

# **Campi Elettromagnetici**

**Corso di Laurea in Ingegneria Informatica,  
Biomedica e delle Telecomunicazioni**

**a.a. 2021-2022 - Laurea “Triennale” – Secondo semestre - Secondo anno**

**Università degli Studi di Napoli “Parthenope”**

**Stefano Perna**

# Color legend

New formulas, important considerations,  
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

# Constitutive relationships

## Time domain - Differential form

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) + \vec{\mathbf{j}}_0(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) + \rho_0(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

Even assuming knowledge of the impressed sources  $\vec{\mathbf{j}}_0(\vec{\mathbf{r}}, t); \rho_0(\vec{\mathbf{r}}, t)$

Number of independent scalar equations: **7**

Number of unknown scalar quantities: **16**

Maxwell equations involve a number of unknowns larger than the number of equations!

The additional missing equations are provided by the **constitutive relationships**, which describe interaction of fields and matter from a macroscopic point of view

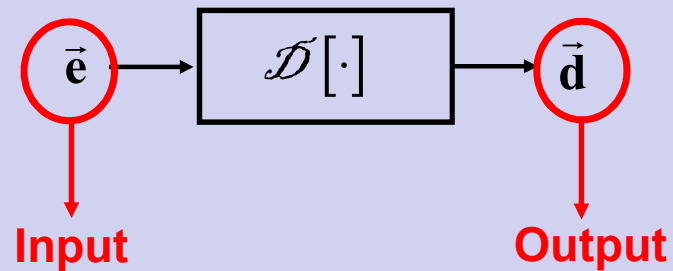
# Constitutive relationships

Linear media that are **NOT** bianisotropic

$$\vec{\mathbf{d}} = \mathcal{D}[\vec{\mathbf{e}}]$$

$$\vec{\mathbf{b}} = \mathcal{B}[\vec{\mathbf{h}}]$$

$$\vec{\mathbf{j}} = \mathcal{J}[\vec{\mathbf{e}}]$$



# Constitutive relationships

## Linear media

### Class

**Time-dispersive**

**Time-nondispersive**

**Time-variant**

**Time-invariant**

### Property

### Effect on the I-O relation

# Constitutive relationships

## Linear media

### Class

**Space-dispersive**

**Space-nondispersive**

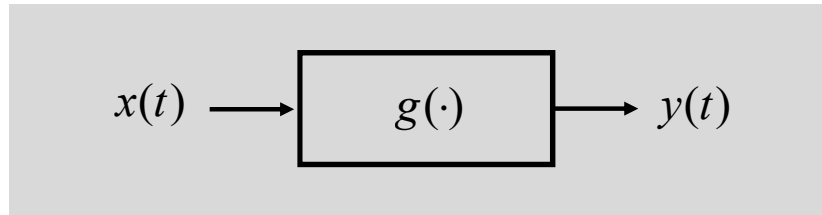
**Space-variant**

**Space-invariant**

### Property

### Effect on the I-O relation

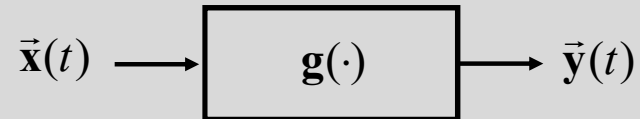
# Memo: linear systems



$x(t)$  and  $y(t)$  are scalar

	Time-dispersive (TD)	Time-nondispersive (TND)
Time-variant (TV)	$y(t) = \int dt' g(t, t') x(t')$	$y(t) = \tilde{g}(t) x(t)$
Time-invariant (TI)	$y(t) = \int dt' g(t - t') x(t')$	$y(t) = \tilde{g} x(t)$

# Linear systems

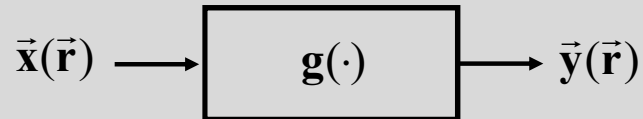


$\vec{x}(t)$  and  $\vec{y}(t)$  are vectors

	Time-dispersive (TD)	Time-nondispersive (TND)
Time-variant (TV)	$\vec{y}(t) = \int dt' \mathbf{g}(t, t') \cdot \vec{x}(t')$	$\vec{y}(t) = \tilde{\mathbf{g}}(t) \cdot \vec{x}(t)$
Time-invariant (TI)	$\vec{y}(t) = \int dt' \mathbf{g}(t - t') \cdot \vec{x}(t')$	$\vec{y}(t) = \tilde{\mathbf{g}} \cdot \vec{x}(t)$



# Linear systems



$\vec{x}(\vec{r})$  and  $\vec{y}(\vec{r})$  are vectors

	Space-dispersive (SD)	Space-nondispersive (SND)
Space-variant (SV)	$\vec{y}(\vec{r}) = \int d\vec{r}' g(\vec{r}, \vec{r}') \cdot \vec{x}(\vec{r}')$	$\vec{y}(\vec{r}) = \tilde{\mathbf{g}}(\vec{r}) \cdot \vec{x}(\vec{r})$
Space-invariant (SI)	$\vec{y}(\vec{r}) = \int d\vec{r}' g(\vec{r} - \vec{r}') \cdot \vec{x}(\vec{r}')$	$\vec{y}(\vec{r}) = \tilde{\mathbf{g}} \cdot \vec{x}(\vec{r})$

	Time-dispersive (TD)	Time-nondispersive (TND)
Time-variant (TV)	$\vec{y}(t) = \int dt' \mathbf{g}(t, t') \cdot \vec{x}(t')$	$\vec{y}(t) = \tilde{\mathbf{g}}(t) \cdot \vec{x}(t)$
Time-invariant (TI)	$\vec{y}(t) = \int dt' \mathbf{g}(t - t') \cdot \vec{x}(t')$	$\vec{y}(t) = \tilde{\mathbf{g}} \cdot \vec{x}(t)$
	Space-dispersive (SD)	Space-nondispersive (SND)
Space-variant (SV)	$\vec{y}(\vec{r}) = \int d\vec{r}' \mathbf{g}(\vec{r}, \vec{r}') \cdot \vec{x}(\vec{r}')$	$\vec{y}(\vec{r}) = \tilde{\mathbf{g}}(\vec{r}) \cdot \vec{x}(\vec{r})$
Space-invariant (SI)	$\vec{y}(\vec{r}) = \int d\vec{r}' \mathbf{g}(\vec{r} - \vec{r}') \cdot \vec{x}(\vec{r}')$	$\vec{y}(\vec{r}) = \tilde{\mathbf{g}} \cdot \vec{x}(\vec{r})$

# MEMO

## Linear media

$$\vec{d}(\vec{r}, t) = \boldsymbol{\varepsilon}(\vec{r}, t) \cdot \vec{e}(\vec{r}, t)$$

$\boldsymbol{\varepsilon}(\vec{r}, t)$ : 3x3 matrix

■ Local (nondispersive) media

■ Anisotropic media

### Class

**Isotropic**

### Property

A **rotation** of the input implies **the same rotation** of the output

### Effect on the I-O relation

$\boldsymbol{\varepsilon}(\vec{r}, t)$  becomes scalar. It is not a matrix anymore!

	Time-dispersive (TD)	Time-nondispersive (TND)
Time-variant (TV)	$\vec{y}(t) = \int dt' g(t, t') \vec{x}(t')$	$\vec{y}(t) = \tilde{g}(t) \vec{x}(t)$
Time-invariant (TI)	$\vec{y}(t) = \int dt' g(t - t') \vec{x}(t')$	$\vec{y}(t) = \tilde{g} \vec{x}(t)$
	Space-dispersive (SD)	Space-nondispersive (SND)
Space-variant (SV)	$\vec{y}(\vec{r}) = \int d\vec{r}' g(\vec{r}, \vec{r}') \vec{x}(\vec{r}')$	$\vec{y}(\vec{r}) = \tilde{g}(\vec{r}) \vec{x}(\vec{r})$
Space-invariant (SI)	$\vec{y}(\vec{r}) = \int d\vec{r}' g(\vec{r} - \vec{r}') \vec{x}(\vec{r}')$	$\vec{y}(\vec{r}) = \tilde{g} \vec{x}(\vec{r})$

	Time-dispersive (TD)	Time-nondispersive (TND)
Time-variant (TV)	$\vec{y}(t) = \int dt' \mathbf{g}(t, t') \cdot \vec{x}(t')$	$\vec{y}(t) = \tilde{\mathbf{g}}(t) \cdot \vec{x}(t)$
Time-invariant (TI)	$\vec{y}(t) = \int dt' \mathbf{g}(t - t') \cdot \vec{x}(t')$	$\vec{y}(t) = \tilde{\mathbf{g}} \cdot \vec{x}(t)$
	Space-dispersive (SD)	Space-nondispersive (SND)
Space-variant (SV)	$\vec{y}(\vec{r}) = \int d\vec{r}' \mathbf{g}(\vec{r}, \vec{r}') \cdot \vec{x}(\vec{r}')$	$\vec{y}(\vec{r}) = \tilde{\mathbf{g}}(\vec{r}) \cdot \vec{x}(\vec{r})$
Space-invariant (SI)	$\vec{y}(\vec{r}) = \int d\vec{r}' \mathbf{g}(\vec{r} - \vec{r}') \cdot \vec{x}(\vec{r}')$	$\vec{y}(\vec{r}) = \tilde{\mathbf{g}} \cdot \vec{x}(\vec{r})$

# Constitutive relationships

Linear & **Anisotropic** & Dispersive media & Space-variant & Time-variant

$$\vec{\mathbf{d}} = \mathcal{D}[\vec{\mathbf{e}}] \quad \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \int d\vec{\mathbf{r}}' \int dt' \mathbf{g}_d(\vec{\mathbf{r}}, \vec{\mathbf{r}}', t, t') \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}}', t')$$

$$\vec{\mathbf{b}} = \mathcal{B}[\vec{\mathbf{h}}] \quad \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = \int d\vec{\mathbf{r}}' \int dt' \mathbf{g}_b(\vec{\mathbf{r}}, \vec{\mathbf{r}}', t, t') \cdot \vec{\mathbf{h}}(\vec{\mathbf{r}}', t')$$

$$\vec{\mathbf{j}} = \mathcal{J}[\vec{\mathbf{e}}] \quad \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) = \int d\vec{\mathbf{r}}' \int dt' \mathbf{g}_j(\vec{\mathbf{r}}, \vec{\mathbf{r}}', t, t') \cdot \vec{\mathbf{e}}(\vec{\mathbf{r}}', t')$$

Linear & **Isotropic** & Dispersive media & Space-variant & Time-variant

$$\vec{\mathbf{d}} = \mathcal{D}[\vec{\mathbf{e}}] \quad \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \int d\vec{\mathbf{r}}' \int dt' g_d(\vec{\mathbf{r}}, \vec{\mathbf{r}}', t, t') \vec{\mathbf{e}}(\vec{\mathbf{r}}', t')$$

$$\vec{\mathbf{b}} = \mathcal{B}[\vec{\mathbf{h}}] \quad \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = \int d\vec{\mathbf{r}}' \int dt' g_b(\vec{\mathbf{r}}, \vec{\mathbf{r}}', t, t') \vec{\mathbf{h}}(\vec{\mathbf{r}}', t')$$

$$\vec{\mathbf{j}} = \mathcal{J}[\vec{\mathbf{e}}] \quad \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) = \int d\vec{\mathbf{r}}' \int dt' g_j(\vec{\mathbf{r}}, \vec{\mathbf{r}}', t, t') \vec{\mathbf{e}}(\vec{\mathbf{r}}', t')$$

# Constitutive relationships

In the following, just for the sake of simplicity,  
we will consider isotropic media

Time: dispersive  
Space: dispersive

	Space-Dispersive (SD) Time-Dispersive (TD)
SV-TV	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r}, \vec{r}', t, t') \vec{e}(\vec{r}', t')$
SV-TI	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r}, \vec{r}', t - t') \vec{e}(\vec{r}', t')$
SI-TV	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r} - \vec{r}', t, t') \vec{e}(\vec{r}', t')$
SI-TI	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r} - \vec{r}', t - t') \vec{e}(\vec{r}', t')$



Time: nondispersive  
Space: dispersive

	Space-Dispersive (SD) Time-Dispersive (TD)	Space-Dispersive (SD) Time-Nondispersive (TND)
SV-TV	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r}, \vec{r}', t, t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \int d\vec{r}' g_d(\vec{r}, \vec{r}', t) \vec{e}(\vec{r}', t)$
SV-TI	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r}, \vec{r}', t - t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \int d\vec{r}' g_d(\vec{r}, \vec{r}') \vec{e}(\vec{r}', t)$
SI-TV	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r} - \vec{r}', t, t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \int d\vec{r}' g_d(\vec{r} - \vec{r}', t) \vec{e}(\vec{r}', t)$
SI-TI	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r} - \vec{r}', t - t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \int d\vec{r}' g_d(\vec{r} - \vec{r}') \vec{e}(\vec{r}', t)$

Time: dispersive  
Space: nondispersive

	Space-Dispersive (SD) Time-Dispersive (TD)	Space-Nondispersive (SND) Time-Dispersive (TD)
SV-TV	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r}, \vec{r}', t, t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \int dt' g_d(\vec{r}, t, t') \vec{e}(\vec{r}, t')$
SV-TI	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r}, \vec{r}', t - t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \int dt' g_d(\vec{r}, t - t') \vec{e}(\vec{r}, t')$
SI-TV	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r} - \vec{r}', t, t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \int dt' g_d(t, t') \vec{e}(\vec{r}, t')$
SI-TI	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r} - \vec{r}', t - t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \int dt' g_d(t - t') \vec{e}(\vec{r}, t')$

# Dispersive (time & space) vs. nondispersive (time & space)

	Space-Dispersive (SD) Time-Dispersive (TD)	Space-Nondispersive (SND) Time-Nondispersive (TND)
SV-TV	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r}, \vec{r}', t, t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \varepsilon(\vec{r}, t) \vec{e}(\vec{r}, t)$
SV-TI	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r}, \vec{r}', t - t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \varepsilon(\vec{r}) \vec{e}(\vec{r}, t)$
SI-TV	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r} - \vec{r}', t, t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \varepsilon(t) \vec{e}(\vec{r}, t)$
SI-TI	$\vec{d}(\vec{r}, t) = \int d\vec{r}' \int dt' g_d(\vec{r} - \vec{r}', t - t') \vec{e}(\vec{r}', t')$	$\vec{d}(\vec{r}, t) = \varepsilon \vec{e}(\vec{r}, t)$

# Permittivity

	SND+TND: Local	
SV-TV	$\vec{d}(\vec{r},t) = \varepsilon(\vec{r},t)\vec{e}(\vec{r},t)$	$[\varepsilon] = ?$
		$[\varepsilon] = \frac{\text{Coulomb}}{m^2} \frac{m}{\text{Volt}} = \frac{\text{Coulomb}}{\text{Volt}} \frac{1}{m} = \frac{\text{Farad}}{m}$
SV-TI	$\vec{d}(\vec{r},t) = \varepsilon(\vec{r})\vec{e}(\vec{r},t)$	
SI-TV	$\vec{d}(\vec{r},t) = \varepsilon(t)\vec{e}(\vec{r},t)$	$[\vec{e}(\vec{r},t)] = \frac{\text{Volt}}{m}$ $[\vec{d}(\vec{r},t)] = \frac{\text{Coulomb}}{m^2}$
SI-TI	$\vec{d}(\vec{r},t) = \varepsilon\vec{e}(\vec{r},t)$	$C = \frac{q}{\Delta v}$ $\text{Farad} = \frac{\text{Coulomb}}{\text{Volt}}$

# Permeability

	SND+TND: Local	
SV-TV	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \mu(\vec{\mathbf{r}},t)\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$	$[\mu] = ?$
SV-TI	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \mu(\vec{\mathbf{r}})\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$	
SI-TV	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \mu(t)\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$	
SI-TI	$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \mu\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$	

$$[\mu] = \frac{\text{Weber}}{m^2} \frac{m}{\text{Ampere}} = \frac{\text{Weber}}{\text{Ampere m}} = \frac{\text{Henry}}{m}$$
  

$$[\vec{\mathbf{b}}(\vec{\mathbf{r}},t)] = \frac{\text{Weber}}{m^2} \quad [\vec{\mathbf{h}}(\vec{\mathbf{r}},t)] = \frac{\text{Ampere}}{m}$$
  

$$L = \frac{\Phi_{\vec{\mathbf{b}}}}{i} \quad \text{Henry} = \frac{\text{Weber}}{\text{Ampere}}$$

# Conductivity

	SND+TND: Local
SV-TV	$\vec{j}(\vec{r}, t) = \sigma(\vec{r}, t) \vec{e}(\vec{r}, t)$
SV-TI	$\vec{j}(\vec{r}, t) = \sigma(\vec{r}) \vec{e}(\vec{r}, t)$
SI-TV	$\vec{j}(\vec{r}, t) = \sigma(t) \vec{e}(\vec{r}, t)$
SI-TI	$\vec{j}(\vec{r}, t) = \sigma \vec{e}(\vec{r}, t)$

$$[\sigma] = ?$$

$$[\sigma] = \frac{\text{Ampere}}{m^2} \frac{m}{\text{Volt}} = \frac{\text{Ampere}}{\text{Volt}} \frac{1}{m} = \frac{1}{\Omega m} = \frac{\text{Siemens}}{m}$$

$$[\vec{j}(\vec{r}, t)] = \frac{\text{Ampere}}{m^2} \quad [\vec{e}(\vec{r}, t)] = \frac{\text{Volt}}{m}$$

$$\Delta v = Ri \quad \Omega = \frac{\text{Volt}}{\text{Ampere}} = \frac{1}{\text{Siemens}}$$

# Exercise n.1

**Linear+ Isotropic + Space-Variant (SV) + Time-Variant (TV)**

**Space-dispersive (SD) +Time-dispersive (TD)**

$$\vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \int d\vec{\mathbf{r}}' \int dt' g_d(\vec{\mathbf{r}}, \vec{\mathbf{r}}', t, t') \vec{\mathbf{e}}(\vec{\mathbf{r}}', t')$$

$$[g_d] = \frac{\text{Farad}}{s \cdot m^4}$$

**Space-dispersive (SD) +Time-nondispersive (TND)**

$$\vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \int d\vec{\mathbf{r}}' g_d(\vec{\mathbf{r}}, \vec{\mathbf{r}}', t) \vec{\mathbf{e}}(\vec{\mathbf{r}}', t)$$

$$[g_d] = \frac{\text{Farad}}{m^4}$$

**Space-nondispersive (SND) +Time-dispersive (TD)**

$$\vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \int dt' g_d(\vec{\mathbf{r}}, t, t') \vec{\mathbf{e}}(\vec{\mathbf{r}}, t')$$

$$[g_d] = \frac{\text{Farad}}{s \cdot m}$$

**Space-nondispersive (SND) +Time-nondispersive (TND)**

$$\vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \varepsilon(\vec{\mathbf{r}}, t) \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)$$

$$[\varepsilon] = \frac{\text{Farad}}{m}$$

# Exercise n.2

Linear+ Isotropic + Space-Variant (SV) + Time-Invariant (TI)

**Space-Dispersive (SD) +Time-Dispersive (TD)**

$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \int d\vec{\mathbf{r}}' \int dt' g_b(\vec{\mathbf{r}},\vec{\mathbf{r}}',t-t') \vec{\mathbf{h}}(\vec{\mathbf{r}}',t')$$

$$[g_b] = \frac{\text{Henry}}{s \cdot m^4}$$

**Space-Dispersive (SD) +Time-Nondispersive (TND)**

$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \int d\vec{\mathbf{r}}' g_b(\vec{\mathbf{r}},\vec{\mathbf{r}}') \vec{\mathbf{h}}(\vec{\mathbf{r}}',t)$$

$$[g_b] = \frac{\text{Henry}}{m^4}$$

**Space-Nondispersive (SND) +Time-Dispersive (TD)**

$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \int dt' g_b(\vec{\mathbf{r}},t-t') \vec{\mathbf{h}}(\vec{\mathbf{r}},t')$$

$$[g_b] = \frac{\text{Henry}}{s \cdot m}$$

**Space-Nondispersive (SND) +Time-Nondispersive (TND)**

$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t) = \mu(\vec{\mathbf{r}}) \vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

$$[\mu] = \frac{\text{Henry}}{m}$$



# Conductors

$$\vec{j}(\vec{r}, t) = \sigma \vec{e}(\vec{r}', t')$$

Metal	Conductivity $\sigma$ [siemens/m]
Silver, 99.98% pure	$6.14 \times 10^7$
Copper, annealed	$5.80 \times 10^7$
Copper, hard drawn	$5.65 \times 10^7$
Gold, pure drawn	$4.10 \times 10^7$
Aluminum, commercial hard drawn	$3.54 \times 10^7$
Magnesium	$2.17 \times 10^7$
Tungsten	$1.81 \times 10^7$
Zinc	$1.74 \times 10^7$
Nickel	$1.28 \times 10^7$
Iron, 99.98% pure	$1.00 \times 10^7$
Steel	$1.00-0.5 \times 10^7$
Lead	$0.48 \times 10^7$
Mercury	$0.10 \times 10^7$

From G.Franceschetti, 'Electromagnetics, Theory, Techniques, and Engineering paradigms', Plenum Press

## Perfect Electric Conductors (PEC)

$$\sigma \rightarrow \infty \quad \Rightarrow \quad \vec{e} = 0 \quad \Rightarrow \quad \vec{h} = 0$$

# Vacuum

$$\vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \varepsilon_0 \vec{\mathbf{e}}(\vec{\mathbf{r}}, t)$$

$$\vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = \mu_0 \vec{\mathbf{h}}(\vec{\mathbf{r}}, t)$$

$$\sigma = 0$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ Henry / m}$$

$$\varepsilon_0 = 8.8 \times 10^{-12} \text{ Farad / m}$$