

# Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

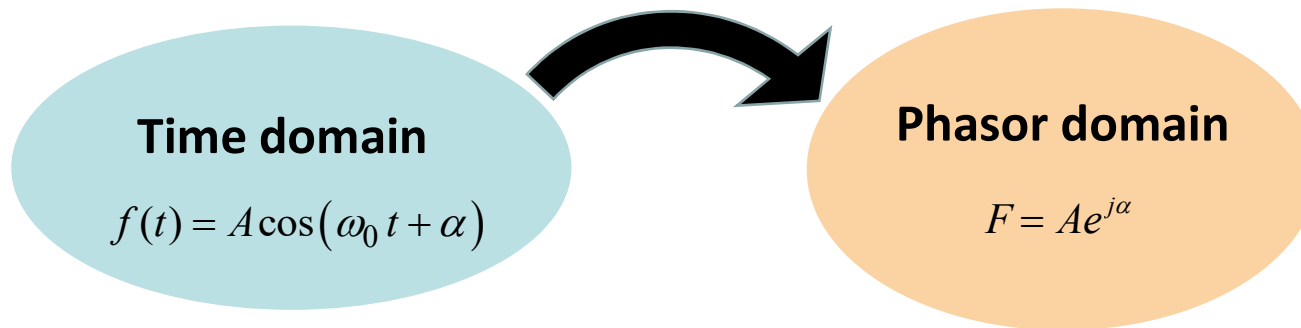
Corso di Campi Elettromagnetici  
a.a. 2021-2022

# Maxwell equations

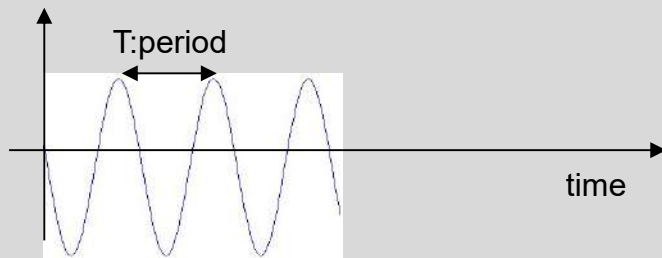
## Time domain & Phasors



# Phasors



## Signals usually adopted in ICT applications

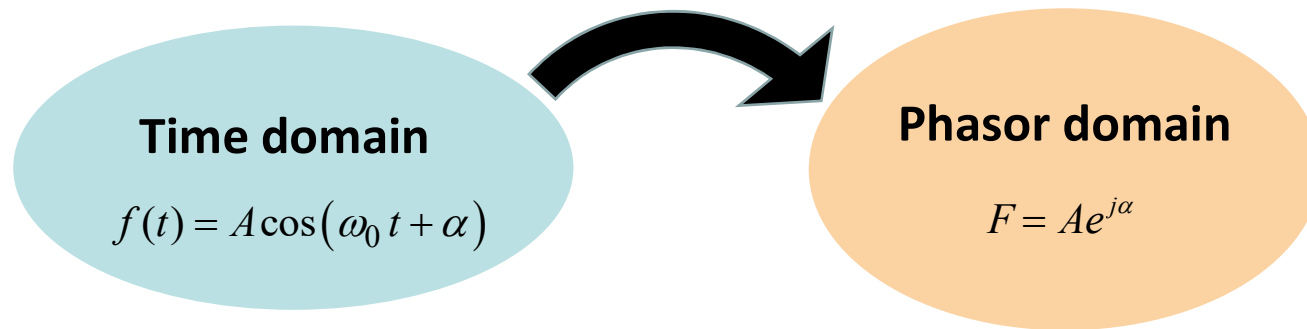


$$f(t) = A \cos(2\pi f_0 t + \alpha)$$

$$f_0 : \text{frequency} = \frac{1}{T}$$

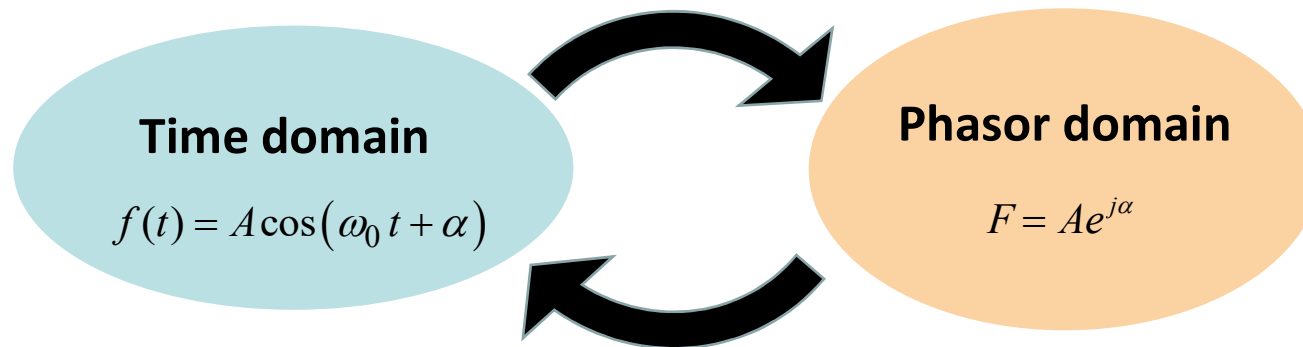
$$\omega_0 : \text{angular frequency} = 2\pi f_0$$

# Phasors



- 1) How to jump back from the Phasor domain to the Time domain
- 2) Time domain derivative and Phasors

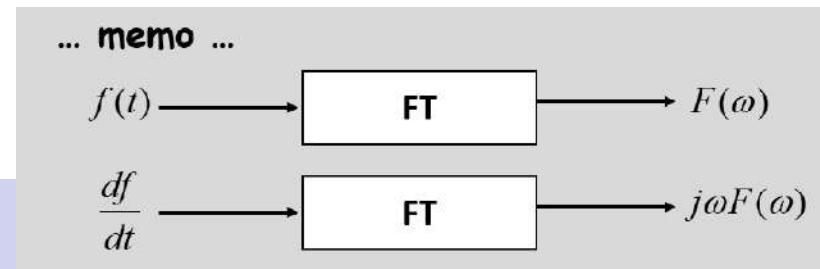
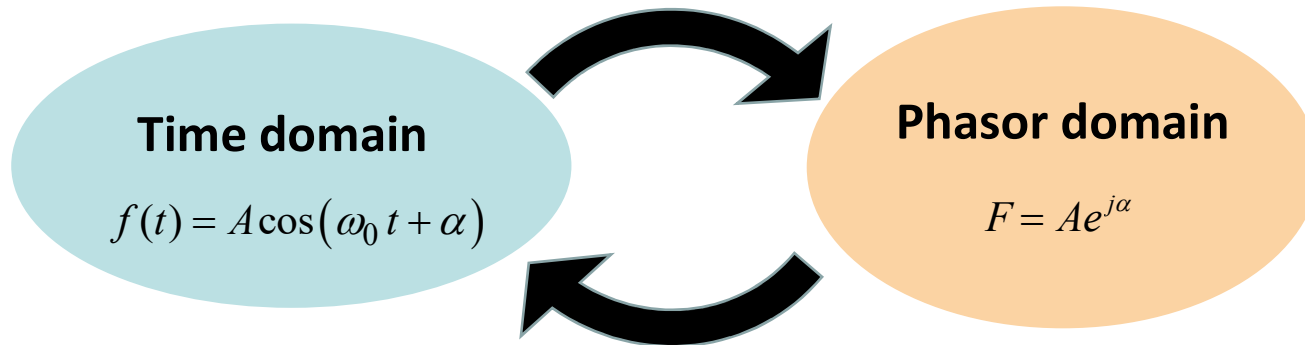
# Phasors



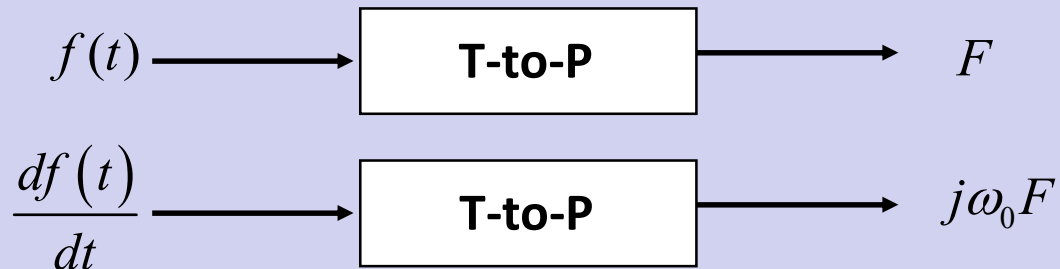
$$f(t) = A \cos(\omega_0 t + \alpha) \longrightarrow \text{T-to-P} \longrightarrow F = A e^{j\alpha}$$

$$F = A e^{j\alpha} \longrightarrow \text{P-to-T} \longrightarrow f(t) = \text{Re}\{F e^{j\omega_0 t}\} = A \cos(\omega_0 t + \alpha)$$

# Phasors



## 2) Time domain derivative and Phasors



$\omega_0$  now is fixed!

# Phasors

- Phasors and functions of  $n$  variables
- Phasors and vector functions
- Phasors and vector functions of  $n$  variables

- 1) How to jump back from the Phasor domain to the Time domain
- 2) Time domain derivative and Phasors

# Phasors

- Phasors and functions of  $n$  variables
- Phasors and vector functions
- **Phasors and vector functions of  $n$  variables**

- 1) How to jump back from the Phasor domain to the Time domain**
- 2) Time domain derivative and Phasors**



# Phasors and vector functions of $n$ variables

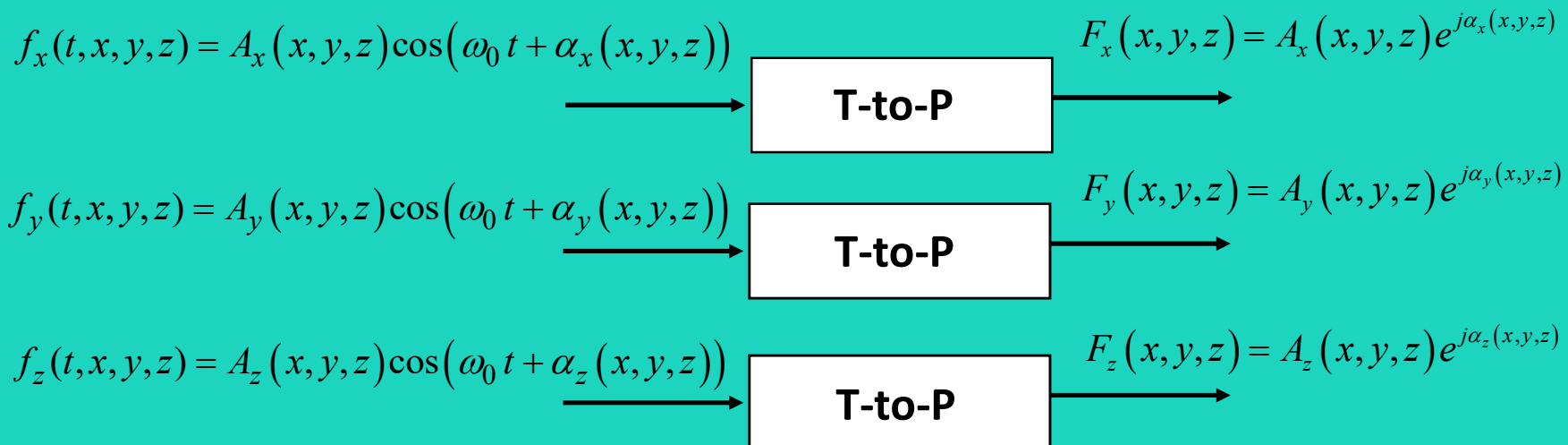
**Time domain**

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



**Phasor domain**

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$



# Phasors and vector functions of $n$ variables

**Time domain**

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$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

$$\vec{\mathbf{f}}(t, x, y, z) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{F}}(x, y, z)$$

$$\begin{aligned}\vec{\mathbf{f}}(t, x, y, z) &= f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z = \\ &= A_x(x, y, z)\cos(\omega_0 t + \alpha_x(x, y, z))\hat{i}_x + \\ &A_y(x, y, z)\cos(\omega_0 t + \alpha_y(x, y, z))\hat{i}_y + \\ &A_z(x, y, z)\cos(\omega_0 t + \alpha_z(x, y, z))\hat{i}_z\end{aligned}$$

$$\begin{aligned}\vec{\mathbf{F}}(x, y, z) &= F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z = \\ &= A_x(x, y, z)e^{j\alpha_x(x, y, z)}\hat{i}_x + A_y(x, y, z)e^{j\alpha_y(x, y, z)}\hat{i}_y + A_z(x, y, z)e^{j\alpha_z(x, y, z)}\hat{i}_z\end{aligned}$$

# Phasors and vector functions of $n$ variables

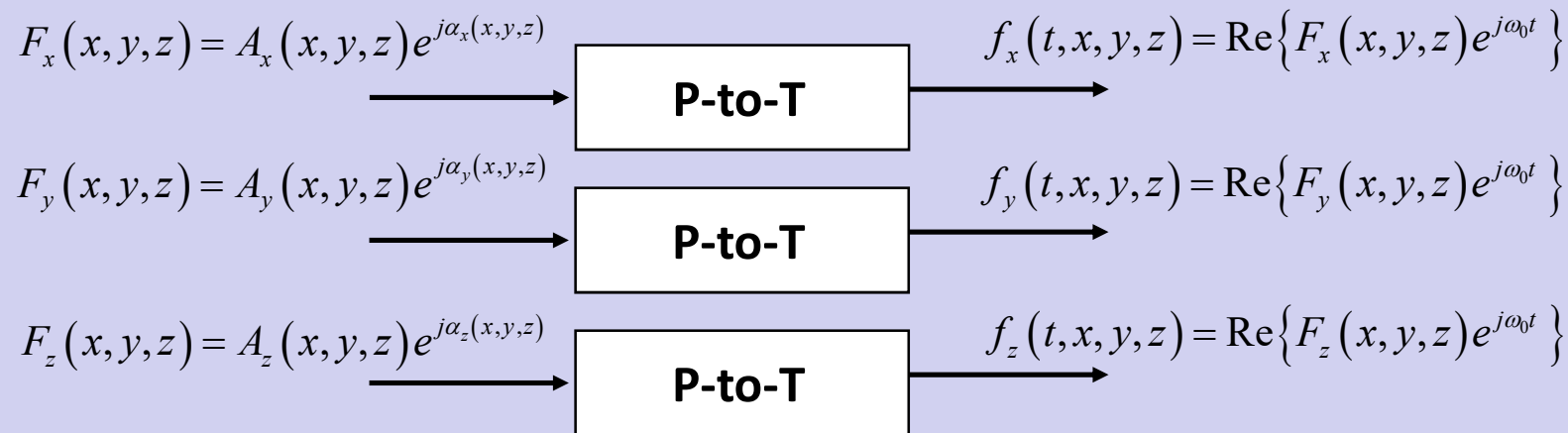
## Time domain

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$

## Phasor domain

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

### 1) How to jump back from the Phasor domain to the Time domain



# Phasors and vector functions of $n$ variables

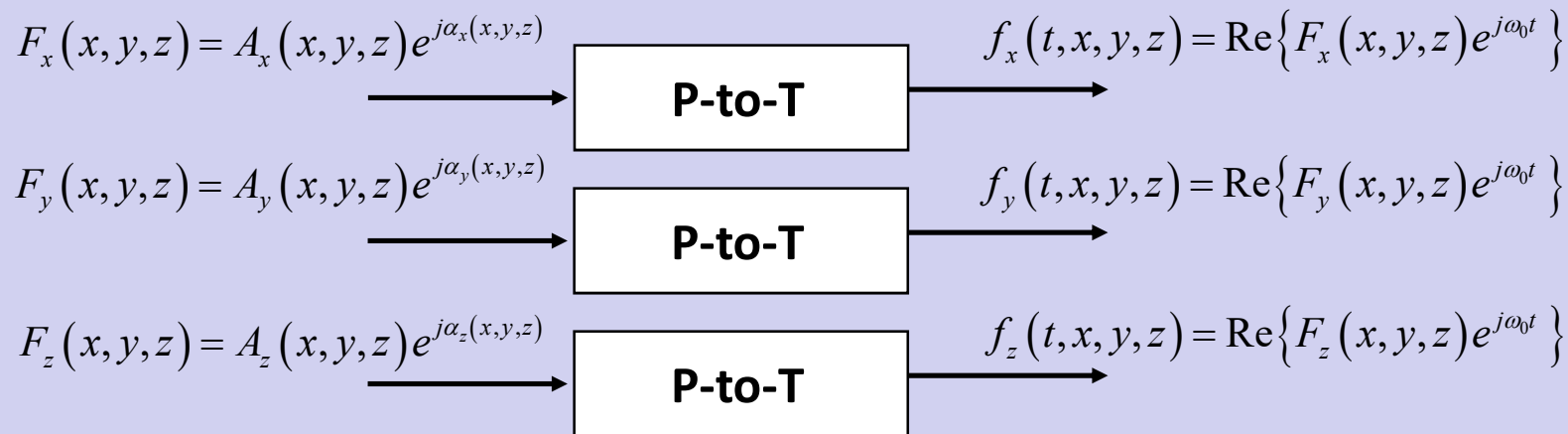
**Time domain**

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$

**Phasor domain**

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## 1) How to jump back from the Phasor domain to the Time domain



# Phasors and vector functions of $n$ variables

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**Phasor domain**

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

## 2) Time domain derivative and Phasors

$$\vec{\mathbf{f}}(t, x, y, z) \xrightarrow{\text{T-to-P}} \vec{\mathbf{F}}(x, y, z)$$

$$\frac{\partial \vec{\mathbf{f}}}{\partial t} \xrightarrow{\text{T-to-P}} j\omega_0 \vec{\mathbf{F}}(x, y, z) = j\omega_0 F_x(x, y, z)\hat{i}_x + j\omega_0 F_y(x, y, z)\hat{i}_y + j\omega_0 F_z(x, y, z)\hat{i}_z$$

$$\frac{\partial \vec{\mathbf{f}}(t, \vec{\mathbf{r}})}{\partial t} = \frac{\partial f_x}{\partial t}\hat{i}_x + \frac{\partial f_y}{\partial t}\hat{i}_y + \frac{\partial f_z}{\partial t}\hat{i}_z$$

$$\frac{\partial f_x(t)}{\partial t} \xrightarrow{\text{T-to-P}} j\omega_0 F_x(x, y, z)$$

$$\frac{\partial f_y(t)}{\partial t} \xrightarrow{\text{T-to-P}} j\omega_0 F_y(x, y, z)$$

$$\frac{\partial f_z(t)}{\partial t} \xrightarrow{\text{T-to-P}} j\omega_0 F_z(x, y, z)$$

# Phasors and vector functions of $n$ variables

**Time domain**

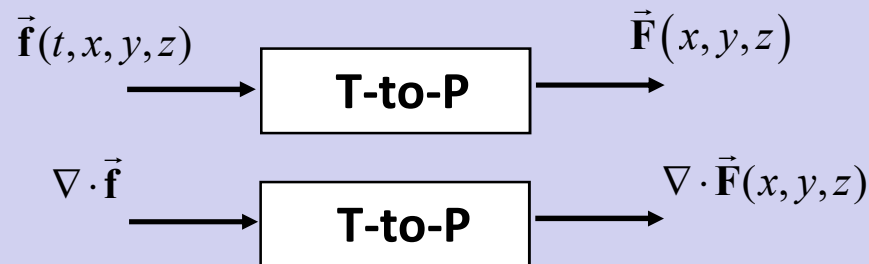
$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



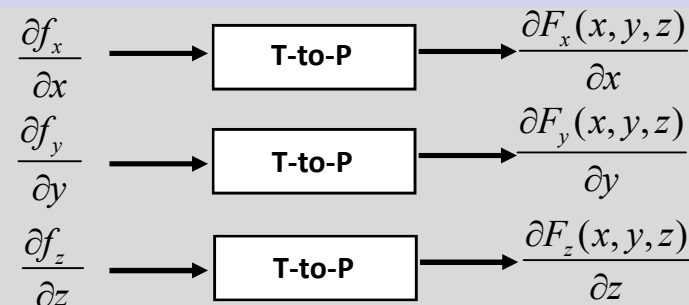
**Phasor domain**

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

## 2) Time domain derivative and Phasors



$$\nabla \cdot \vec{\mathbf{f}}(t, \vec{\mathbf{r}}) = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}$$



# Phasors and vector functions of $n$ variables

**Time domain**

$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



**Phasor domain**

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## 2) Time domain derivative and Phasors

$$\vec{\mathbf{f}}(t, x, y, z) \longrightarrow \vec{\mathbf{F}}(x, y, z)$$

**T-to-P**

$$\nabla \times \vec{\mathbf{f}} \longrightarrow \nabla \times \vec{\mathbf{F}}(x, y, z)$$

**T-to-P**

$$\nabla \times \vec{\mathbf{f}}(t, \vec{\mathbf{r}}) = \left( \frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z} \right) \hat{i}_x + \left( \frac{\partial f_x}{\partial z} - \frac{\partial f_z}{\partial x} \right) \hat{i}_y + \left( \frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y} \right) \hat{i}_z$$

# Phasors and vector functions of $n$ variables

**Time domain**

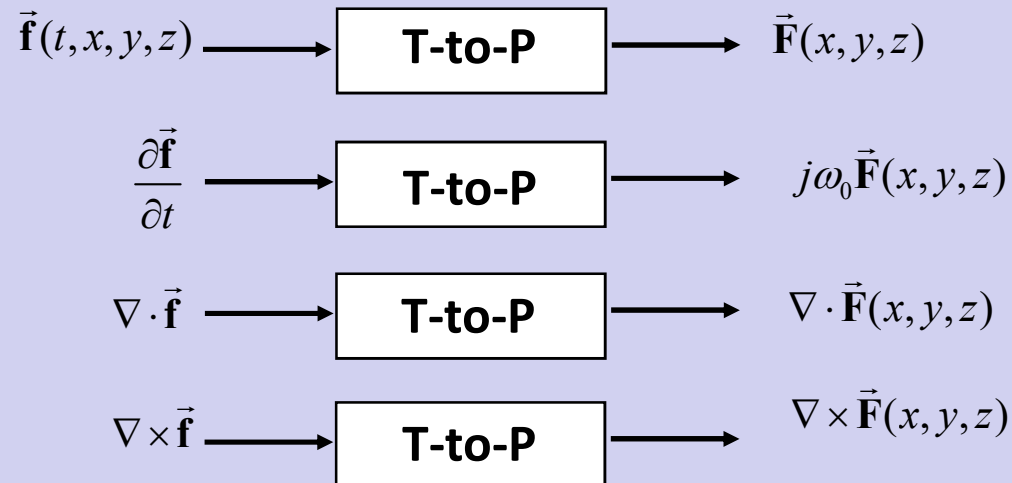
$$\vec{\mathbf{f}}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$



**Phasor domain**

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

## 2) Time domain derivative and Phasors





# Phasors and vector functions of $n$ variables

**Time domain**

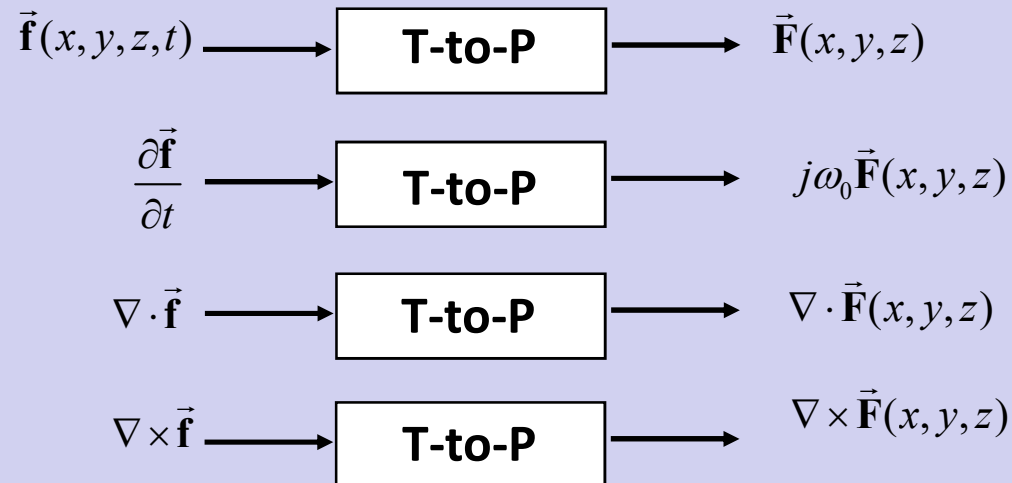
$$\vec{\mathbf{f}}(x, y, z, t) = f_x(x, y, z, t)\hat{i}_x + f_y(x, y, z, t)\hat{i}_y + f_z(x, y, z, t)\hat{i}_z$$



**Phasor domain**

$$\vec{\mathbf{F}}(x, y, z) = F_x(x, y, z)\hat{i}_x + F_y(x, y, z)\hat{i}_y + F_z(x, y, z)\hat{i}_z$$

## 2) Time domain derivative and Phasors





# Maxwell equations

## Time domain & Phasor domain

### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$



# Maxwell equations

## Time domain & Phasor domain

### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

### Phasor domain





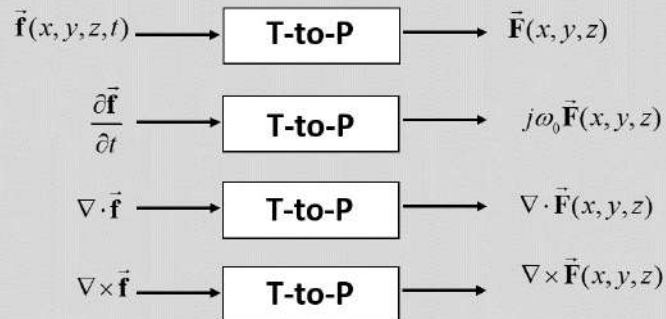
# Maxwell equations

## Time domain & Phasor domain

### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

### Phasor domain





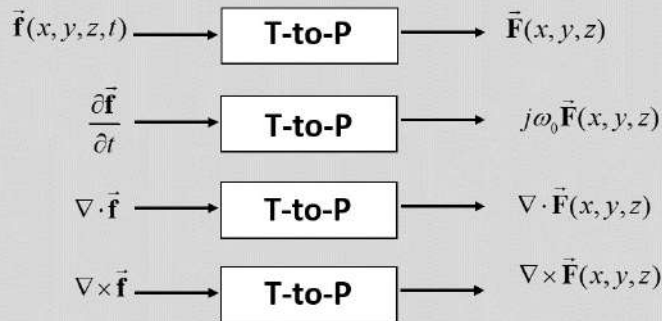
# Maxwell equations

## Time domain & Phasor domain

### Time domain

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{cases}$$

### Phasor domain





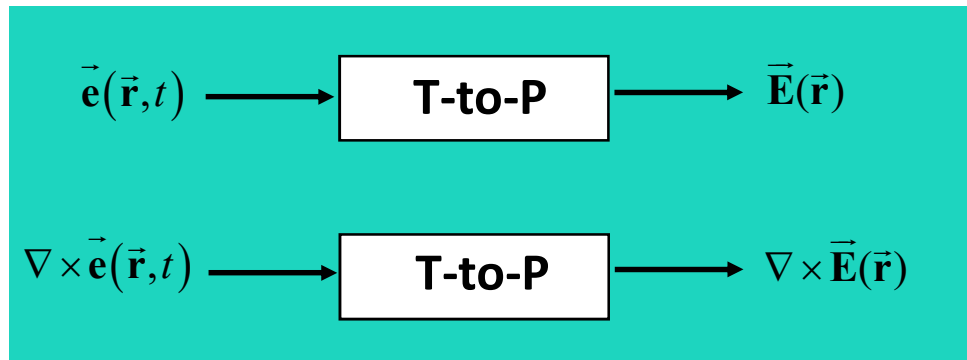
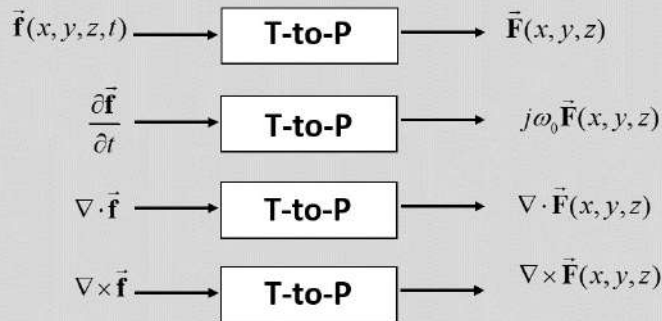
# Maxwell equations

## Time domain & Phasor domain

### Time domain

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### Phasor domain





# Maxwell equations

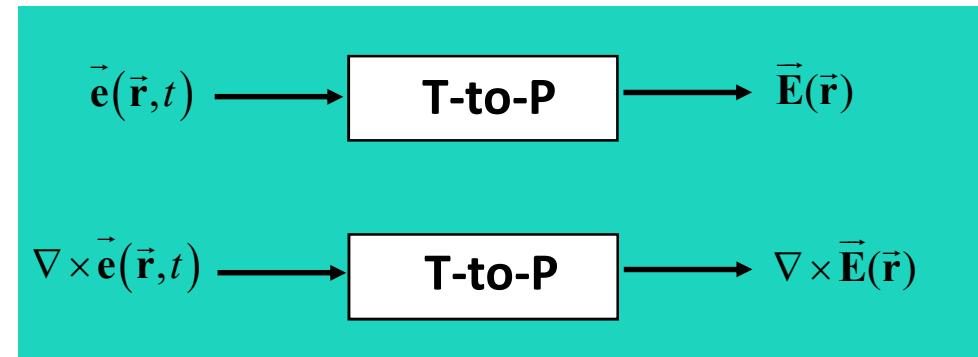
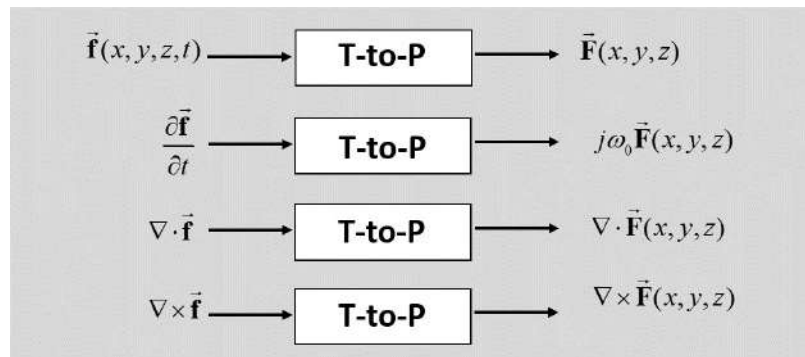
## Time domain & Phasor domain

### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

### Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}) \end{array} \right.$$





# Maxwell equations

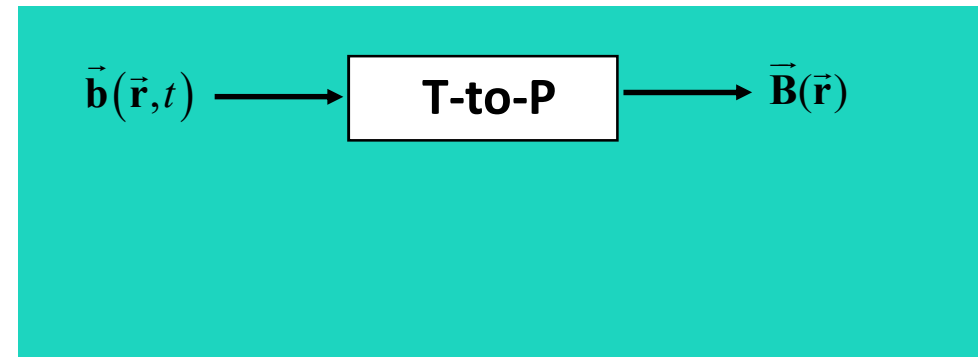
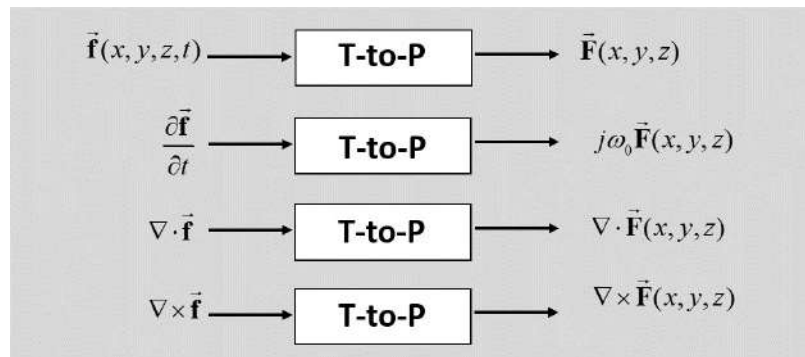
## Time domain & Phasor domain

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### Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) \\ \vdots \end{array} \right.$$







# Maxwell equations

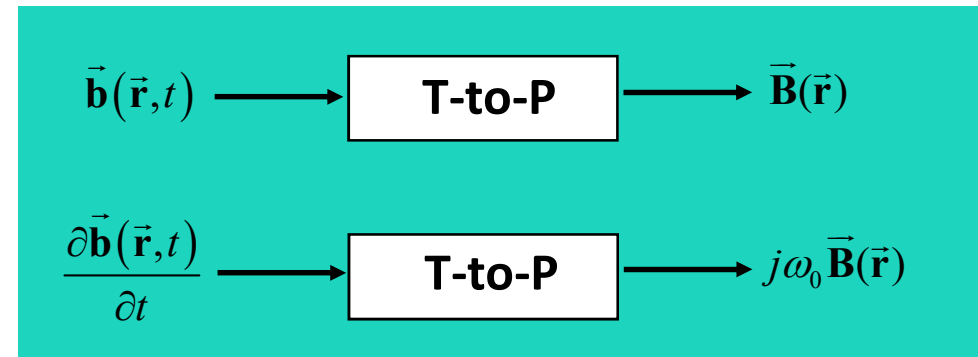
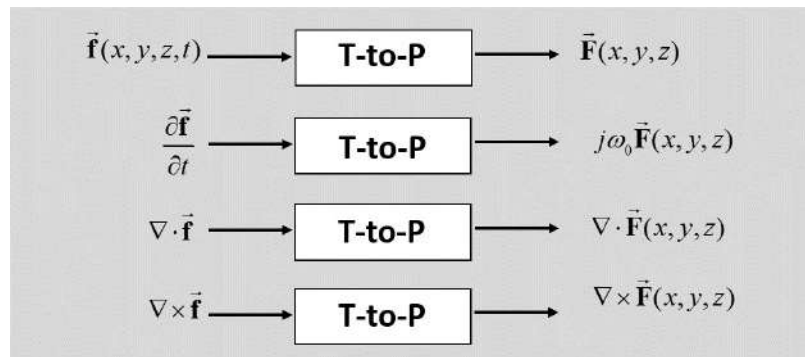
## Time domain & Phasor domain

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# Maxwell equations

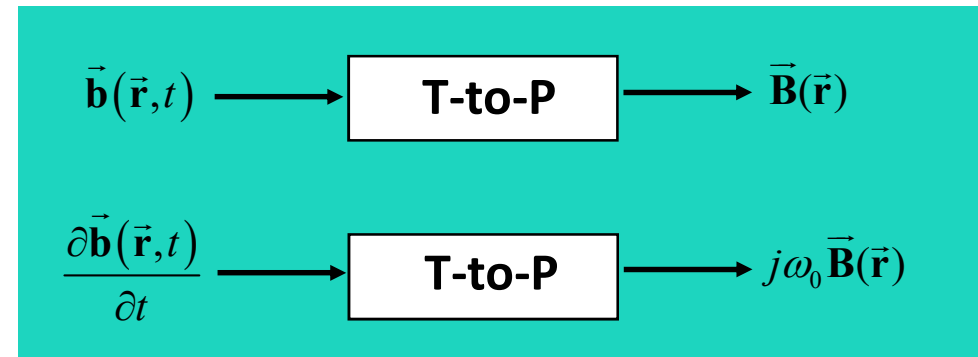
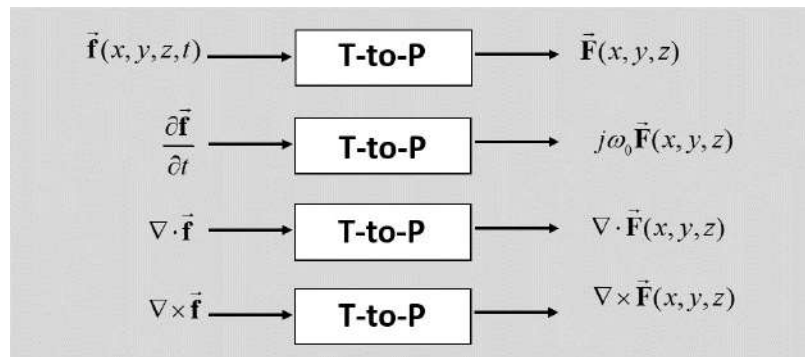
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### Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \end{array} \right.$$





# Maxwell equations

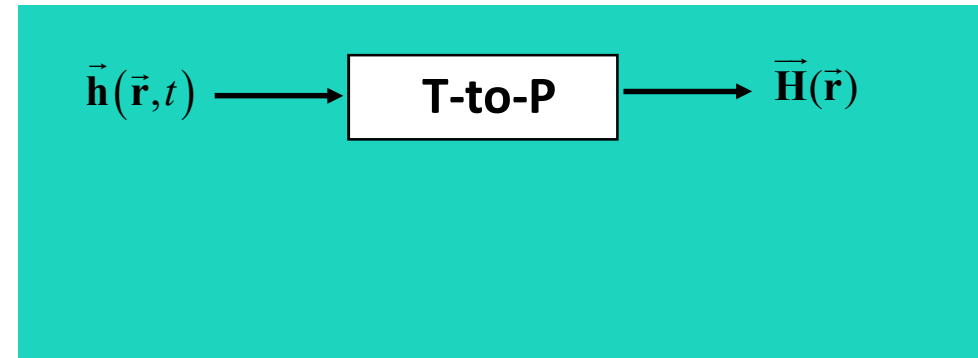
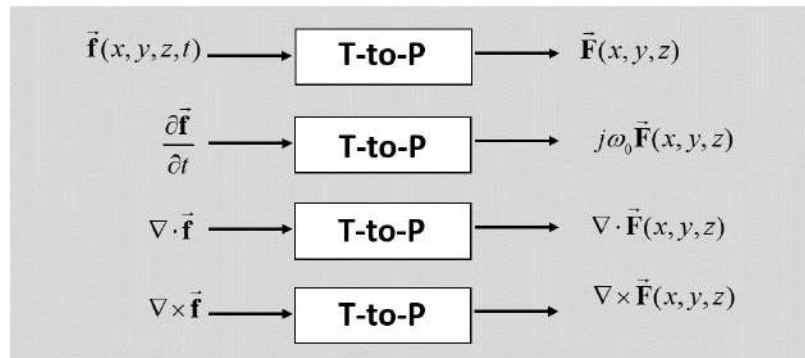
## Time domain & Phasor domain

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# Maxwell equations

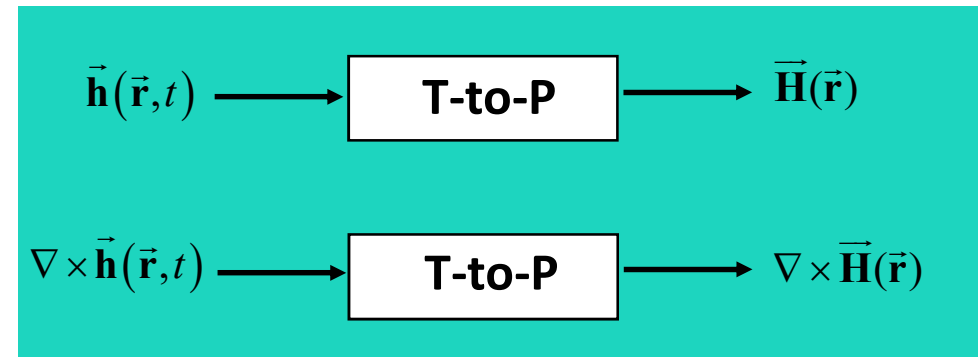
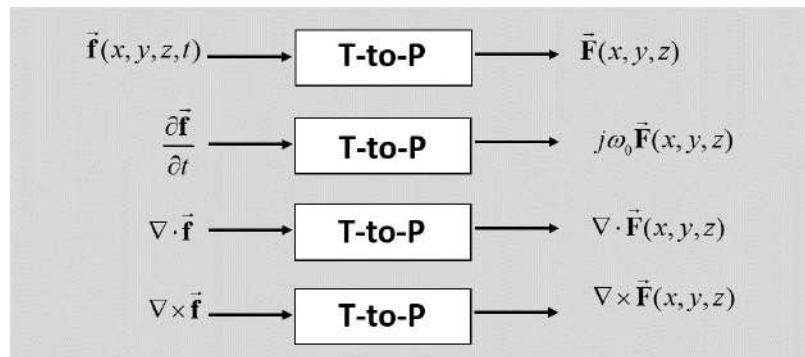
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# Maxwell equations

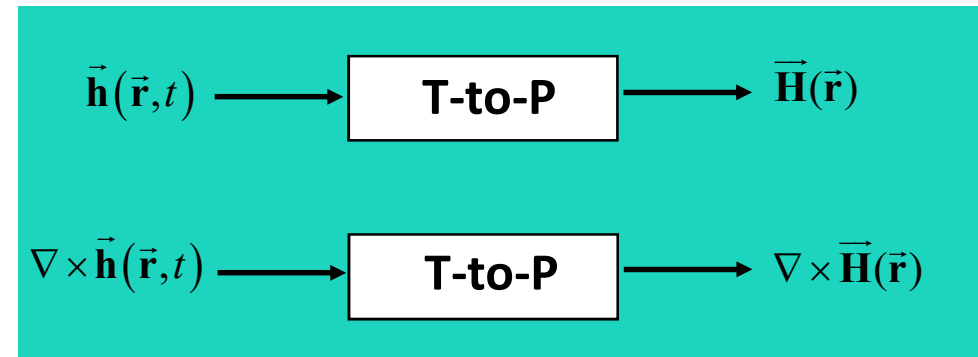
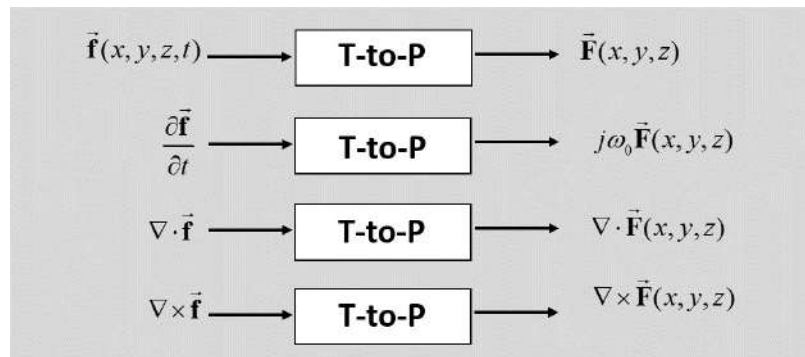
## Time domain & Phasor domain

### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

### Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) \end{array} \right.$$





# Maxwell equations

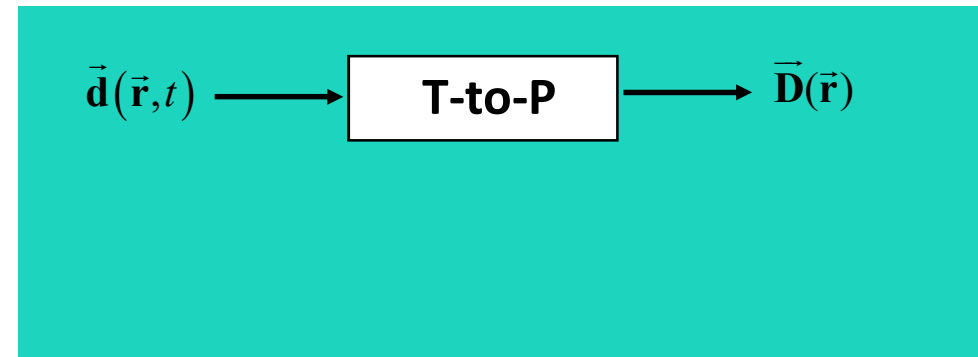
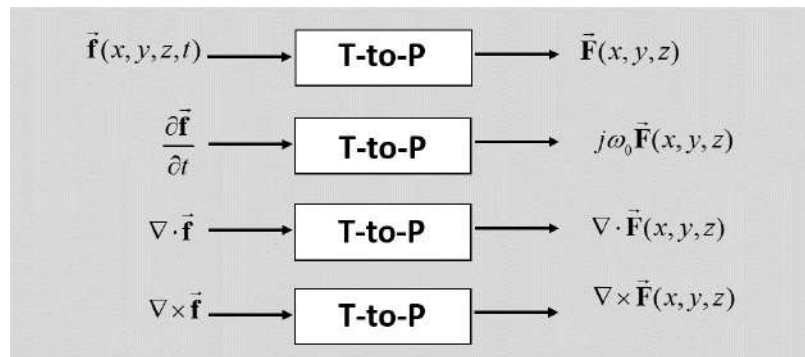
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# Maxwell equations

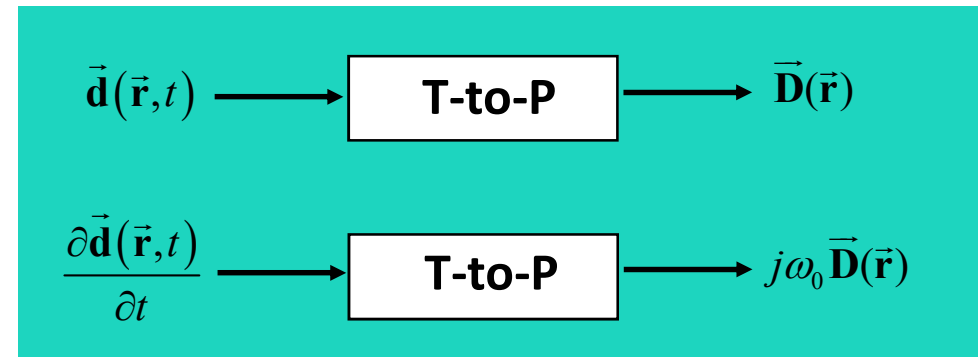
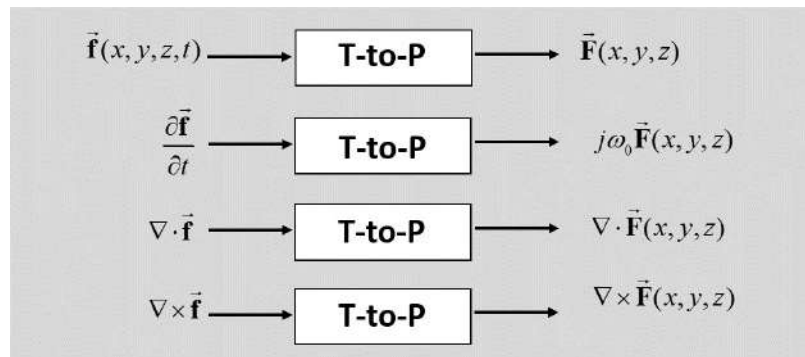
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# Maxwell equations

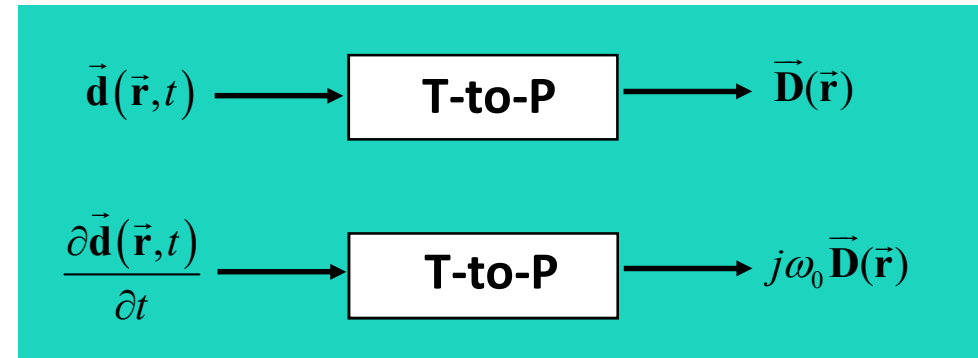
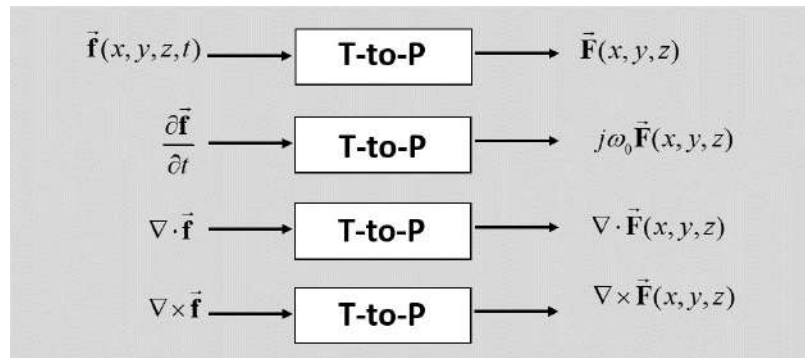
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# Maxwell equations

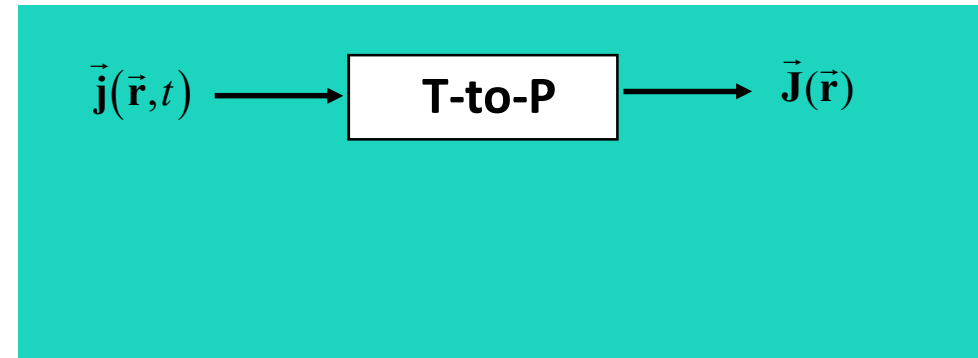
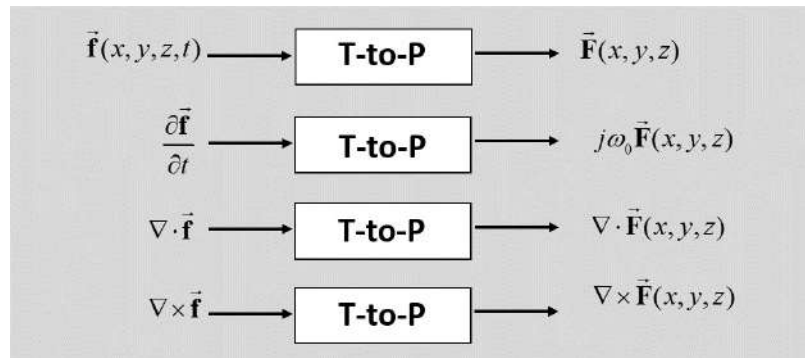
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# Maxwell equations

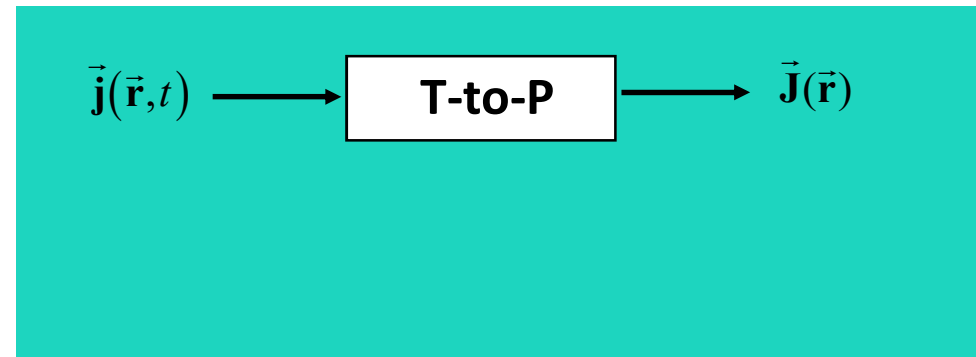
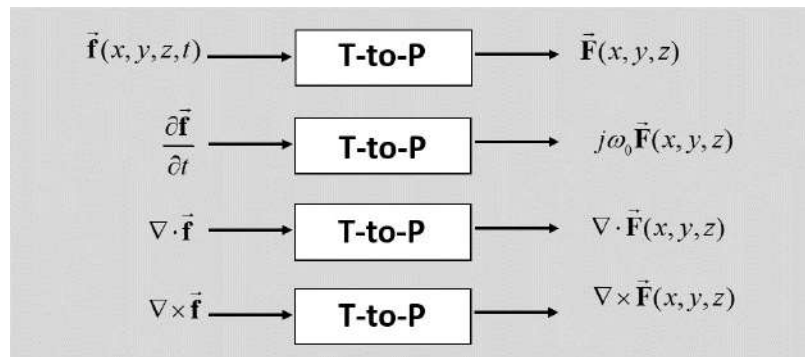
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$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \end{array} \right.$$





# Maxwell equations

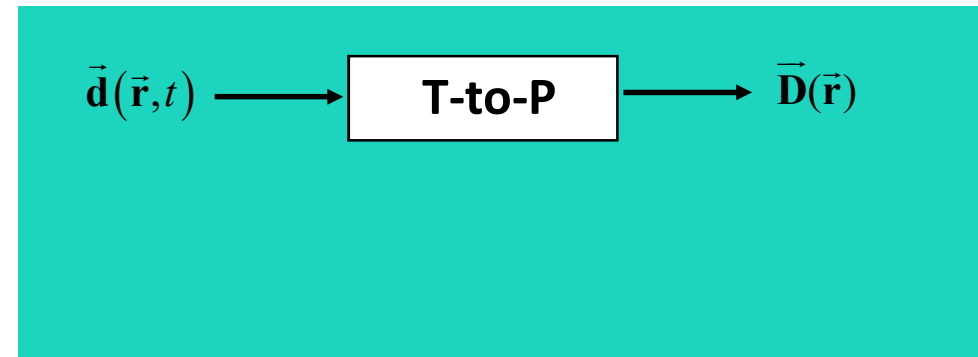
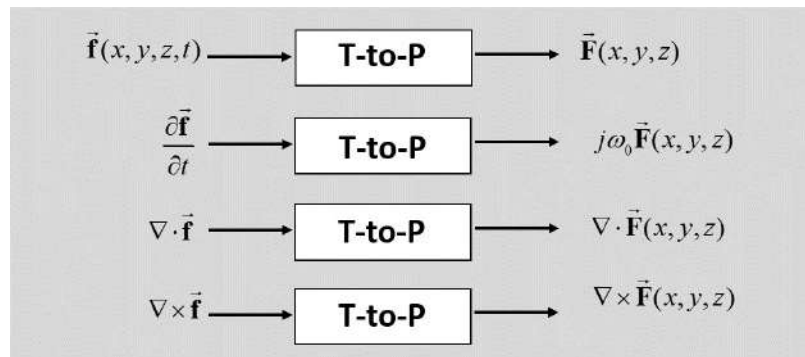
## Time domain & Phasor domain

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# Maxwell equations

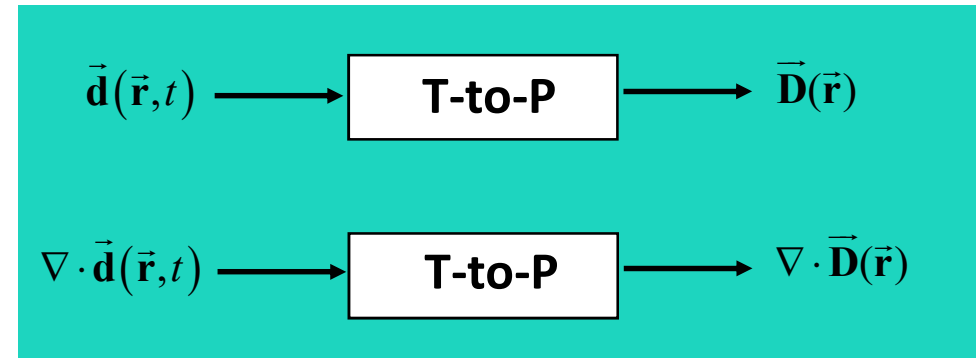
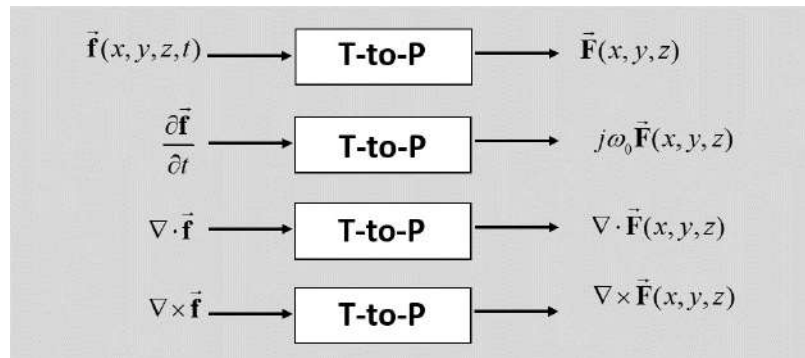
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# Maxwell equations

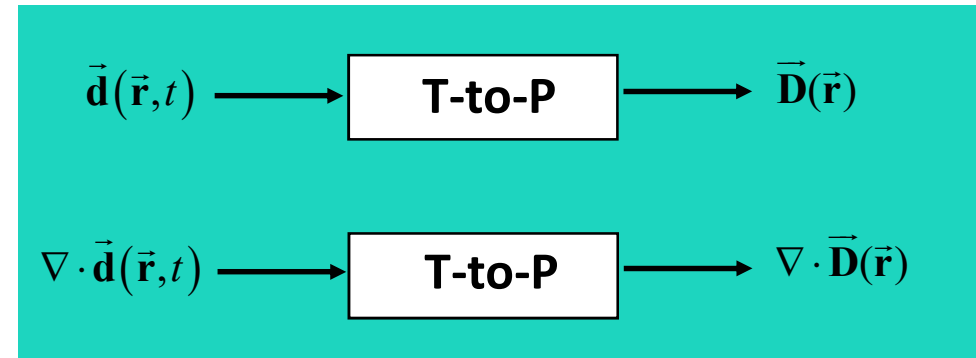
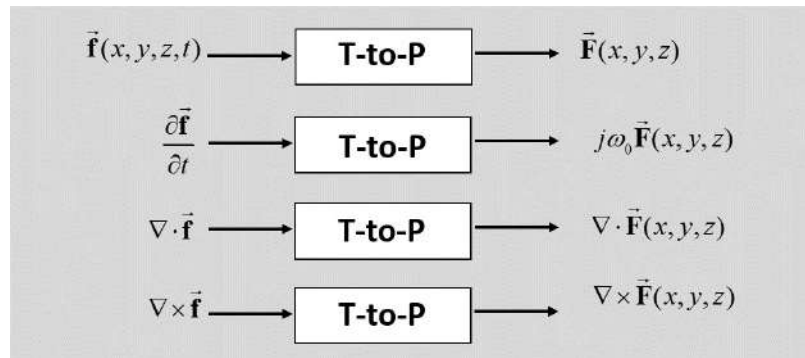
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# Maxwell equations

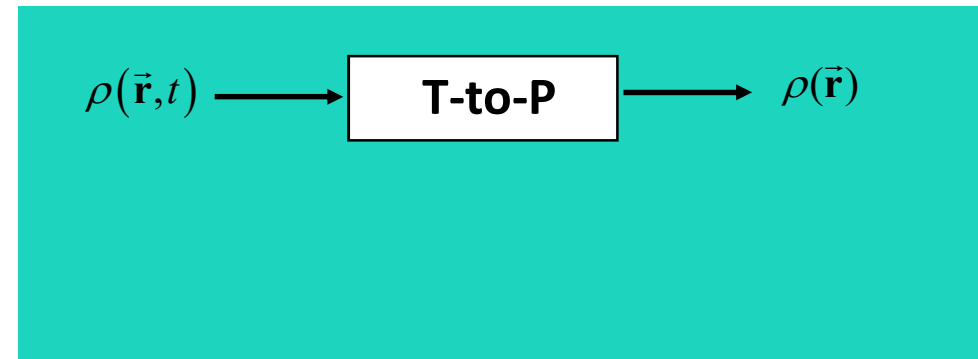
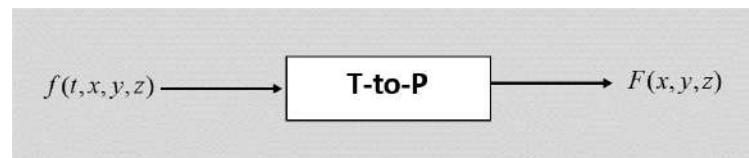
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# Maxwell equations

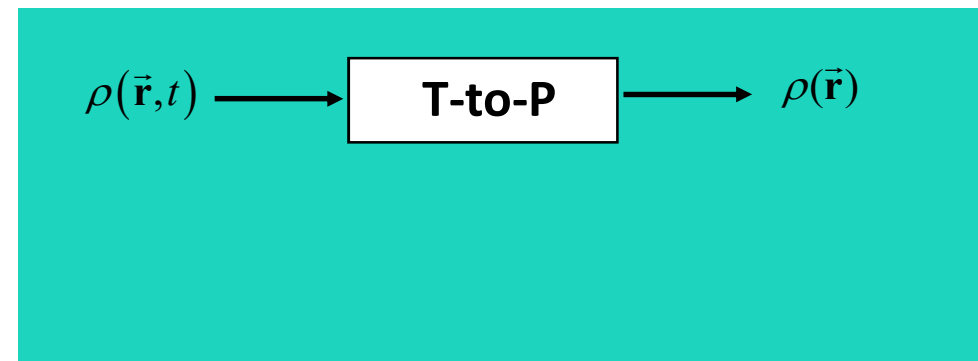
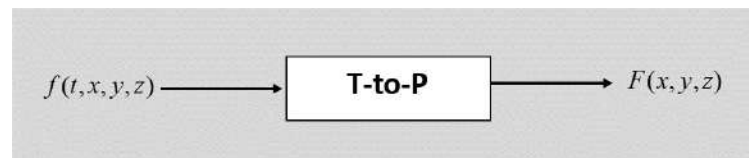
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# Maxwell equations

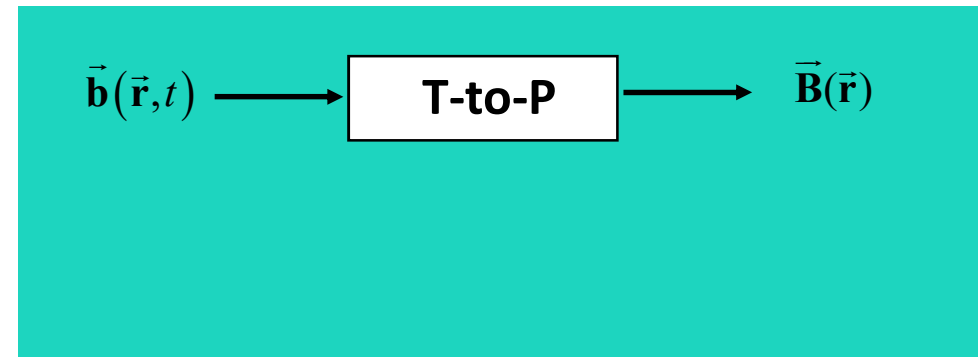
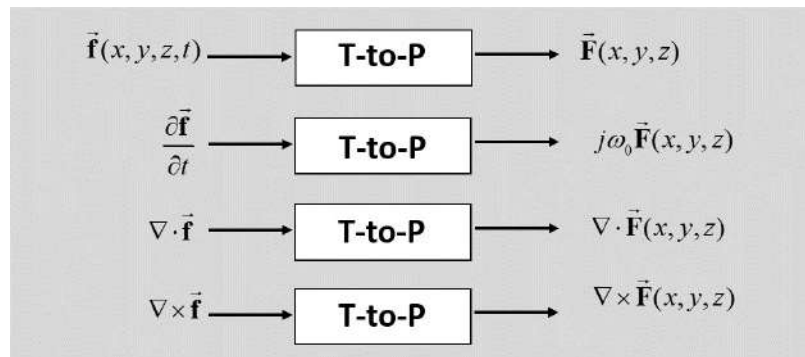
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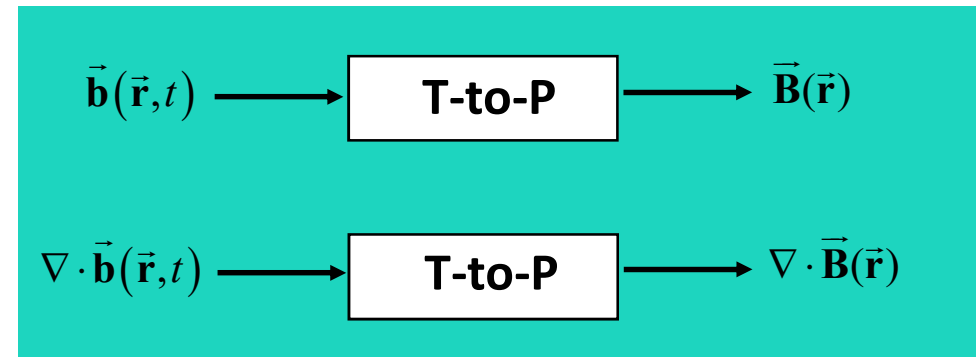
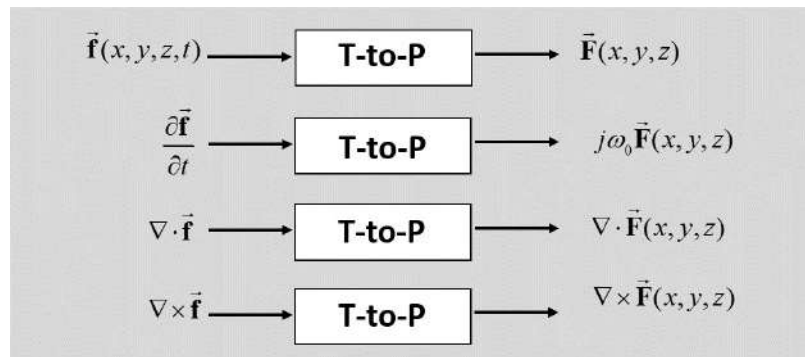
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# Maxwell equations

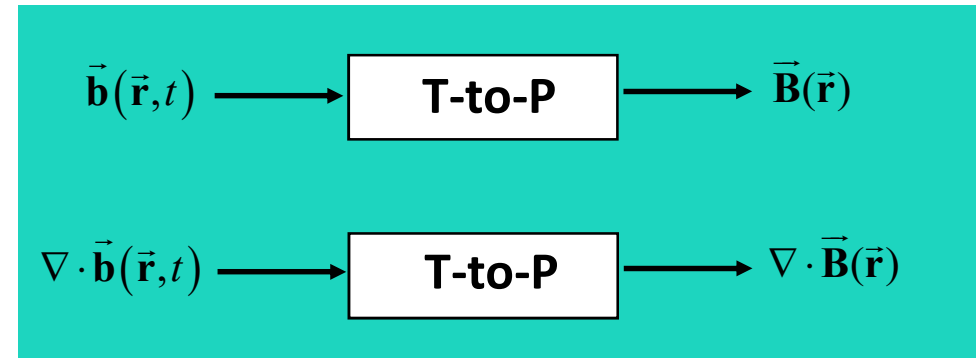
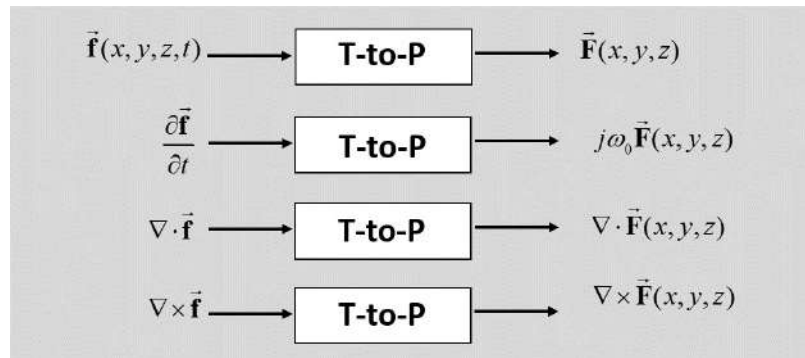
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$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m <sup>2</sup>
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m <sup>2</sup>
$\vec{j}(\vec{r}, t)$	Ampere/m <sup>2</sup>
$\rho(\vec{r}, t)$	Coulomb/m <sup>3</sup>



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$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

### Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) + \vec{J}(\vec{r}) \\ \nabla \cdot \vec{D}(\vec{r}) = \rho(\vec{r}) \\ \nabla \cdot \vec{B}(\vec{r}) = 0 \end{array} \right.$$

$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m <sup>2</sup>
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m <sup>2</sup>
$\vec{j}(\vec{r}, t)$	Ampere/m <sup>2</sup>
$\rho(\vec{r}, t)$	Coulomb/m <sup>3</sup>

$\vec{E}(\vec{r})$
$\vec{D}(\vec{r})$
$\vec{H}(\vec{r})$
$\vec{B}(\vec{r})$
$\vec{J}(\vec{r})$
$\rho(\vec{r})$



# Maxwell equations

## Time domain & Phasor domain

### Time domain

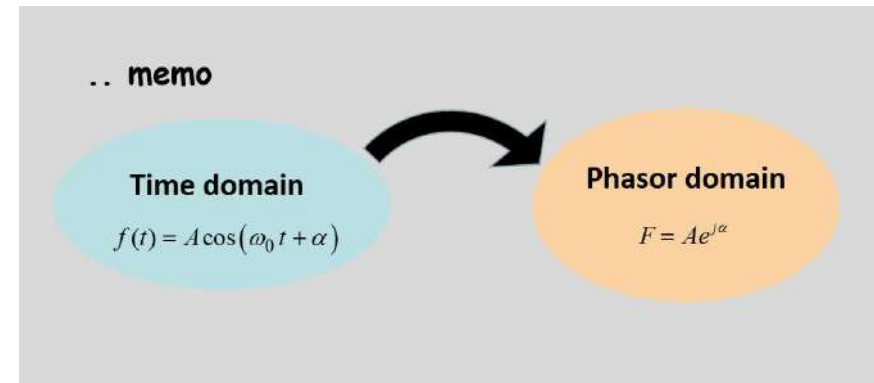
$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m <sup>2</sup>
$\vec{h}(\vec{r}, t)$	Ampere/m
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$\vec{j}(\vec{r}, t)$	Ampere/m <sup>2</sup>
$\rho(\vec{r}, t)$	Coulomb/m <sup>3</sup>

### Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) + \vec{J}(\vec{r}) \\ \nabla \cdot \vec{D}(\vec{r}) = \rho(\vec{r}) \\ \nabla \cdot \vec{B}(\vec{r}) = 0 \end{array} \right.$$

$\vec{E}(\vec{r})$





# Maxwell equations

## Time domain & Phasor domain

### Time domain

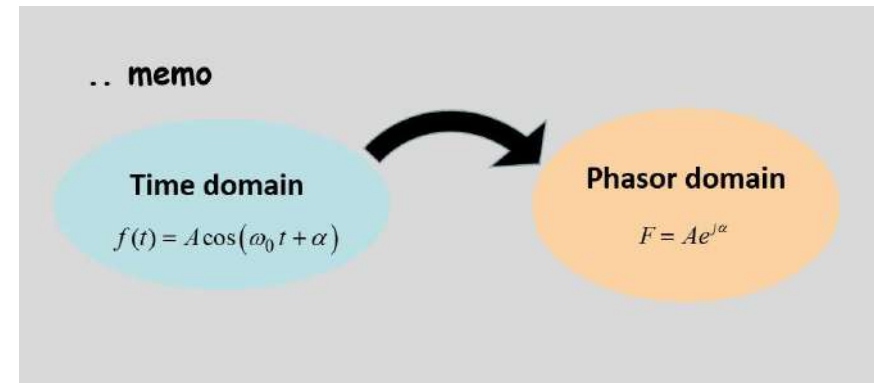
$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m <sup>2</sup>
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m <sup>2</sup>
$\vec{j}(\vec{r}, t)$	Ampere/m <sup>2</sup>
$\rho(\vec{r}, t)$	Coulomb/m <sup>3</sup>

### Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) + \vec{J}(\vec{r}) \\ \nabla \cdot \vec{D}(\vec{r}) = \rho(\vec{r}) \\ \nabla \cdot \vec{B}(\vec{r}) = 0 \end{array} \right.$$

$\vec{E}(\vec{r})$  Volt/m







# Maxwell equations

## Time domain & Phasor domain

### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

### Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) + \vec{J}(\vec{r}) \\ \nabla \cdot \vec{D}(\vec{r}) = \rho(\vec{r}) \\ \nabla \cdot \vec{B}(\vec{r}) = 0 \end{array} \right.$$

$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m <sup>2</sup>
$\vec{h}(\vec{r}, t)$	Ampere/m
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$\vec{j}(\vec{r}, t)$	Ampere/m <sup>2</sup>
$\rho(\vec{r}, t)$	Coulomb/m <sup>3</sup>

$\vec{E}(\vec{r})$	Volt/m
$\vec{D}(\vec{r})$	Coulomb/m <sup>2</sup>
$\vec{H}(\vec{r})$	Ampere/m
$\vec{B}(\vec{r})$	Weber/m <sup>2</sup>
$\vec{J}(\vec{r})$	Ampere/m <sup>2</sup>
$\rho(\vec{r})$	Coulomb/m <sup>3</sup>



# Maxwell equations

## Frequency domain & Phasor domain

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}, \omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) = \rho(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) = 0 \end{array} \right.$$

### Phasor domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}) = -j\omega_0 \vec{\mathbf{B}}(\vec{\mathbf{r}}) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}) = j\omega_0 \vec{\mathbf{D}}(\vec{\mathbf{r}}) + \vec{\mathbf{J}}(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}) = \rho(\vec{\mathbf{r}}) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}) = 0 \end{array} \right.$$

The Maxwell equations in the Fourier domain and Phasor domain are **formally** equivalent.

However, they exhibit noticeable differences:

- i) The dimensions of the involved quantities (f.i.,  $\vec{\mathbf{E}}$ ) are different in the two domains.
- ii) In the Frequency domain  $\omega$  is an independent variable, whereas in the Phasor domain  $\omega_0$  is fixed.

# Color legend

New formulas, important considerations,  
important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics



# Maxwell equations

## Time domain & Phasor domain

### Time domain

$$\begin{cases} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{cases}$$

$$\frac{\partial \rho(\vec{r}, t)}{\partial t} + \nabla \cdot \vec{j}(\vec{r}, t) = 0$$

### Frequency domain

$$\begin{cases} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{cases}$$

$$j\omega \rho(\vec{r}, \omega) + \nabla \cdot \vec{J}(\vec{r}, \omega) = 0$$

### Phasor domain

$$\begin{cases} \nabla \times \vec{E}(\vec{r}) = -j\omega_0 \vec{B}(\vec{r}) \\ \nabla \times \vec{H}(\vec{r}) = j\omega_0 \vec{D}(\vec{r}) + \vec{J}(\vec{r}) \\ \nabla \cdot \vec{D}(\vec{r}) = \rho(\vec{r}) \\ \nabla \cdot \vec{B}(\vec{r}) = 0 \end{cases}$$

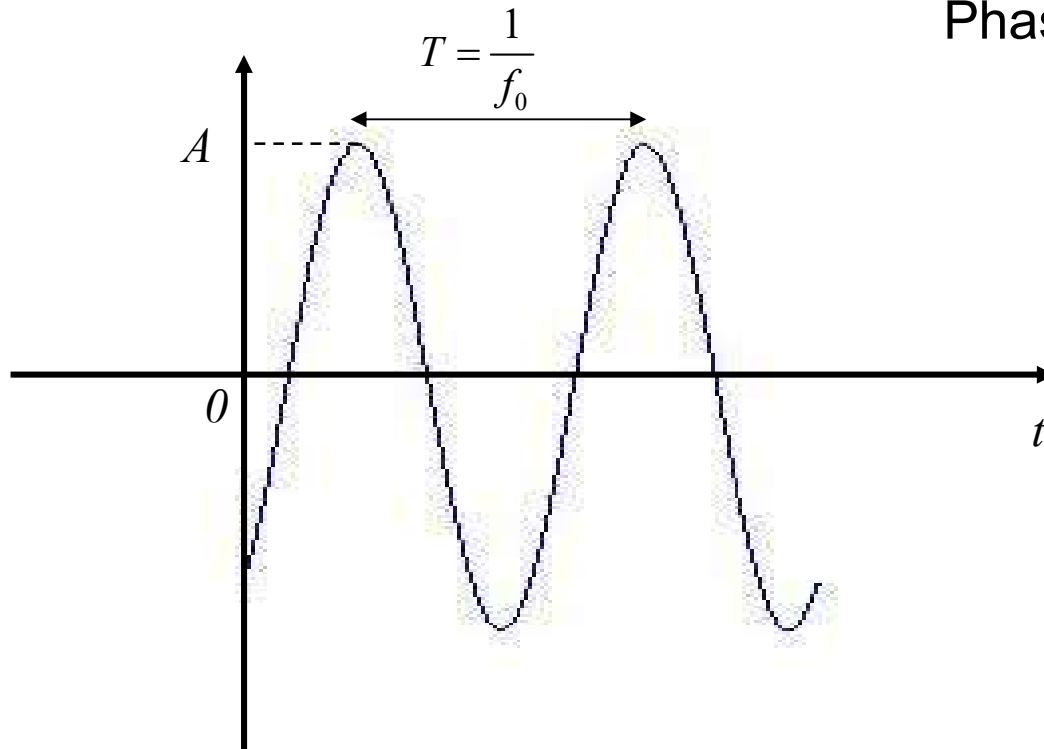
$$j\omega_0 \rho(\vec{r}) + \nabla \cdot \vec{J}(\vec{r}) = 0$$

# Memo: Phasors

# Phasors

$$v(t) = A \cos(2\pi f_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

Phasor (complex number)



# Phasors

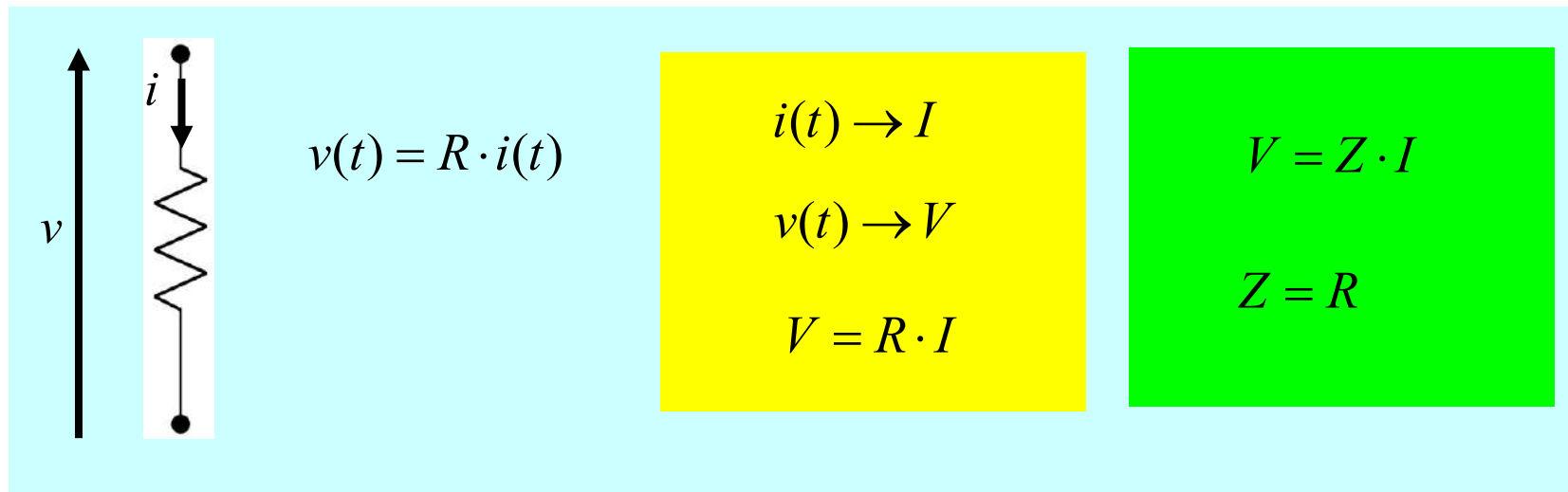
$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$k \cdot v(t) \longrightarrow k \cdot V = k \cdot Ae^{j\alpha}$$

# Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$k \cdot v(t) \longrightarrow k \cdot V = k \cdot Ae^{j\alpha}$$





# Phasors

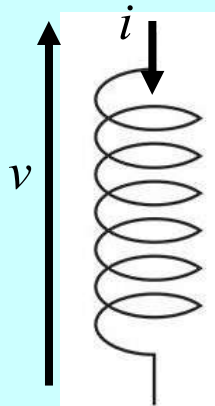
$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{dv(t)}{dt} \longrightarrow j\omega_0 V = j\omega_0 Ae^{j\alpha}$$

# Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{dv(t)}{dt} \longrightarrow j\omega_0 V = j\omega_0 Ae^{j\alpha}$$



$$v(t) = L \frac{di(t)}{dt}$$

$$i(t) \rightarrow I$$

$$v(t) \rightarrow V$$

$$V = j\omega_0 LI$$

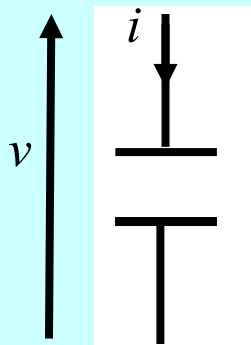
$$V = Z \cdot I$$

$$Z = j\omega_0 L$$

# Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$\frac{dv(t)}{dt} \longrightarrow j\omega_0 V = j\omega_0 Ae^{j\alpha}$$



$$i(t) = C \frac{dv(t)}{dt}$$

$$i(t) \rightarrow I$$

$$v(t) \rightarrow V$$

$$I = j\omega_0 CV$$

$$V = Z \cdot I$$

$$Z = -j \frac{1}{\omega_0 C}$$

# Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$i(t) = B \cos(\omega_0 t + \beta) \longrightarrow I = Be^{j\beta}$$

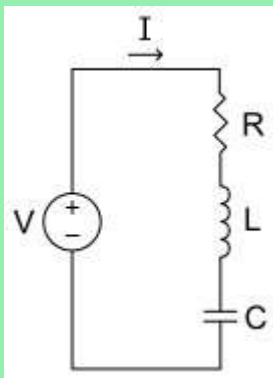
$$\langle v(t) \cdot i(t) \rangle = \frac{1}{T} \int_0^T v(t) \cdot i(t) dt = \frac{1}{2} \operatorname{Re}\{V \cdot I^*\}$$

# Phasors

$$v(t) = A \cos(\omega_0 t + \alpha) \longrightarrow V = Ae^{j\alpha}$$

$$i(t) = B \cos(\omega_0 t + \beta) \longrightarrow I = Be^{j\beta}$$

$$\langle v(t) \cdot i(t) \rangle = \frac{1}{T} \int_0^T v(t) \cdot i(t) dt = \frac{1}{2} \operatorname{Re}\{V \cdot I^*\}$$



$$P = \frac{1}{2} V \cdot I^* = P_1 + jP_2$$

$$P = \frac{1}{2} V \cdot I^* = \frac{1}{2} (Z_R + Z_L + Z_C) I \cdot I^* = \frac{1}{2} \left( R + j\omega_0 L - \frac{j}{\omega_0 C} \right) |I|^2$$

$$P_1 = \frac{1}{2} R |I|^2 ; \quad P_2 = \frac{1}{2} \left( \omega_0 L - \frac{1}{\omega_0 C} \right) |I|^2$$

# Phasors and vector functions of $n$ variables

$$\vec{\mathbf{f}}_1(x, y, z, t) \longrightarrow \vec{\mathbf{F}}_1(x, y, z)$$

$$\vec{\mathbf{f}}_2(x, y, z, t) \longrightarrow \vec{\mathbf{F}}_2(x, y, z)$$

# Phasors and vector functions of $n$ variables

$$\vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \longrightarrow \vec{\mathbf{F}}_1(\vec{\mathbf{r}})$$

$$\vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) \longrightarrow \vec{\mathbf{F}}_2(\vec{\mathbf{r}})$$

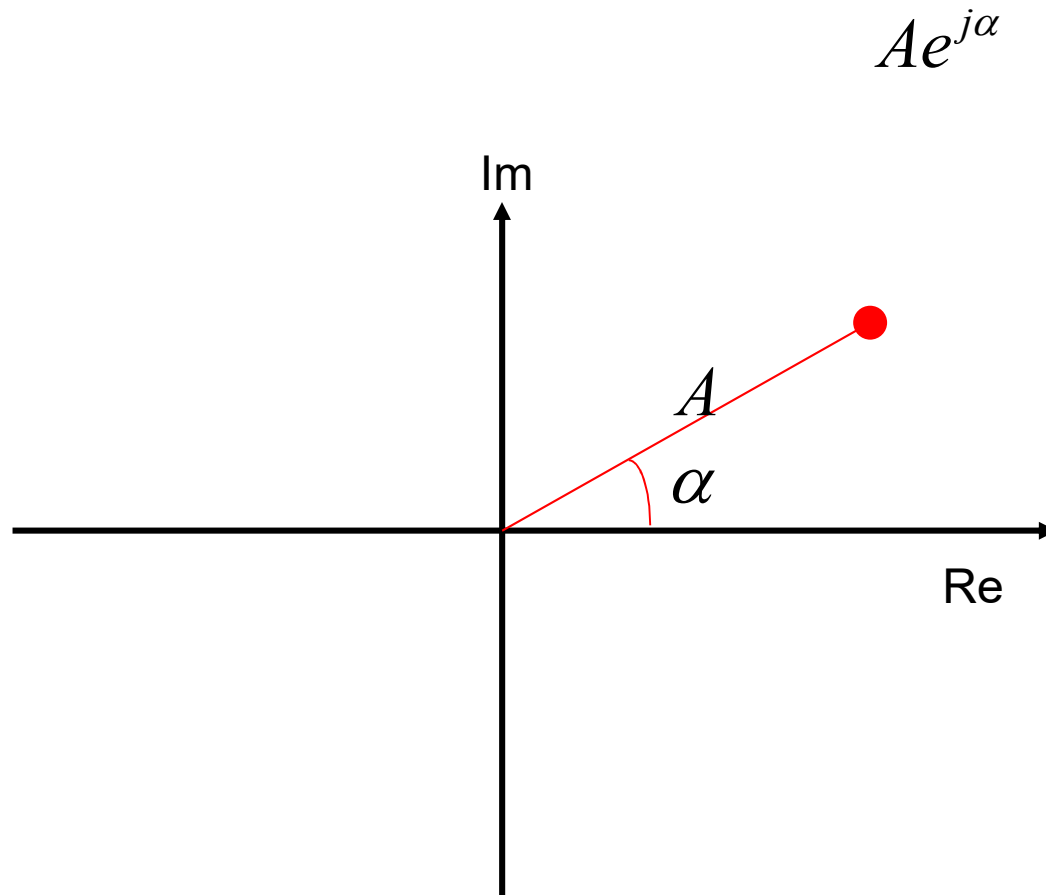
$$\langle \vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \cdot \vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) \rangle = \frac{1}{T} \int_0^T \vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \cdot \vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) dt = \frac{1}{2} \operatorname{Re} \{ \vec{\mathbf{F}}_1(\vec{\mathbf{r}}) \cdot \vec{\mathbf{F}}_2^*(\vec{\mathbf{r}}) \}$$

$$\langle \vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \times \vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) \rangle = \frac{1}{T} \int_0^T \vec{\mathbf{f}}_1(\vec{\mathbf{r}}, t) \times \vec{\mathbf{f}}_2(\vec{\mathbf{r}}, t) dt = \frac{1}{2} \operatorname{Re} \{ \vec{\mathbf{F}}_1(\vec{\mathbf{r}}) \times \vec{\mathbf{F}}_2^*(\vec{\mathbf{r}}) \}$$

# Memo: complex numbers

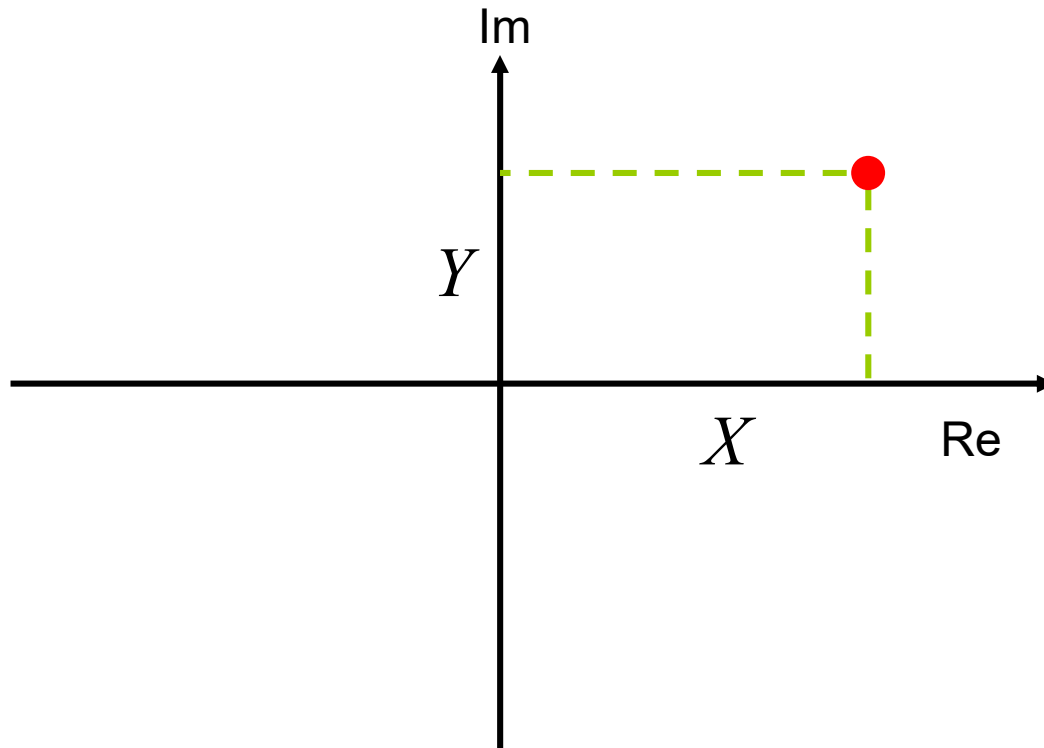


# Complex numbers



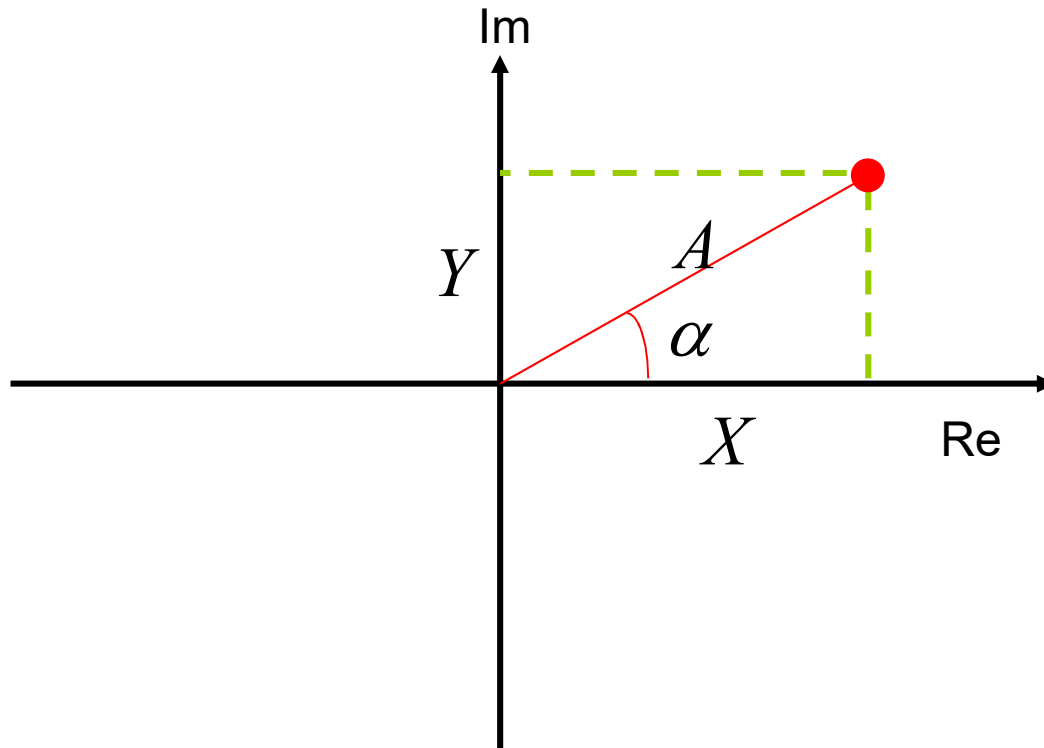
# Complex numbers

$$Ae^{j\alpha} = X + jY$$



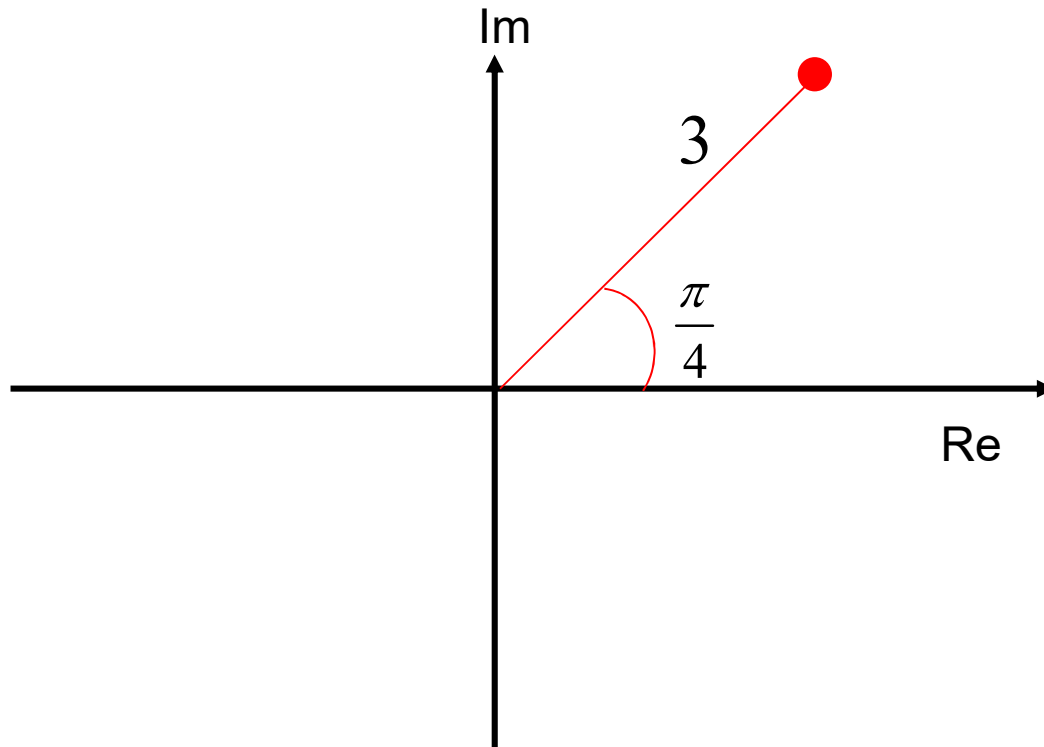
# Complex numbers

$$Ae^{j\alpha} = X + jY$$

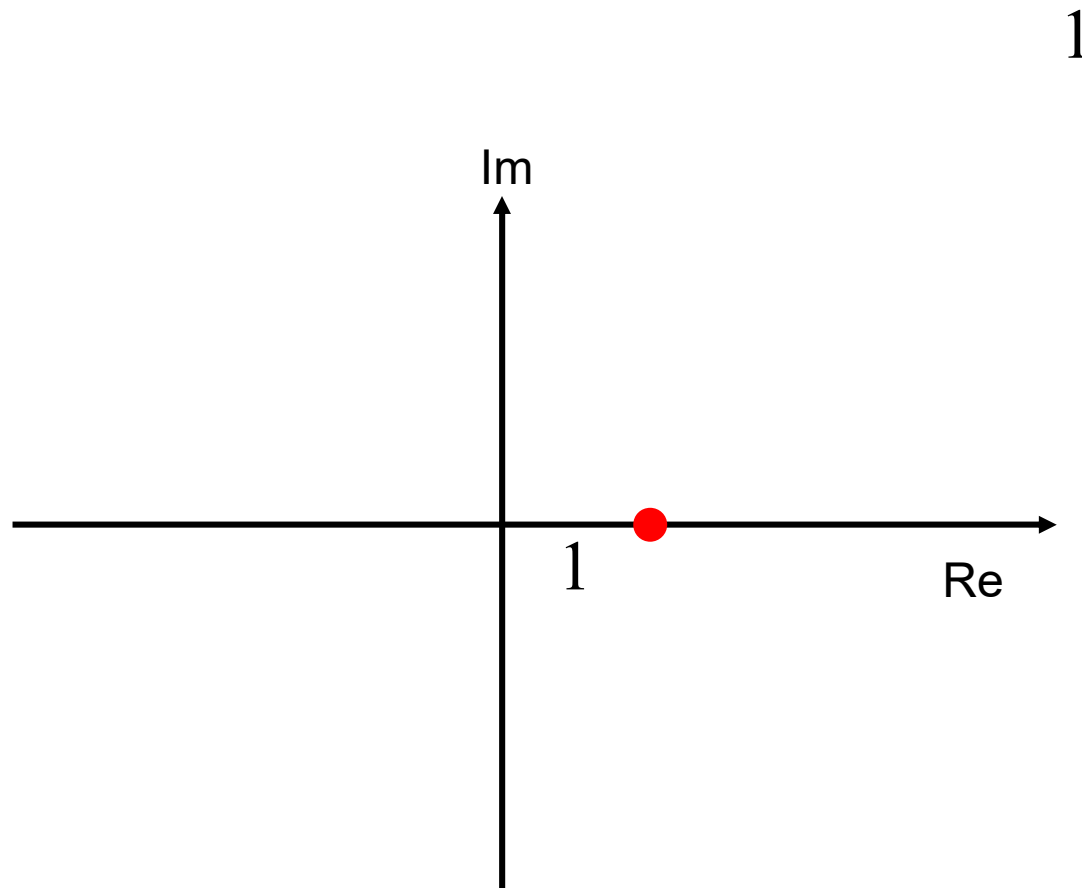


# Complex numbers: some examples

$$3e^{j\frac{\pi}{4}}$$

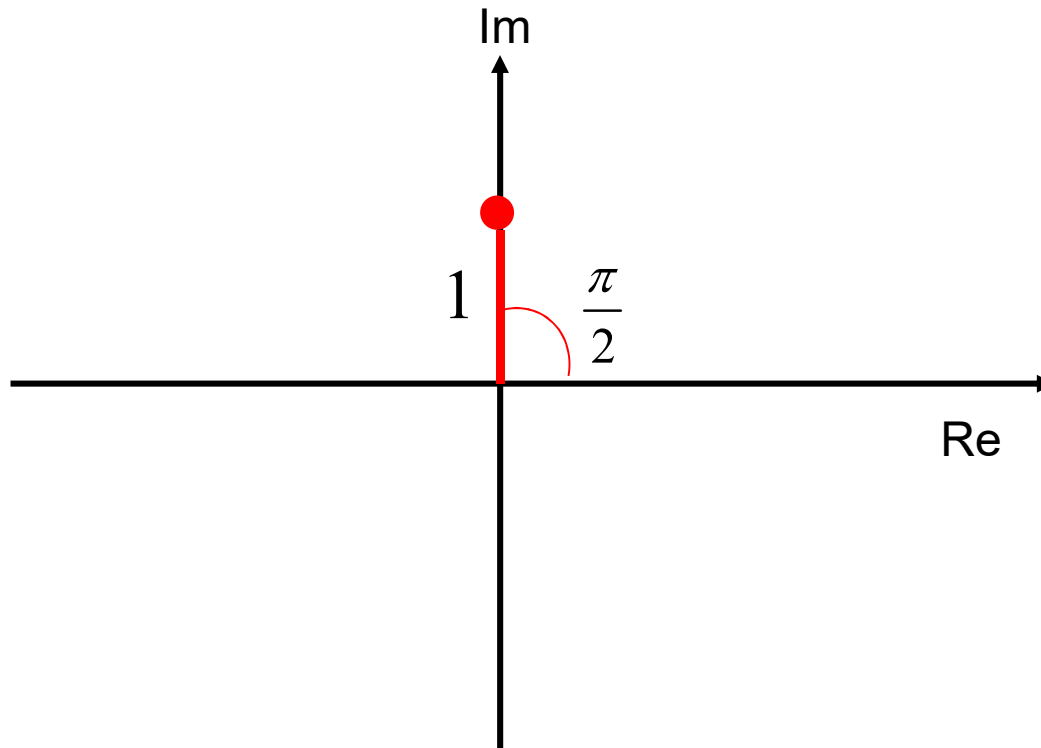


# Complex numbers: some examples



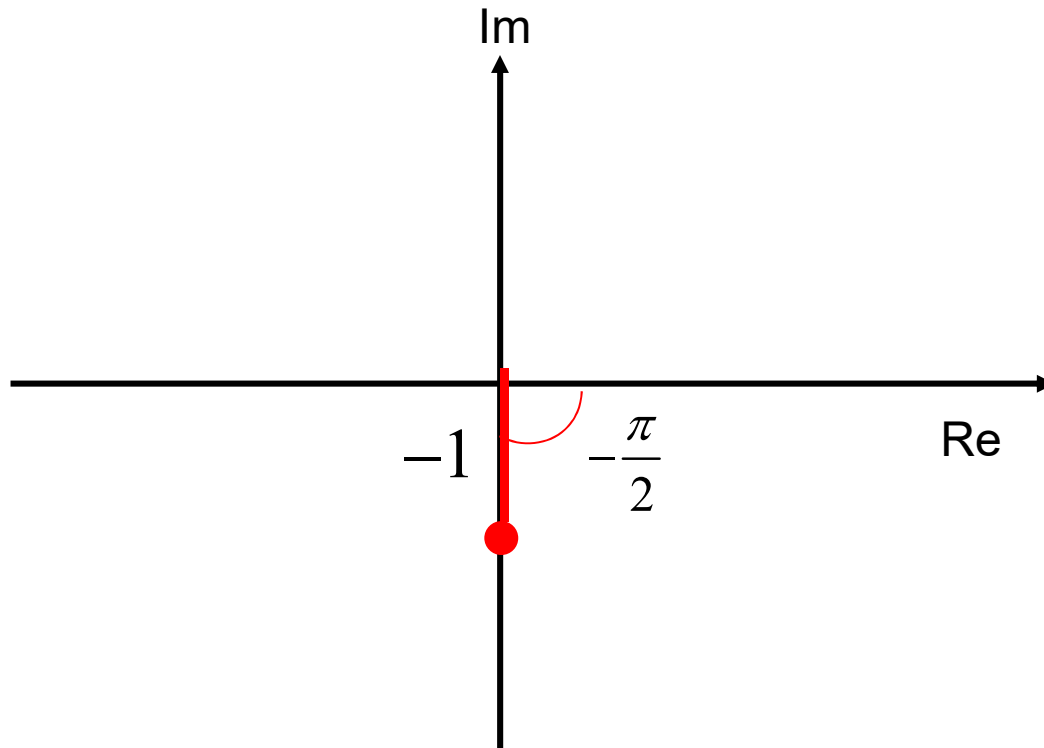
# Complex numbers: some examples

$$j = e^{j\frac{\pi}{2}} = e^{j\left(\frac{\pi}{2} + 2\pi\right)}$$



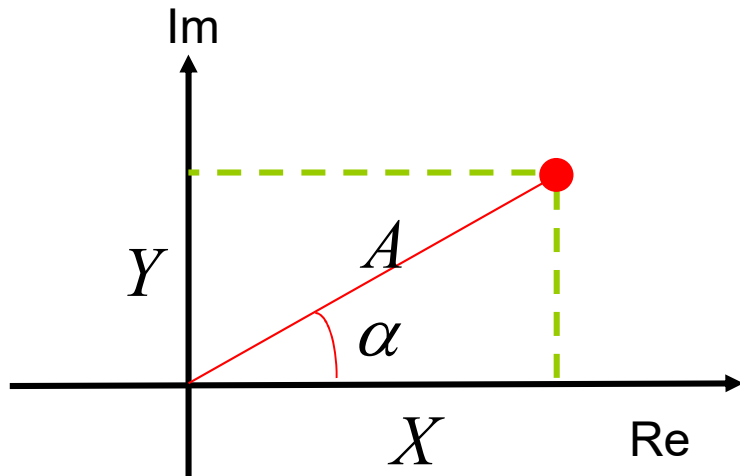
# Complex numbers: some examples

$$-j = e^{-j\frac{\pi}{2}}$$



# Complex numbers: conversion formulas

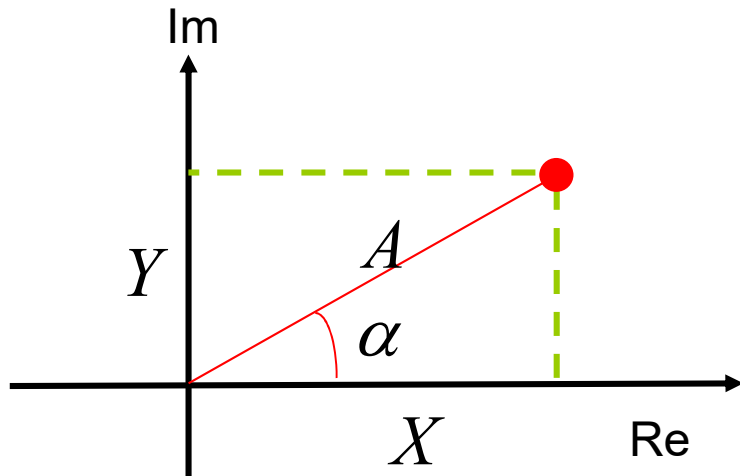
$$Ae^{j\alpha} = X + jY$$





# Complex numbers: conversion formulas

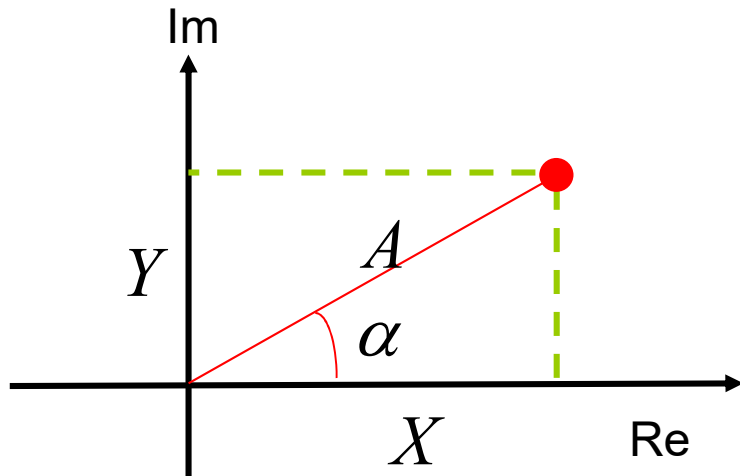
$$Ae^{j\alpha} = X + jY$$



$$Ae^{j\alpha} \longrightarrow X + jY$$

# Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



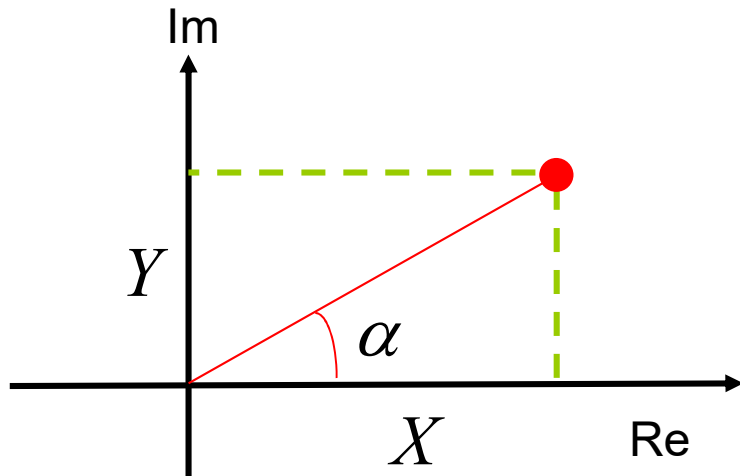
$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

# Complex numbers: conversion formulas

$$e^{j\alpha} = \cos \alpha + j \sin \alpha$$



$$Ae^{j\alpha} \rightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

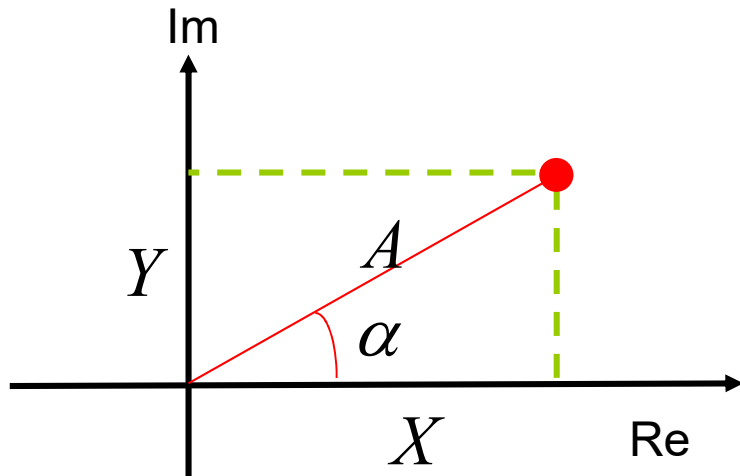
## Eulero's formulas

$$\cos \alpha = \frac{e^{j\alpha} + e^{-j\alpha}}{2}$$

$$\sin \alpha = \frac{e^{j\alpha} - e^{-j\alpha}}{2j}$$

# Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



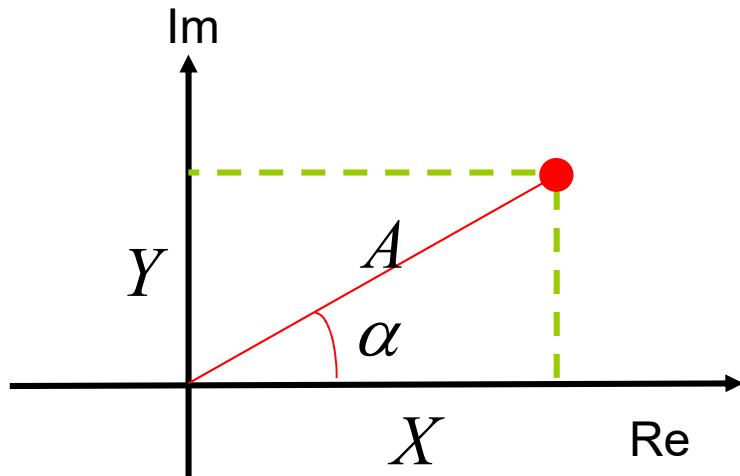
$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

# Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



$$X + jY \rightarrow Ae^{j\alpha}$$

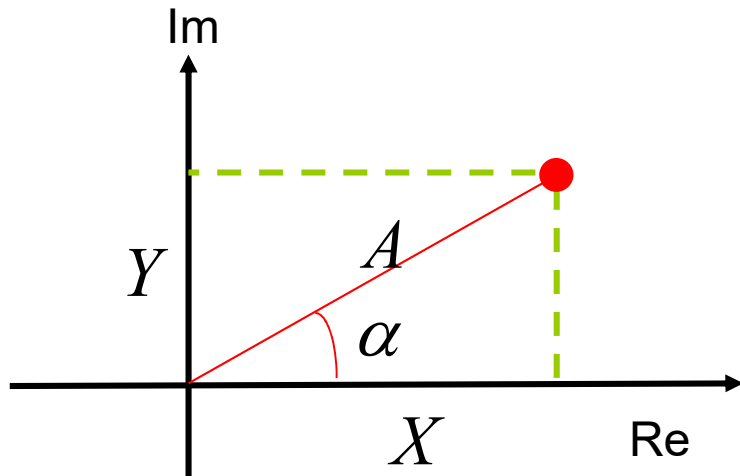
$$Ae^{j\alpha} \rightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

# Complex numbers: conversion formulas

$$Ae^{j\alpha} = X + jY$$



$$Ae^{j\alpha} \rightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

$$X + jY \rightarrow Ae^{j\alpha}$$

$$A = \sqrt{X^2 + Y^2}$$

$$\begin{cases} \alpha = \arccos\left(\frac{X}{A}\right) & \text{se } Y \geq 0 \\ \alpha = -\arccos\left(\frac{X}{A}\right) & \text{se } Y < 0 \end{cases}$$

# Complex numbers: conversion formulas

## Some examples

$$Ae^{j\alpha} \longrightarrow X + jY$$

$$X = A \cos \alpha$$

$$Y = A \sin \alpha$$

$$2e^{j\frac{3}{4}\pi} = -\sqrt{2} + j\sqrt{2}$$

# Complex numbers: conversion formulas

## Some examples

$$X + jY \rightarrow Ae^{j\alpha}$$

$$A = \sqrt{X^2 + Y^2}$$

$$\begin{cases} \alpha = \arccos\left(\frac{X}{A}\right) & \text{se } Y \geq 0 \\ \alpha = -\arccos\left(\frac{X}{A}\right) & \text{se } Y < 0 \end{cases}$$

$$\sqrt{3} + j = 2e^{j\frac{\pi}{6}}$$

$$\sqrt{3} - j = 2e^{-j\frac{\pi}{6}}$$



# Complex numbers: summation

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 + Z_2 = (X_1 + X_2) + j(Y_1 + Y_2)$$

# Complex numbers: product

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = (X_1 \cdot X_2 - Y_1 \cdot Y_2) + j(X_1 \cdot Y_2 + Y_1 \cdot X_2)$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

# Complex numbers: product

## Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = 3e^{j\frac{\pi}{6}}$$

$$z_2 = 4e^{j\frac{\pi}{4}}$$

$$z_1 \cdot z_2 = 12e^{j\left(\frac{\pi}{6} + \frac{\pi}{4}\right)} = 12e^{j\left(\frac{5}{12}\pi\right)}$$

# Complex numbers: product

## Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = \sqrt{3} + j = 2e^{j\frac{\pi}{6}}$$

$$z_2 = -\sqrt{2} + j\sqrt{2} = 2e^{j\frac{3\pi}{4}}$$

$$z_1 \cdot z_2 = 4e^{j\left(\frac{\pi}{6} + \frac{3\pi}{4}\right)} = 4e^{j\left(\frac{11}{12}\pi\right)}$$

# Complex numbers: product

## Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

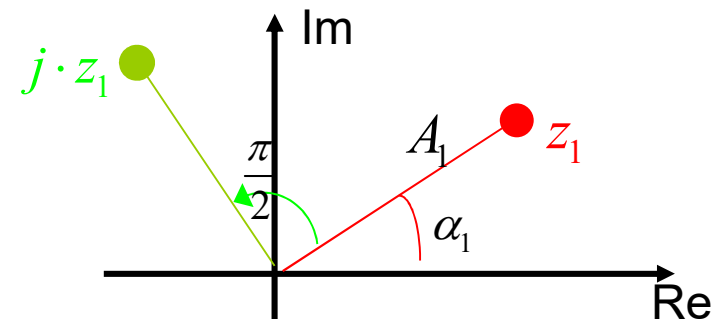
$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = A_1 e^{j\alpha_1}$$

$$j \cdot z_1 = A_1 e^{j\left(\alpha_1 + \frac{\pi}{2}\right)}$$

$$z_2 = j = e^{j\frac{\pi}{2}}$$



# Complex numbers: product

## Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

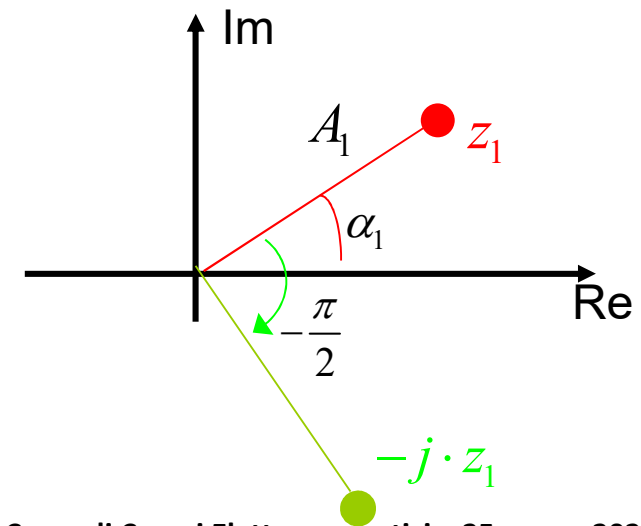
$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = A_1 e^{j\alpha_1}$$

$$-j \cdot z_1 = A_1 e^{j\left(\alpha_1 - \frac{\pi}{2}\right)}$$

$$z_2 = -j = e^{-j\frac{\pi}{2}}$$



# Complex numbers: product

## Some examples

$$Z_1 = A_1 e^{j\alpha_1} = X_1 + jY_1$$

$$Z_2 = A_2 e^{j\alpha_2} = X_2 + jY_2$$

$$Z_1 \cdot Z_2 = A_1 \cdot A_2 e^{j(\alpha_1 + \alpha_2)}$$

$$z_1 = A_1 e^{j\alpha_1}$$

$$-z_1 = A_1 e^{j(\alpha_1 + \pi)}$$

$$z_2 = -1 = e^{j\pi}$$

