(x, y, z, t) \cdot \hat{i}_x$$

$$= \int_{\Delta C} dy \int_{\Delta l} dz \vec{j}_s(x, y, z=0, t) \delta(z) \cdot \hat{i}_x = \int_{\Delta C} dy \vec{j}_s(x, y, z=0, t) \cdot \hat{i}_x$$



$$\Delta l \rightarrow 0$$

$$\iint_{\Delta S} dS \vec{j}(\vec{r}, t) \cdot \hat{s}$$

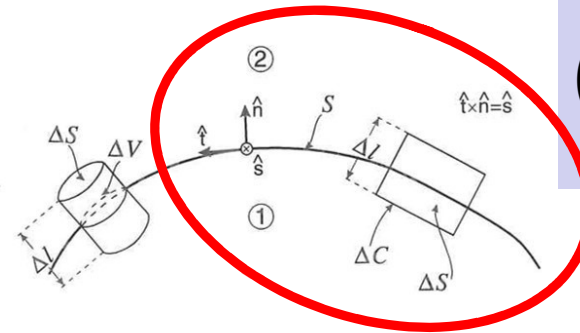
When a localized current is present over the boundary, so that \vec{j} is infinite there

$$\oint_C d\vec{c} \vec{h}(\vec{r}, t) \cdot \hat{c} = \int_{\Delta C} d\vec{c} (\hat{n} \times [\vec{h}_2(\vec{r}, t) - \vec{h}_1(\vec{r}, t)]) \cdot \hat{s}$$

Fields at boundaries

$$\vec{j}(x, y, z, t) = \vec{j}_s(x, y, z=0, t) \delta(z)$$

$$\begin{aligned} \iint_{\Delta S} dS \vec{j}(x, y, z, t) \cdot \hat{i}_x &= \int_{\Delta C} dy \int_{\Delta l} dz \vec{j}(x, y, z, t) \cdot \hat{i}_x \\ &= \int_{\Delta C} dy \int_{\Delta l} dz \vec{j}_s(x, y, z=0, t) \delta(z) \cdot \hat{i}_x = \int_{\Delta C} dy \vec{j}_s(x, y, z=0, t) \cdot \hat{i}_x \end{aligned}$$



$$(\vec{d}_2 - \vec{d}_1) \cdot \hat{n} = \rho_s$$

$$(\vec{b}_2 - \vec{b}_1) \cdot \hat{n} = 0$$

$$(\vec{j}_2 - \vec{j}_1) \cdot \hat{n} = -\frac{\partial \rho_s}{\partial t}$$

$$\Delta l \rightarrow 0$$

$$\iint_{\Delta S} dS \vec{j}(\vec{r}, t) \cdot \hat{s} = \int_{\Delta C} dc \vec{j}_s(\vec{r}, t) \cdot \hat{s}$$

When a localized current is present over the boundary, so that \vec{j} is infinite there

$$\oint_C dc \vec{h}(\vec{r}, t) \cdot \hat{c} = \int_{\Delta C} dc (\hat{n} \times [\vec{h}_2(\vec{r}, t) - \vec{h}_1(\vec{r}, t)]) \cdot \hat{s}$$

$$[\vec{j}]: \frac{\text{Ampere}}{m^2}$$

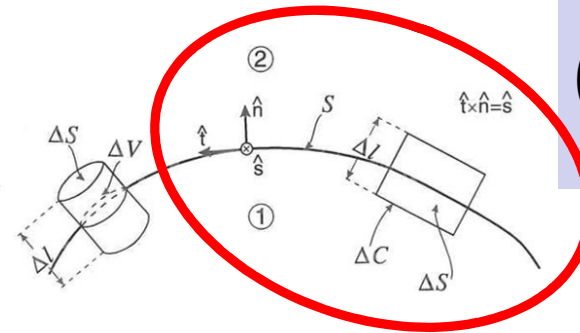
$$[\delta(z)]: \frac{1}{m}$$

$$[\vec{j}_s]: \frac{\text{Ampere}}{m}$$

Fields at boundaries

Integral form

$$\left\{ \begin{aligned} \oint_C dc \vec{e}(\vec{r}, t) \cdot \hat{c} &= -\frac{d}{dt} \iint_S dS \vec{b}(\vec{r}, t) \cdot \hat{n} \\ \oint_C dc \vec{h}(\vec{r}, t) \cdot \hat{c} &= \frac{d}{dt} \iint_S dS \vec{d}(\vec{r}, t) \cdot \hat{n} + \iint_S dS \vec{j}(\vec{r}, t) \cdot \hat{n} \\ \oiint_S dS \vec{d}(\vec{r}, t) \cdot \hat{n} &= \iiint_V dV \rho(\vec{r}, t) \\ \oiint_S dS \vec{b}(\vec{r}, t) \cdot \hat{n} &= 0 \end{aligned} \right.$$



$$(\vec{d}_2 - \vec{d}_1) \cdot \hat{n} = \rho_s$$

$$(\vec{b}_2 - \vec{b}_1) \cdot \hat{n} = 0$$

$$(\vec{j}_2 - \vec{j}_1) \cdot \hat{n} = -\frac{\partial \rho_s}{\partial t}$$

$$\Delta l \rightarrow 0$$

$$\iint_{\Delta S} dS \vec{d}(\vec{r}, t) \cdot \hat{s} = 0$$

When a localized current is present over the boundary, so that \vec{j} is infinite there

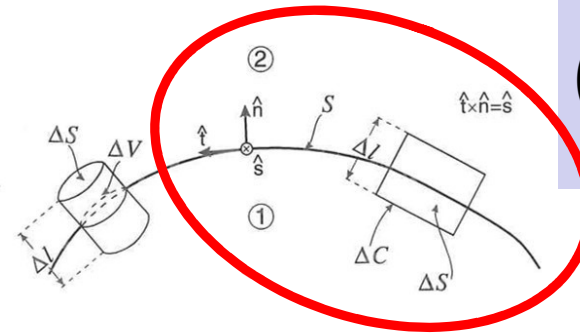
$$\oint_C dc \vec{h}(\vec{r}, t) \cdot \hat{c} = \int_{\Delta C} dc (\hat{n} \times [\vec{h}_2(\vec{r}, t) - \vec{h}_1(\vec{r}, t)]) \cdot \hat{s}$$

$$\iint_{\Delta S} dS \vec{j}(\vec{r}, t) \cdot \hat{s} = \int_{\Delta C} dc \vec{j}_s(\vec{r}, t) \cdot \hat{s}$$

Fields at boundaries

Integral form

$$\left\{ \begin{array}{l} \oint_C d\mathbf{c} \, \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = -\frac{d}{dt} \iint_S dS \, \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oint_C d\mathbf{c} \, \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = \frac{d}{dt} \iint_S dS \, \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} + \iint_S dS \, \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oiint_S dS \, \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = \iiint_V dV \, \rho(\vec{\mathbf{r}}, t) \\ \oiint_S dS \, \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = 0 \end{array} \right.$$



$$(\vec{\mathbf{d}}_2 - \vec{\mathbf{d}}_1) \cdot \hat{\mathbf{n}} = \rho_s$$

$$(\vec{\mathbf{b}}_2 - \vec{\mathbf{b}}_1) \cdot \hat{\mathbf{n}} = 0$$

$$(\vec{\mathbf{j}}_2 - \vec{\mathbf{j}}_1) \cdot \hat{\mathbf{n}} = -\frac{\partial \rho_s}{\partial t}$$

$$\Delta l \rightarrow 0$$

$$\int_{\Delta C} d\mathbf{c} \left(\hat{\mathbf{n}} \times [\vec{\mathbf{h}}_2(\vec{\mathbf{r}}, t) - \vec{\mathbf{h}}_1(\vec{\mathbf{r}}, t)] \right) \cdot \hat{\mathbf{s}} = \int_{\Delta C} d\mathbf{c} \, \vec{\mathbf{j}}_s(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{s}}$$

When a localized current is present over the boundary, so that \mathbf{j} is infinite there

$$\oint_C d\mathbf{c} \, \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = \int_{\Delta C} d\mathbf{c} \left(\hat{\mathbf{n}} \times [\vec{\mathbf{h}}_2(\vec{\mathbf{r}}, t) - \vec{\mathbf{h}}_1(\vec{\mathbf{r}}, t)] \right) \cdot \hat{\mathbf{s}}$$

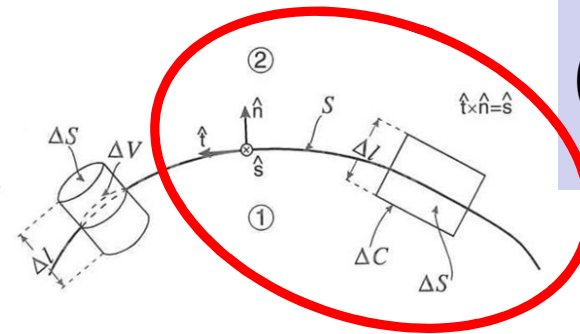
$$\iint_{\Delta S} dS \, \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{s}} = \int_{\Delta C} d\mathbf{c} \, \vec{\mathbf{j}}_s(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{s}}$$

$$\iint_{\Delta S} dS \, \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{s}} = 0$$

Fields at boundaries

Integral form

$$\left\{ \begin{array}{l} \oint_C d\mathbf{c} \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = -\frac{d}{dt} \iint_S dS \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oint_C d\mathbf{c} \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = \frac{d}{dt} \iint_S dS \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} + \iint_S dS \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \\ \oiint_S dS \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = \iiint_V dV \rho(\vec{\mathbf{r}}, t) \\ \oiint_S dS \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = 0 \end{array} \right.$$



$$(\vec{\mathbf{d}}_2 - \vec{\mathbf{d}}_1) \cdot \hat{\mathbf{n}} = \rho_s$$

$$(\vec{\mathbf{b}}_2 - \vec{\mathbf{b}}_1) \cdot \hat{\mathbf{n}} = 0$$

$$(\vec{\mathbf{j}}_2 - \vec{\mathbf{j}}_1) \cdot \hat{\mathbf{n}} = -\frac{\partial \rho_s}{\partial t}$$

$$\Delta l \rightarrow 0$$

$$\int_{\Delta C} d\mathbf{c} \left(\hat{\mathbf{n}} \times [\vec{\mathbf{h}}_2(\vec{\mathbf{r}}, t) - \vec{\mathbf{h}}_1(\vec{\mathbf{r}}, t)] \right) \cdot \hat{\mathbf{s}} = \int_{\Delta C} d\mathbf{c} \vec{\mathbf{j}}_s(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{s}}$$

$$\hat{\mathbf{n}} \times (\vec{\mathbf{h}}_2 - \vec{\mathbf{h}}_1) = \vec{\mathbf{j}}_s$$

When a localized current is present over the boundary, so that \mathbf{j} is infinite there

Fields at boundaries

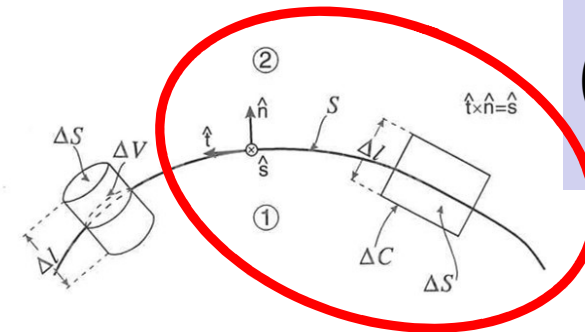
Integral form

$$\oint_C d\mathbf{c} \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = -\frac{d}{dt} \iint_S dS \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}}$$

$$\oint_C d\mathbf{c} \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = \frac{d}{dt} \iint_S dS \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} + \iint_S dS \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}}$$

$$\oiint_S dS \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = \iiint_V dV \rho(\vec{\mathbf{r}}, t)$$

$$\oiint_S dS \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} = 0$$



$$(\vec{\mathbf{d}}_2 - \vec{\mathbf{d}}_1) \cdot \hat{\mathbf{n}} = \rho_s$$

$$(\vec{\mathbf{b}}_2 - \vec{\mathbf{b}}_1) \cdot \hat{\mathbf{n}} = 0$$

$$(\vec{\mathbf{j}}_2 - \vec{\mathbf{j}}_1) \cdot \hat{\mathbf{n}} = -\frac{\partial \rho_s}{\partial t}$$

$$\oint_C d\mathbf{c} \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = \frac{d}{dt} \iint_S dS \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} + \iint_S dS \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \quad \longrightarrow \quad \hat{\mathbf{n}} \times (\vec{\mathbf{h}}_2 - \vec{\mathbf{h}}_1) = \vec{\mathbf{j}}_s$$

$$\oint_C d\mathbf{c} \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{c}} = -\frac{d}{dt} \iint_S dS \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) \cdot \hat{\mathbf{n}} \quad \longrightarrow \quad \hat{\mathbf{n}} \times (\vec{\mathbf{e}}_2 - \vec{\mathbf{e}}_1) = \mathbf{0}$$

Fields at boundaries

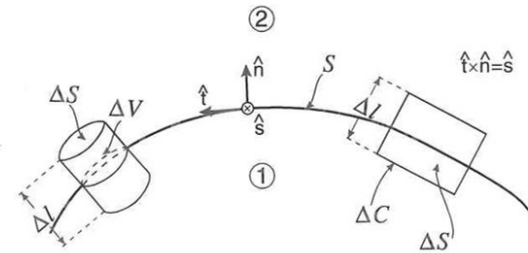
$$\hat{\mathbf{n}} \times (\vec{\mathbf{e}}_2 - \vec{\mathbf{e}}_1) = \mathbf{0}$$

$$\hat{\mathbf{n}} \times (\vec{\mathbf{h}}_2 - \vec{\mathbf{h}}_1) = \vec{\mathbf{j}}_s$$

$$(\vec{\mathbf{d}}_2 - \vec{\mathbf{d}}_1) \cdot \hat{\mathbf{n}} = \rho_s$$

$$(\vec{\mathbf{b}}_2 - \vec{\mathbf{b}}_1) \cdot \hat{\mathbf{n}} = 0$$

$$(\vec{\mathbf{j}}_2 - \vec{\mathbf{j}}_1) \cdot \hat{\mathbf{n}} = -\frac{\partial \rho_s}{\partial t}$$



Fields at boundaries

One example: the medium 1 is a PEC

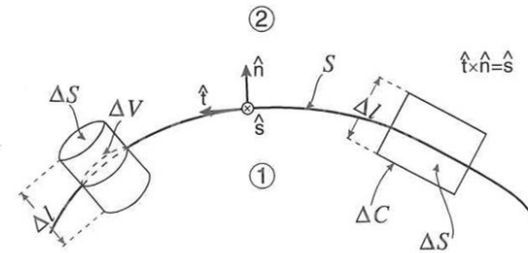
$$\hat{\mathbf{n}} \times (\vec{\mathbf{e}}_2 - \vec{\mathbf{e}}_1) = \mathbf{0}$$

$$\hat{\mathbf{n}} \times (\vec{\mathbf{h}}_2 - \vec{\mathbf{h}}_1) = \vec{\mathbf{j}}_s$$

$$(\vec{\mathbf{d}}_2 - \vec{\mathbf{d}}_1) \cdot \hat{\mathbf{n}} = \rho_s$$

$$(\vec{\mathbf{b}}_2 - \vec{\mathbf{b}}_1) \cdot \hat{\mathbf{n}} = 0$$

$$(\vec{\mathbf{j}}_2 - \vec{\mathbf{j}}_1) \cdot \hat{\mathbf{n}} = -\frac{\partial \rho_s}{\partial t}$$



$$\vec{\mathbf{e}}_1 = \mathbf{0}$$

$$\vec{\mathbf{h}}_1 = \mathbf{0}$$

$$\vec{\mathbf{d}}_1 = \mathbf{0}$$

$$\vec{\mathbf{b}}_1 = \mathbf{0}$$

Fields at boundaries

One example: the medium 1 is a PEC

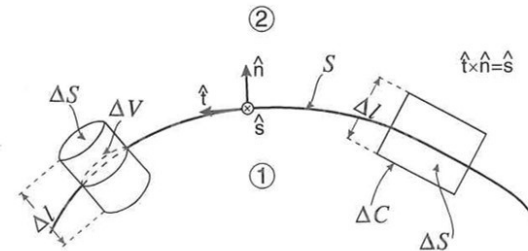
$$\hat{\mathbf{n}} \times (\vec{\mathbf{e}}_2 - \vec{\mathbf{e}}_1) = \mathbf{0}$$

$$\hat{\mathbf{n}} \times (\vec{\mathbf{h}}_2 - \vec{\mathbf{h}}_1) = \vec{\mathbf{j}}_s$$

$$(\vec{\mathbf{d}}_2 - \vec{\mathbf{d}}_1) \cdot \hat{\mathbf{n}} = \rho_s$$

$$(\vec{\mathbf{b}}_2 - \vec{\mathbf{b}}_1) \cdot \hat{\mathbf{n}} = 0$$

$$(\vec{\mathbf{j}}_2 - \vec{\mathbf{j}}_1) \cdot \hat{\mathbf{n}} = -\frac{\partial \rho_s}{\partial t}$$



$$\vec{\mathbf{e}}_1 = \mathbf{0}$$

$$\vec{\mathbf{h}}_1 = \mathbf{0}$$

$$\vec{\mathbf{d}}_1 = \mathbf{0}$$

$$\vec{\mathbf{b}}_1 = \mathbf{0}$$

$$\hat{\mathbf{n}} \times \vec{\mathbf{e}}_2 = \mathbf{0}$$

$$\hat{\mathbf{n}} \times \vec{\mathbf{h}}_2 = \vec{\mathbf{j}}_s$$

$$\vec{\mathbf{d}}_2 \cdot \hat{\mathbf{n}} = \rho_s$$

$$\vec{\mathbf{b}}_2 \cdot \hat{\mathbf{n}} = 0$$