Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2021-2022 - Laurea "Triennale" – Secondo semestre - Secondo anno

Università degli Studi di Napoli "Parthenope"

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Color legend

New formulas, important considerations, important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Mathematical tools that we will exploit today

$$\nabla \cdot \left(\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}}) \right) = 0$$

Maxwell equations

Time domain - Differential form

$$
\begin{cases}
\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\
\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0
\end{cases}
$$

James Clerk Maxwell 1831-1879

Maxwell equations

Time domain - Differential form

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\begin{cases}
\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\
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\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) + \rho_0(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0
\end{cases}
$$

James Clerk Maxwell 1831-1879

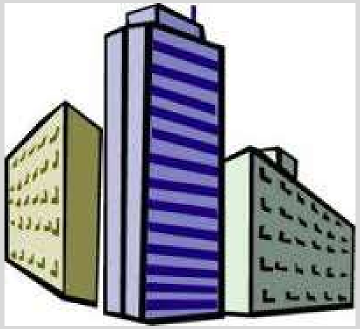
Maxwell equations

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) + \rho_0(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

$$\begin{cases} \vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\ \rho_0(\vec{\mathbf{r}},t) \end{cases}$$ **Prescribed sources**

$$\begin{cases} \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \rho(\vec{\mathbf{r}},t) \end{cases}$$ **Induced sources**

Complex scenario



The independence of the Maxwell equations

Time domain - Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) + \rho_0(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

$$\nabla \cdot \left(\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right) = \nabla \cdot \left(-\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}\right)$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$0 \qquad = -\frac{\partial\left(\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t)\right)}{\partial t}$$

$$\nabla \cdot \left(\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}})\right) = 0$$

The independence of the Maxwell equations

Time domain - Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) + \rho_0(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

$$\nabla \cdot \left(\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right) = \nabla \cdot \left(-\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}\right)$$

⬇ ⬇

$$0 = -\frac{\partial\left(\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t)\right)}{\partial t}$$

$\nabla \cdot \vec{\mathbf{b}}$ is independent of time. If the fields are equal to zero before a given time, then $\nabla \cdot \vec{\mathbf{b}} = 0$ for all times, thus recovering the last Maxwell equation

The independence of the Maxwell equations

Time domain - Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) + \rho_0(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

Number of independent scalar equations:

3+3+1=7

Let us assume knowledge of the impressed sources $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t); \rho_0(\vec{\mathbf{r}},t)$

Number of unknown scalar quantities:

$\vec{\mathbf{e}}(\vec{\mathbf{r}},t); \vec{\mathbf{d}}(\vec{\mathbf{r}},t); \vec{\mathbf{h}}(\vec{\mathbf{r}},t); \vec{\mathbf{b}}(\vec{\mathbf{r}},t); \vec{\mathbf{j}}(\vec{\mathbf{r}},t); \rho(\vec{\mathbf{r}},t)$

3 +3 +3 +3 +3 +1

16

The independence of the Maxwell equations

Time domain - Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) + \rho_0(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

Even assuming knowledge of the impressed sources $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t);\rho_0(\vec{\mathbf{r}},t)$

Number of independent scalar equations: 7

Number of unknown scalar quantities: 16

Maxwell equations involve a number of unknowns larger than the number of equations!

Constitutive relationships

Time domain - Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) + \rho_0(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

Even assuming knowledge of the impressed sources $\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t);\rho_0(\vec{\mathbf{r}},t)$

Number of independent scalar equations: 7

Number of unknown scalar quantities: 16

Maxwell equations involve a number of unknowns larger than the number of equations!

The additional missing equations are provided by the **constitutive relationships**, which describe interaction of fields and matter from a macroscopic point of view

Memo

Vacuum

$$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\varepsilon_0\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$
$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\mu_0\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$
$$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\sigma\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$\mu_0 = 4\pi \times 10^{-7}$ *Permeability* $[Henry/m]$

$\varepsilon_0 = 8.8\times 10^{-12}$ *Permittivity* $[Farad/m]$

$\sigma = 0$ *Conductivity* $[Siemens/m]$

These relationships depend on the particular medium that we have considered, that is, the vacuum.
More generally, similar relationships, can be found also in other media. They depend upon the characteristics of the medium in which the electromagnetic field is considered, and are called **CONSTITUTIVE RELATIONSHPS**

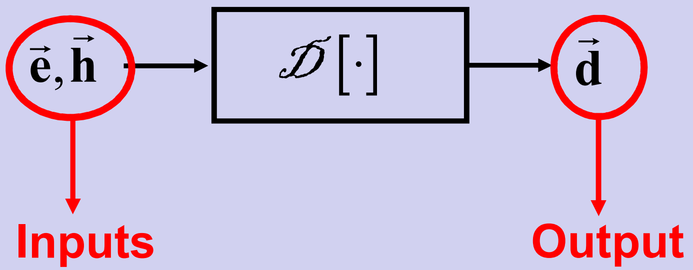
Constitutive relationships

Inductions and currents must be represented in terms of fields

$$\vec{\mathbf{d}} = \mathscr{D}\left[\vec{\mathbf{e}},\vec{\mathbf{h}}\right]$$

$$\vec{\mathbf{b}} = \mathscr{B}\left[\vec{\mathbf{e}},\vec{\mathbf{h}}\right]$$

$$\vec{\mathbf{j}} = \mathscr{J}\left[\vec{\mathbf{e}},\vec{\mathbf{h}}\right]$$



$\mathscr{D}[\cdot], \mathscr{B}[\cdot]$ and $\mathscr{J}[\cdot]$ are functionals which depend upon the medium in which the electromagnetic field is considered and upon the fields themselves.

Constitutive relationships

Linear media

$$\vec{\mathbf{d}}_1 = \mathscr{D}\left[\vec{\mathbf{e}}_1, \vec{\mathbf{h}}_1\right]$$

$$\vec{\mathbf{d}}_2 = \mathscr{D}\left[\vec{\mathbf{e}}_2, \vec{\mathbf{h}}_2\right]$$

$$\Rightarrow \quad \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2 = \mathscr{D}\left[\vec{\mathbf{e}}_1 + \vec{\mathbf{e}}_2, \vec{\mathbf{h}}_1 + \vec{\mathbf{h}}_2\right]$$

Constitutive relationships

In the following we will consider linear media

Constitutive relationships

Linear media

Example 1

$$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\boldsymbol{\varepsilon}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)+\boldsymbol{\chi}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

where

$\vec{\mathbf{d}}(\vec{\mathbf{r}},t);\ \vec{\mathbf{e}}(\vec{\mathbf{r}},t);\ \vec{\mathbf{h}}(\vec{\mathbf{r}},t):$ 3x1 column vectors

$\boldsymbol{\varepsilon}(\vec{\mathbf{r}},t);\ \boldsymbol{\chi}(\vec{\mathbf{r}},t):$ 3x3 matrices

- **Local (nondispersive) media**
- **Bianisotropic media**

Constitutive relationships

Linear media

Example 2

$$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\boldsymbol{\varepsilon}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

where

$\vec{\mathbf{d}}(\vec{\mathbf{r}},t);\ \vec{\mathbf{e}}(\vec{\mathbf{r}},t);\ \vec{\mathbf{h}}(\vec{\mathbf{r}},t):$ 3x1 column vectors

$\boldsymbol{\varepsilon}(\vec{\mathbf{r}},t):$ 3x3 matrices

- **Local (nondispersive) media**
- **Anisotropic media**

Constitutive relationships

Linear media

Example 2

$$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)=\boldsymbol{\varepsilon}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$
$$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)=\boldsymbol{\mu}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$
$$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)=\boldsymbol{\sigma}(\vec{\mathbf{r}},t)\cdot\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

- **Local (nondispersive) media**
- **Anisotropic media**

where

$\vec{\mathbf{d}}(\vec{\mathbf{r}},t);\vec{\mathbf{b}}(\vec{\mathbf{r}},t);\vec{\mathbf{j}}(\vec{\mathbf{r}},t);\vec{\mathbf{e}}(\vec{\mathbf{r}},t);\vec{\mathbf{h}}(\vec{\mathbf{r}},t):$ 3x1 column vectors

$\boldsymbol{\varepsilon}(\vec{\mathbf{r}},t);\boldsymbol{\mu}(\vec{\mathbf{r}},t);\boldsymbol{\sigma}(\vec{\mathbf{r}},t):$ 3x3 matrices

$\boldsymbol{\varepsilon}(\vec{\mathbf{r}},t)$: permittivity [Farad/m]

$\boldsymbol{\mu}(\vec{\mathbf{r}},t)$: permeability [Henry/m]

$\boldsymbol{\sigma}(\vec{\mathbf{r}},t)$: conductivity [Siemens/m]

Constitutive relationships

Linear & Anisotropic media

$$\vec{\mathbf{d}} = \mathscr{D}\left[\vec{\mathbf{e}}\right]$$

$$\vec{\mathbf{b}} = \mathscr{B}\left[\vec{\mathbf{h}}\right]$$

$$\vec{\mathbf{j}} = \mathscr{J}\left[\vec{\mathbf{e}}\right]$$