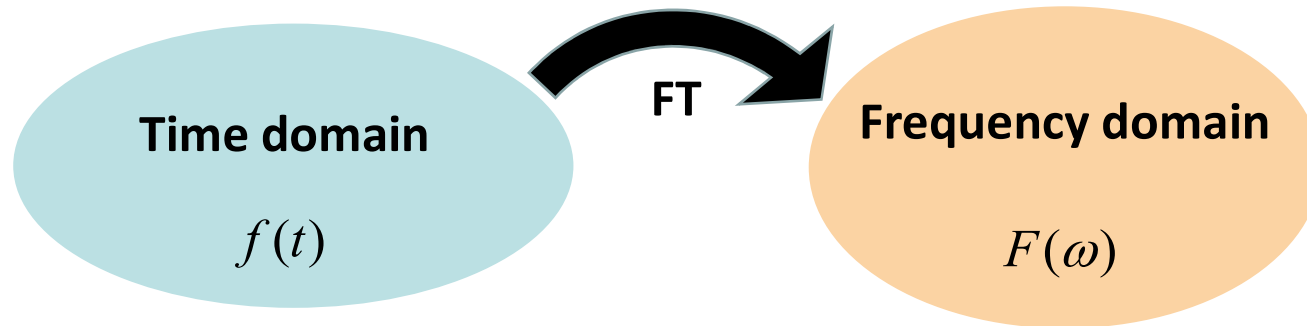


# Maxwell equations: Time domain, Frequency domain, Phasors



# Frequency domain



$$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

**Fourier Transform (FT)**

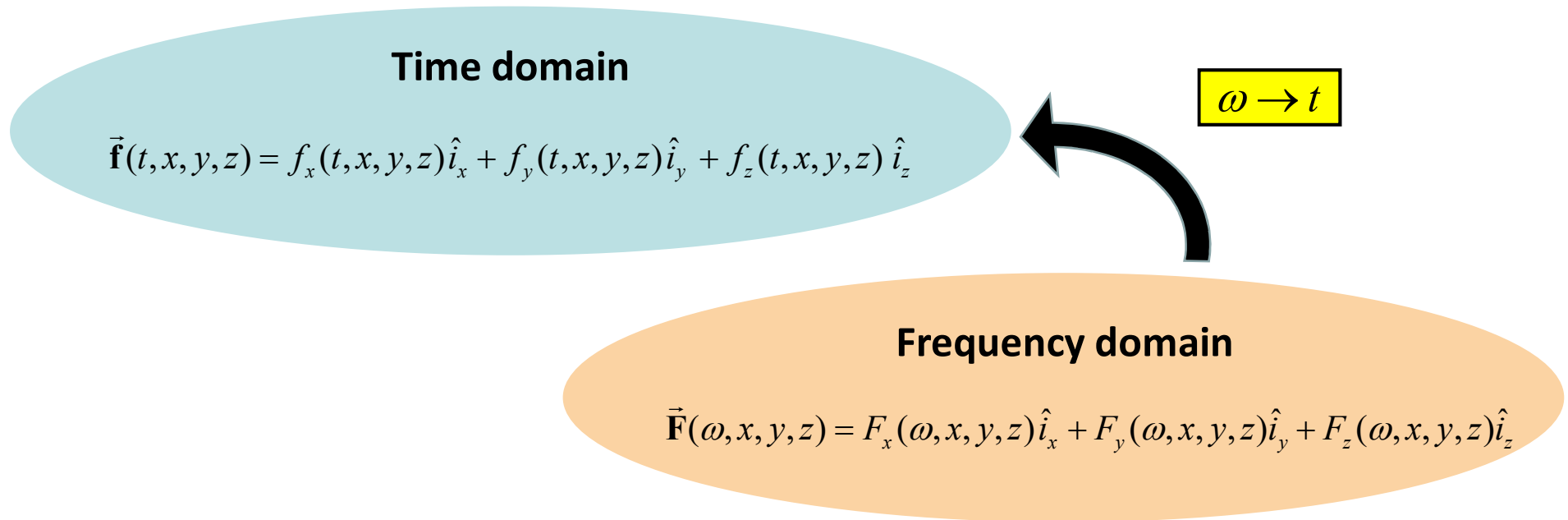
- 1) How to jump back from the Frequency domain to the Time domain**
- 2) Time domain derivative and Fourier Transform**

# Frequency domain

- **Fourier Transform and functions of  $n$  variables**
- **Fourier Transform and vector functions**
- **Fourier Transform and vector functions of  $n$  variables**

- 1) How to jump back from the Frequency domain to the Time domain**
- 2) Time domain derivative and Fourier Transform**

# Fourier Transform and vector functions of $n$ variables



## 1) How to jump back from the Spectral domain to the Time domain

$$F_x(\omega, x, y, z) \xrightarrow{\text{1D-IFT}} f_x(t, x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F_x(\omega, x, y, z) e^{j\omega t} d\omega$$

$$F_y(\omega, x, y, z) \xrightarrow{\text{1D-IFT}} f_y(t, x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F_y(\omega, x, y, z) e^{j\omega t} d\omega$$

$$F_z(\omega, x, y, z) \xrightarrow{\text{1D-IFT}} f_z(t, x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F_z(\omega, x, y, z) e^{j\omega t} d\omega$$

# Fourier Transform and vector functions of $n$ variables

Time domain

$$\vec{f}(t, x, y, z) = f_x(t, x, y, z)\hat{i}_x + f_y(t, x, y, z)\hat{i}_y + f_z(t, x, y, z)\hat{i}_z$$

$t \rightarrow \omega$

Frequency domain

$$\vec{F}(\omega, x, y, z) = F_x(\omega, x, y, z)\hat{i}_x + F_y(\omega, x, y, z)\hat{i}_y + F_z(\omega, x, y, z)\hat{i}_z$$

## 2) Time domain derivative and Fourier Transform

$t \rightarrow \omega$

$$\vec{f}(t, x, y, z) \longrightarrow \boxed{\text{FT}} \longrightarrow \vec{F}(\omega, x, y, z)$$

$$\frac{\partial \vec{f}}{\partial t} \longrightarrow \boxed{\text{FT}} \longrightarrow j\omega \vec{F}(\omega, x, y, z)$$

$$\nabla \cdot \vec{f} \longrightarrow \boxed{\text{FT}} \longrightarrow \nabla \cdot \vec{F}(\omega, x, y, z)$$

$$\nabla \times \vec{f} \longrightarrow \boxed{\text{FT}} \longrightarrow \nabla \times \vec{F}(\omega, x, y, z)$$

# Fourier Transform and vector functions of $n$ variables

Time domain

$$\vec{f}(x, y, z, t) = f_x(x, y, z, t)\hat{i}_x + f_y(x, y, z, t)\hat{i}_y + f_z(x, y, z, t)\hat{i}_z$$

$t \rightarrow \omega$

Frequency domain

$$\vec{F}(x, y, z, \omega) = F_x(x, y, z, \omega)\hat{i}_x + F_y(x, y, z, \omega)\hat{i}_y + F_z(x, y, z, \omega)\hat{i}_z$$

## 2) Time domain derivative and Fourier Transform

$t \rightarrow \omega$

$$\vec{f}(x, y, z, t) \xrightarrow{\text{FT}} \vec{F}(x, y, z, \omega)$$

$$\frac{\partial \vec{f}}{\partial t} \xrightarrow{\text{FT}} j\omega \vec{F}(x, y, z, \omega)$$

$$\nabla \cdot \vec{f} \xrightarrow{\text{FT}} \nabla \cdot \vec{F}(x, y, z, \omega)$$

$$\nabla \times \vec{f} \xrightarrow{\text{FT}} \nabla \times \vec{F}(x, y, z, \omega)$$



# Maxwell equations

## Time domain & Frequency domain

### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$



# Maxwell equations

## Time domain & Frequency domain

**Time domain**

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

**Frequency domain**





# Maxwell equations

## Time domain & Frequency domain

### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$\vec{\mathbf{f}}(x, y, z, t)$	→	<b>FT</b>	→	$\vec{\mathbf{F}}(x, y, z, \omega)$	$t \rightarrow \omega$
$\frac{\partial \vec{\mathbf{f}}}{\partial t}$	→	<b>FT</b>	→	$j\omega \vec{\mathbf{F}}(x, y, z, \omega)$	
$\nabla \cdot \vec{\mathbf{f}}$	→	<b>FT</b>	→	$\nabla \cdot \vec{\mathbf{F}}(x, y, z, \omega)$	
$\nabla \times \vec{\mathbf{f}}$	→	<b>FT</b>	→	$\nabla \times \vec{\mathbf{F}}(x, y, z, \omega)$	



# Maxwell equations

## Time domain & Frequency domain

### Time domain

$$\begin{cases} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{cases}$$

$t \rightarrow \omega$

### Frequency domain

$\vec{f}(x, y, z, t)$	→	<b>FT</b>	→	$\vec{F}(x, y, z, \omega)$	$t \rightarrow \omega$
$\frac{\partial \vec{f}}{\partial t}$	→	<b>FT</b>	→	$j\omega \vec{F}(x, y, z, \omega)$	
$\nabla \cdot \vec{f}$	→	<b>FT</b>	→	$\nabla \cdot \vec{F}(x, y, z, \omega)$	
$\nabla \times \vec{f}$	→	<b>FT</b>	→	$\nabla \times \vec{F}(x, y, z, \omega)$	

$$\vec{e}(\vec{r}, t) \longrightarrow \text{FT} \longrightarrow \vec{E}(\vec{r}, \omega)$$



# Maxwell equations

## Time domain & Frequency domain

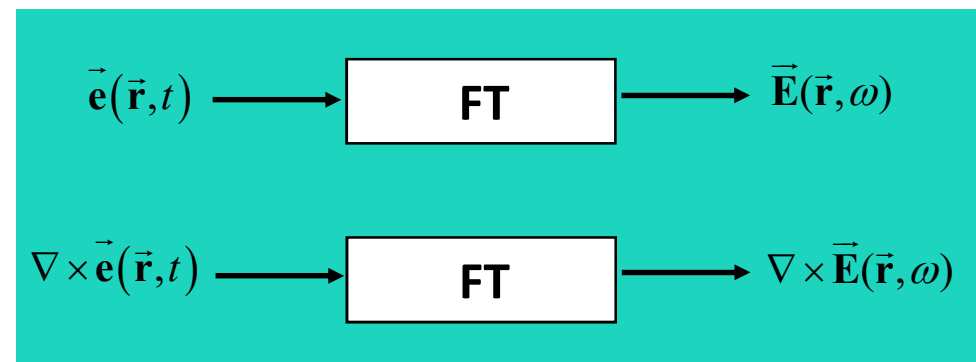
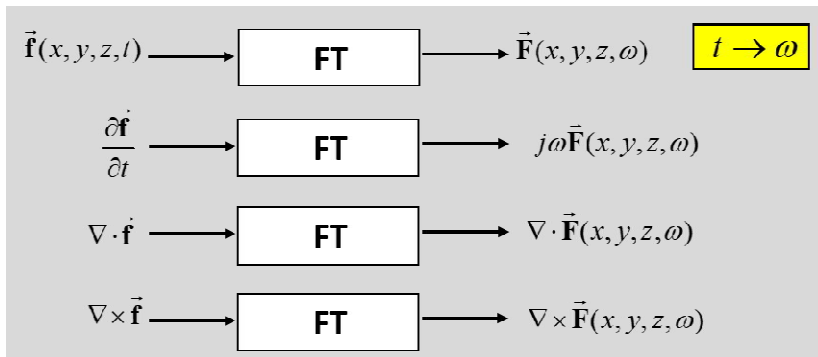
**Time domain**

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

**Frequency domain**

Empty space reserved for the frequency domain equations.





# Maxwell equations

## Time domain & Frequency domain

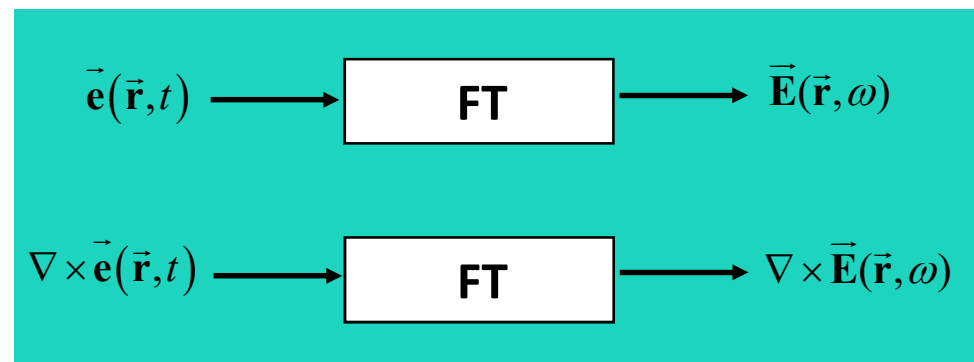
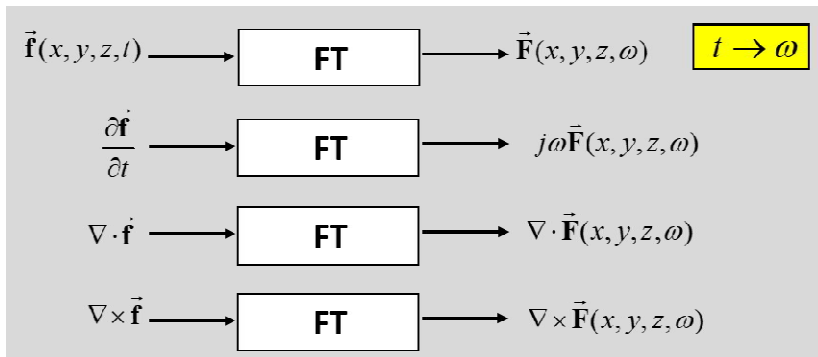
**Time domain**

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

**Frequency domain**

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

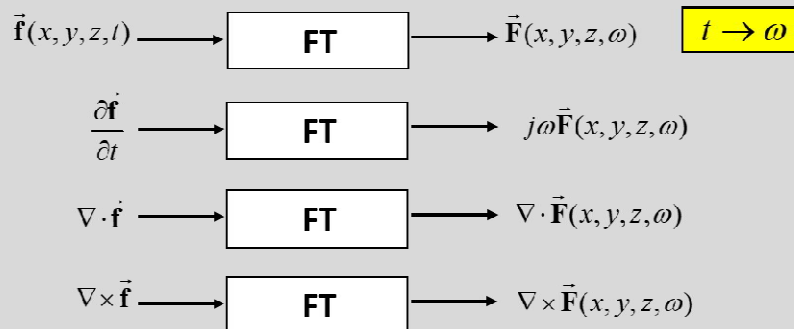
### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

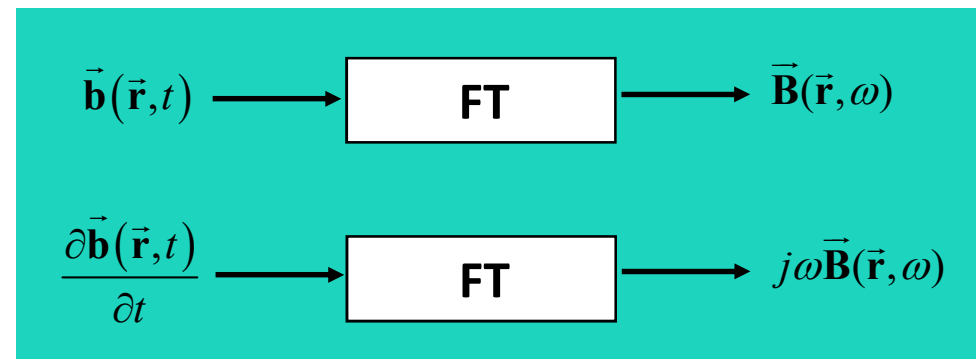
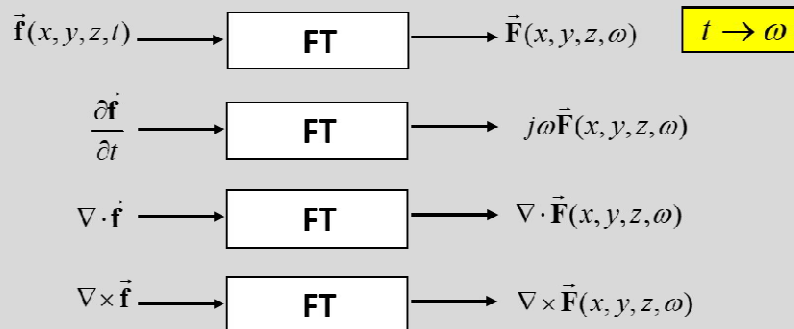
### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = \vec{j}(\vec{r}, \omega) + j\omega \vec{D}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

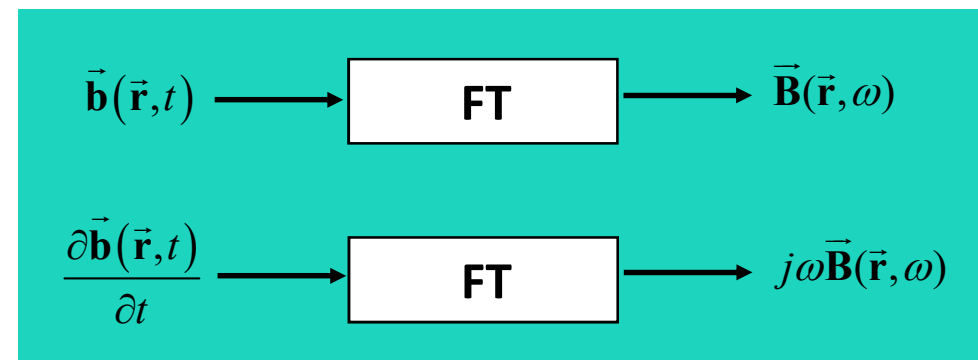
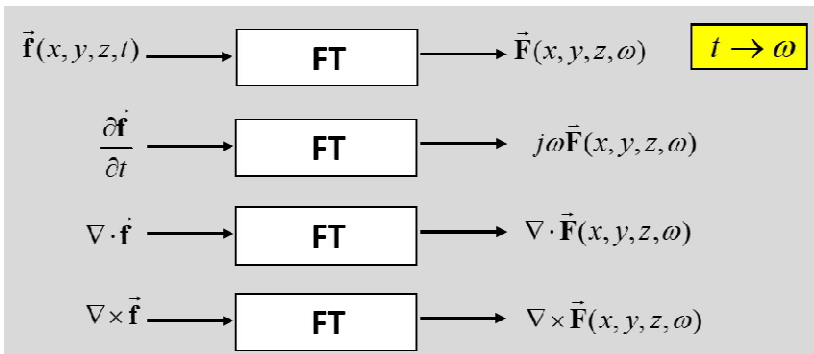
### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

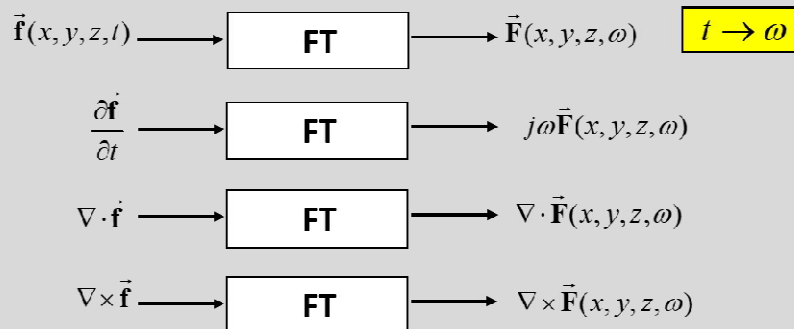
### Time domain

$$\begin{cases} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{cases}$$

$t \rightarrow \omega$

### Frequency domain

$$\begin{cases} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \end{cases}$$







# Maxwell equations

## Time domain & Frequency domain

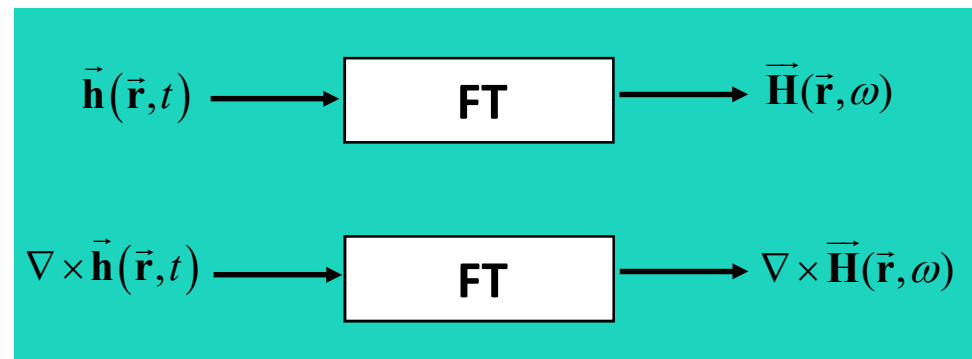
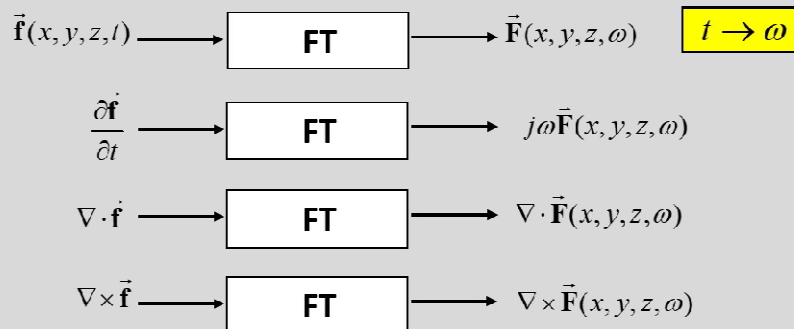
### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

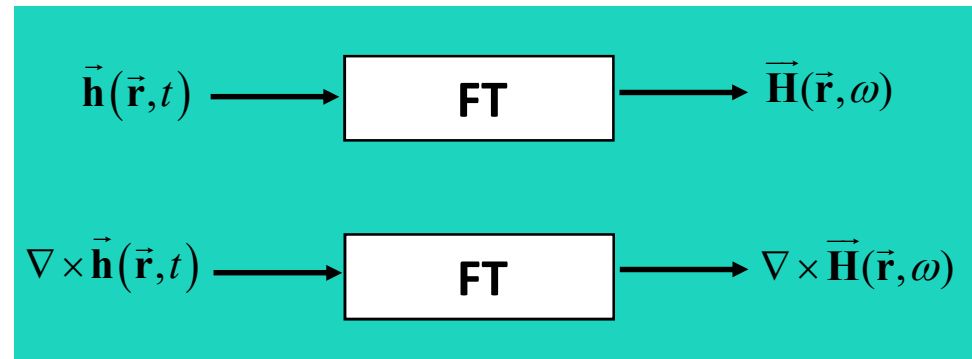
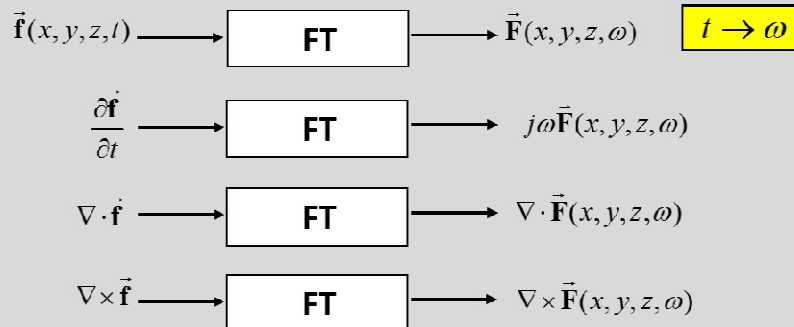
### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

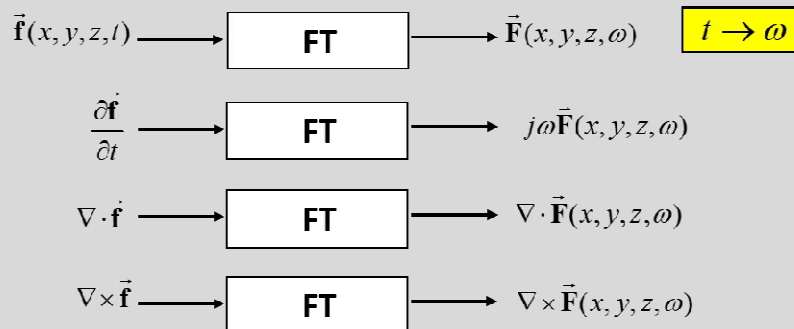
### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

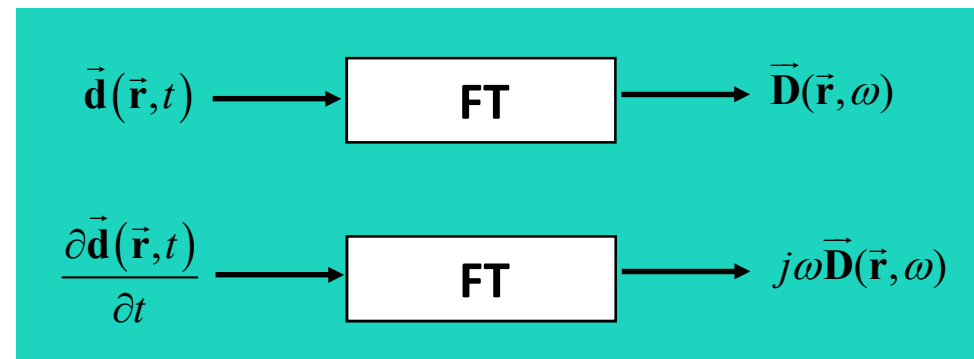
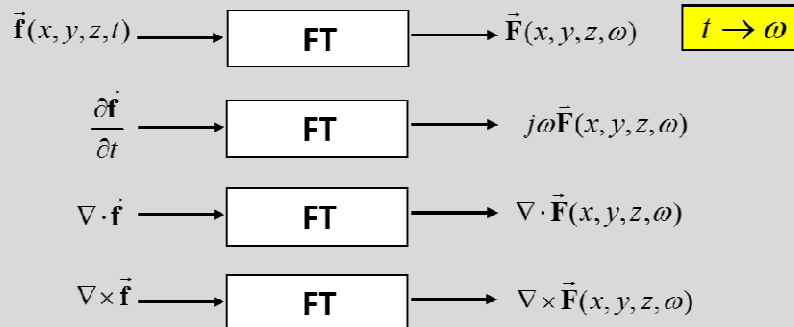
### Time domain

$$\begin{cases} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{cases}$$

$t \rightarrow \omega$

### Frequency domain

$$\begin{cases} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) \end{cases}$$





# Maxwell equations

## Time domain & Frequency domain

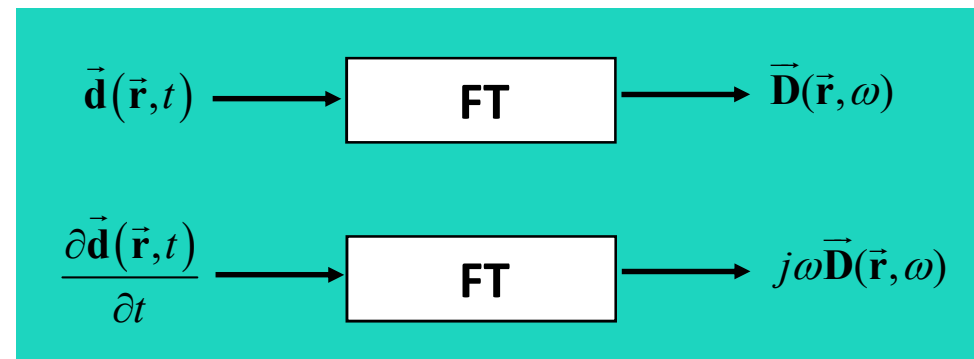
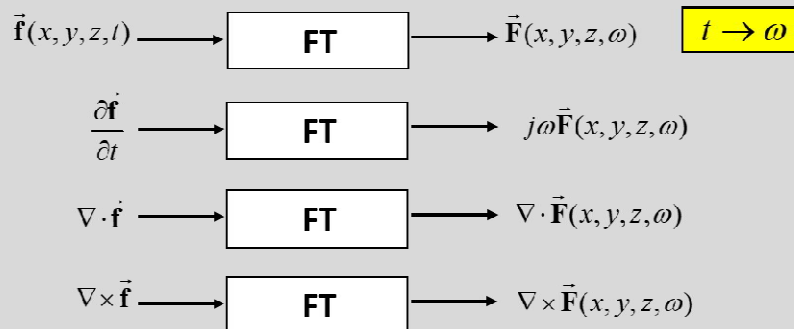
### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

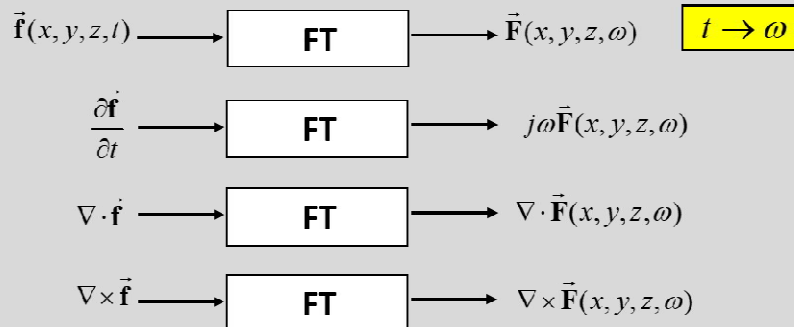
### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

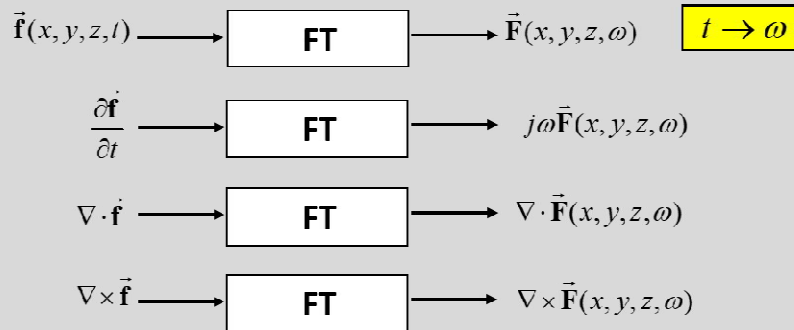
### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

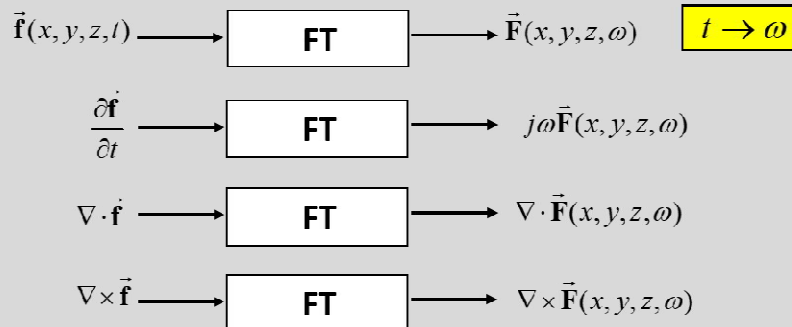
### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \end{array} \right.$$







# Maxwell equations

## Time domain & Frequency domain

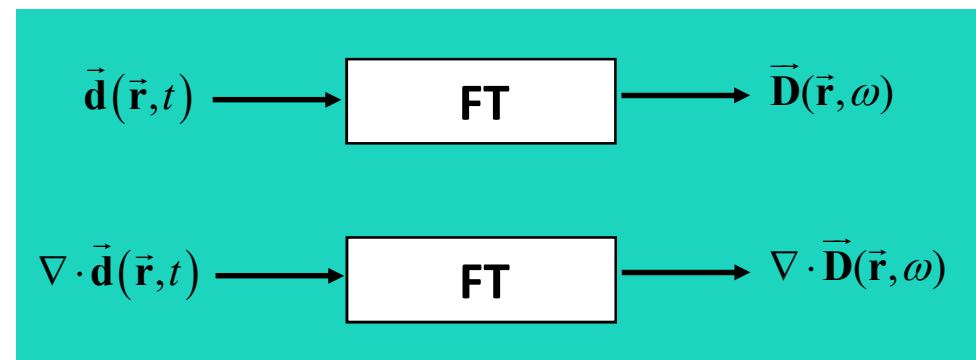
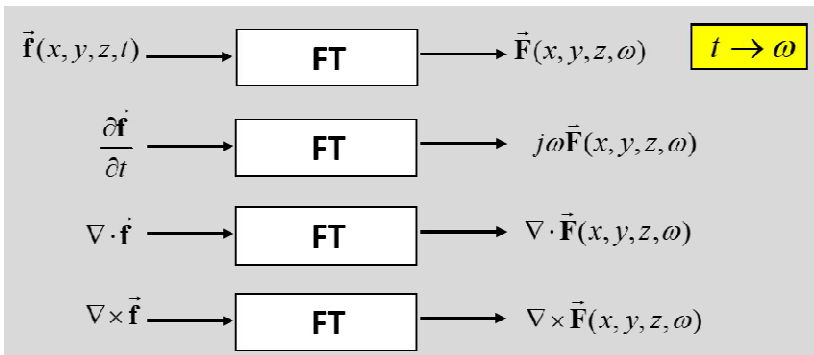
### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

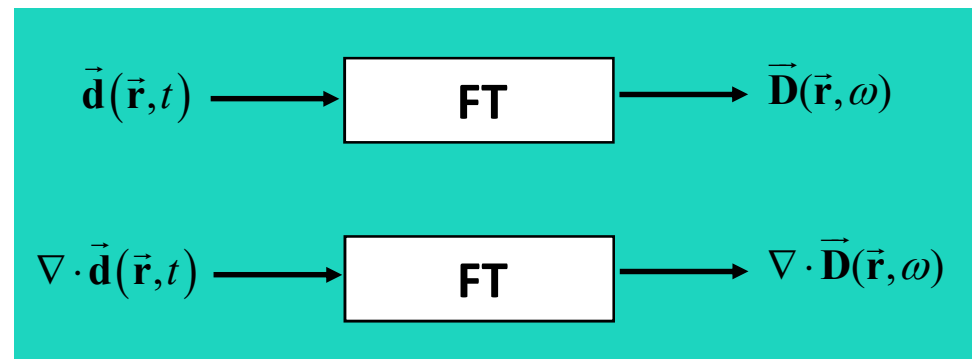
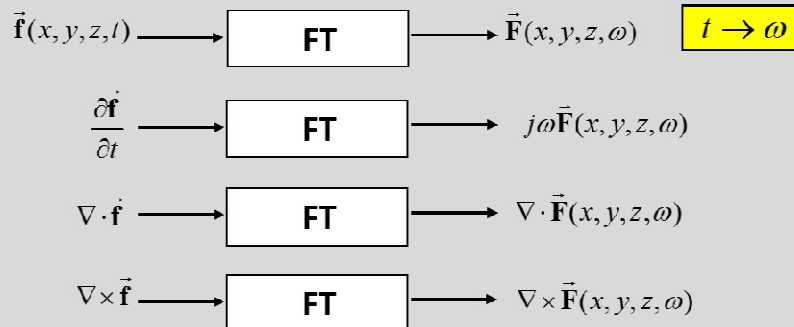
### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

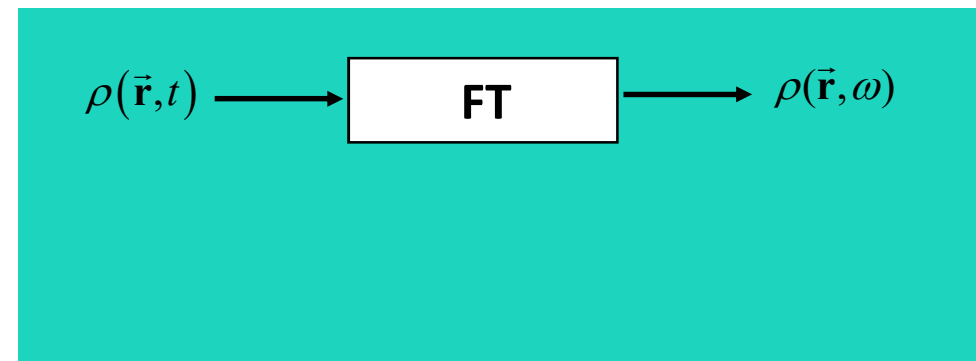
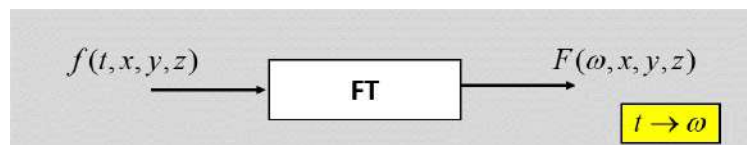
### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}, \omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

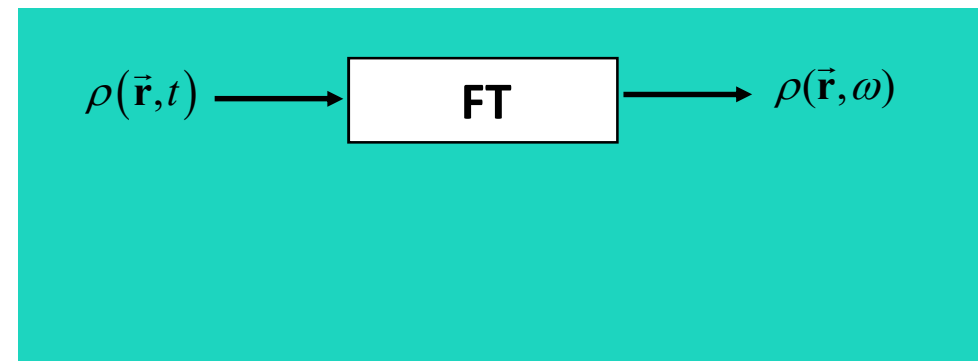
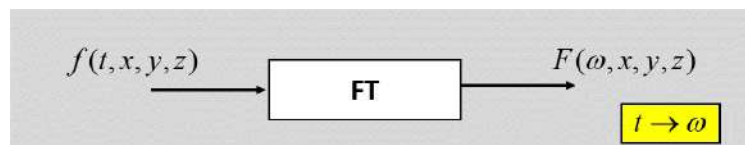
### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}, \omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) = \rho(\vec{\mathbf{r}}, \omega) \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

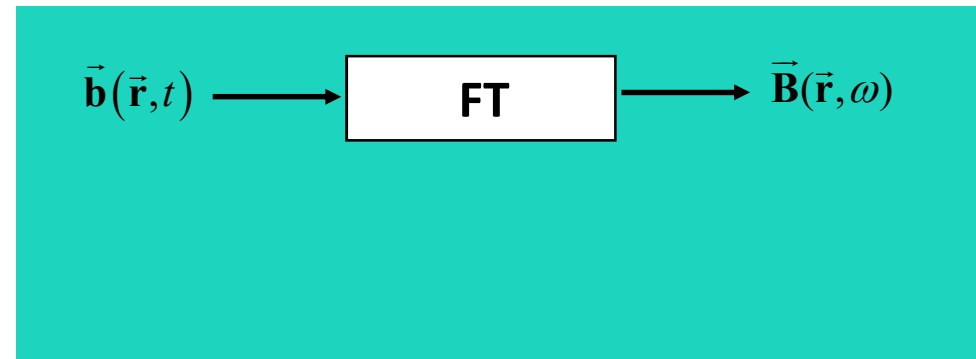
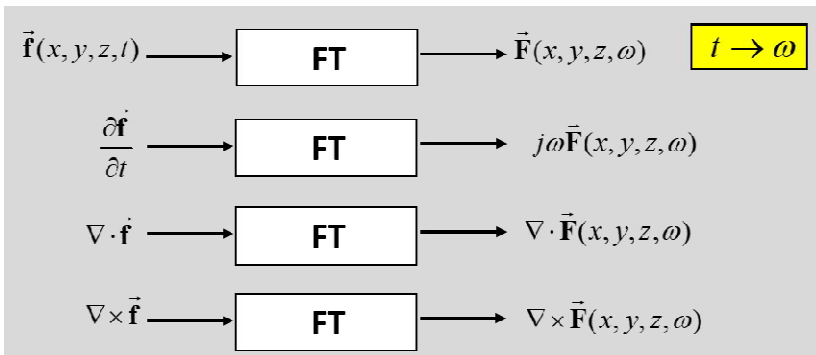
### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

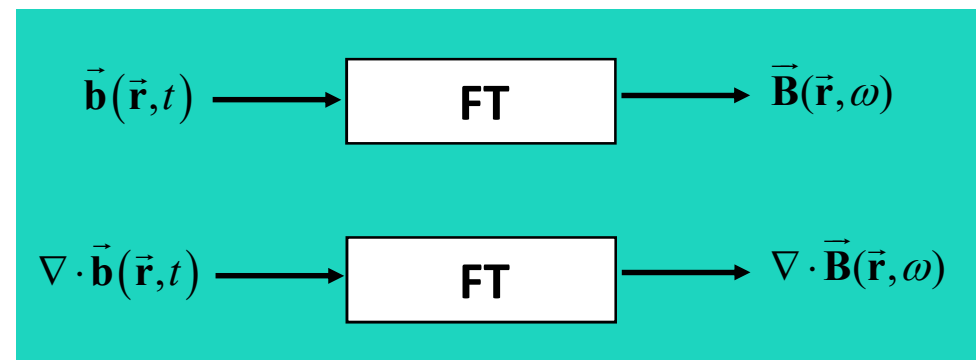
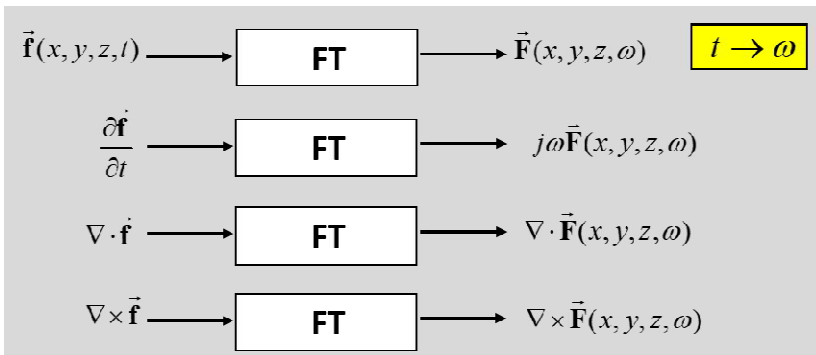
### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

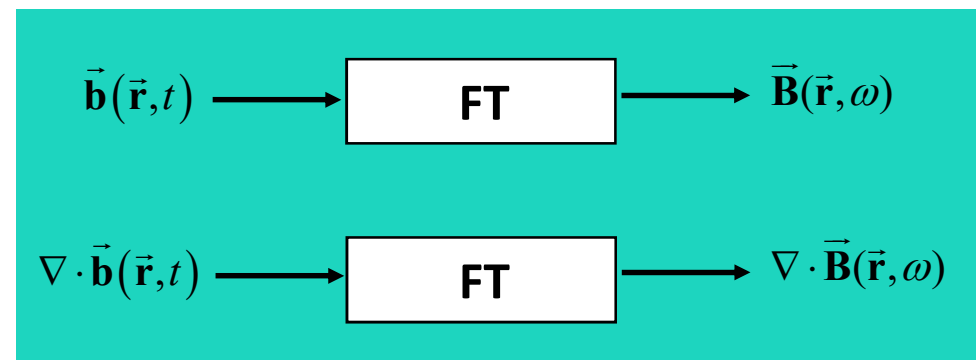
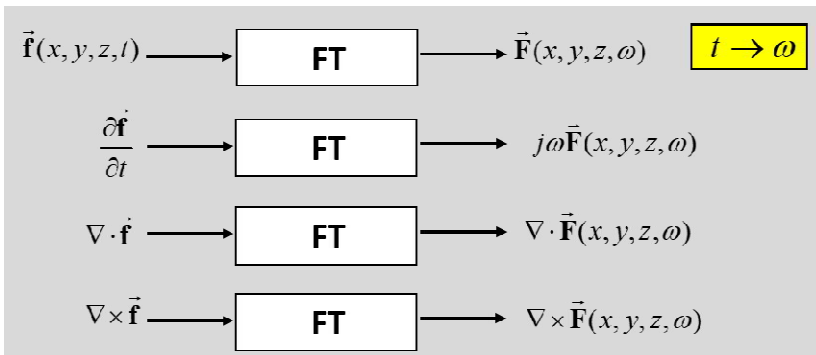
### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$t \rightarrow \omega$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}, \omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) = \rho(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) = 0 \end{array} \right.$$





# Maxwell equations

## Time domain & Frequency domain

### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$$t \rightarrow \omega$$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}, \omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) = \rho(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) = 0 \end{array} \right.$$



# Maxwell equations

## Time domain & Frequency domain

Time domain		Frequency domain
$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$	$t \rightarrow \omega$	$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{array} \right.$

$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m <sup>2</sup>
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m <sup>2</sup>
$\vec{j}(\vec{r}, t)$	Ampere/m <sup>2</sup>
$\rho(\vec{r}, t)$	Coulomb/m <sup>3</sup>



# Maxwell equations

## Time domain & Frequency domain

Time domain		Frequency domain
$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$	$t \rightarrow \omega$	$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{array} \right.$

$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m <sup>2</sup>
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m <sup>2</sup>
$\vec{j}(\vec{r}, t)$	Ampere/m <sup>2</sup>
$\rho(\vec{r}, t)$	Coulomb/m <sup>3</sup>

$\vec{E}(\vec{r}, \omega)$
$\vec{D}(\vec{r}, \omega)$
$\vec{H}(\vec{r}, \omega)$
$\vec{B}(\vec{r}, \omega)$
$\vec{J}(\vec{r}, \omega)$
$\rho(\vec{r}, \omega)$

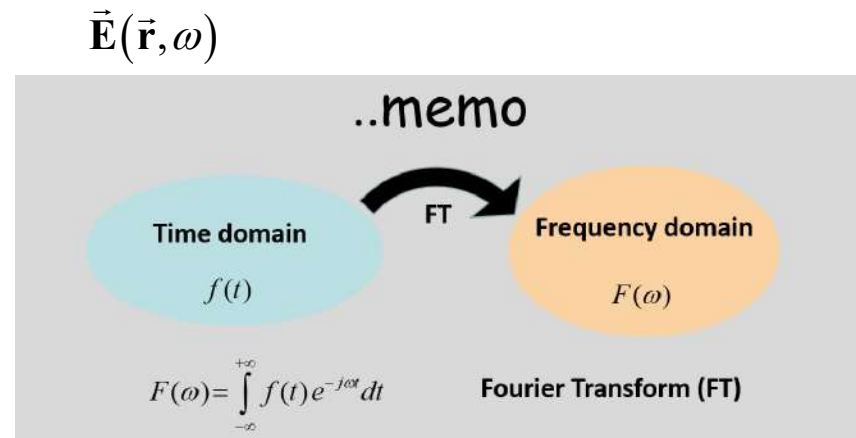


# Maxwell equations

## Time domain & Frequency domain

Time domain		Frequency domain
$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$	<div style="background-color: yellow; border: 1px solid black; padding: 2px; display: inline-block;"><math>t \rightarrow \omega</math></div>	$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{array} \right.$

- $\vec{e}(\vec{r}, t)$  Volt/m
- $\vec{d}(\vec{r}, t)$  Coulomb/m<sup>2</sup>
- $\vec{h}(\vec{r}, t)$  Ampere/m
- $\vec{b}(\vec{r}, t)$  Weber/m<sup>2</sup>
- $\vec{j}(\vec{r}, t)$  Ampere/m<sup>2</sup>
- $\rho(\vec{r}, t)$  Coulomb/m<sup>3</sup>





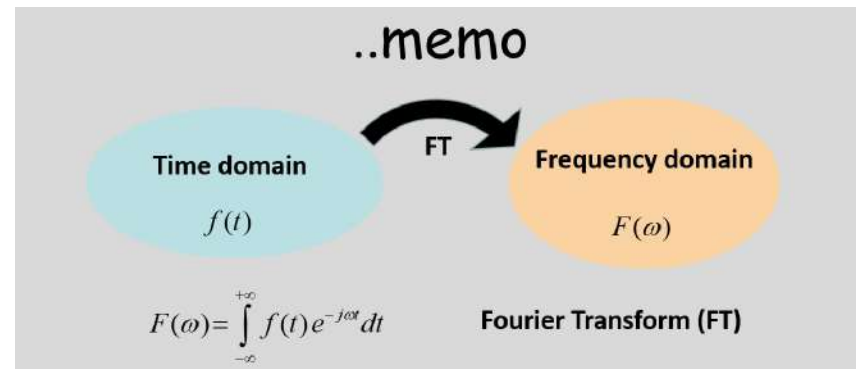
# Maxwell equations

## Time domain & Frequency domain

Time domain		Frequency domain
$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$	<div style="border: 1px solid black; background-color: yellow; padding: 2px; display: inline-block;"><math>t \rightarrow \omega</math></div>	$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{array} \right.$

- $\vec{e}(\vec{r}, t)$  Volt/m
- $\vec{d}(\vec{r}, t)$  Coulomb/m<sup>2</sup>
- $\vec{h}(\vec{r}, t)$  Ampere/m
- $\vec{b}(\vec{r}, t)$  Weber/m<sup>2</sup>
- $\vec{j}(\vec{r}, t)$  Ampere/m<sup>2</sup>
- $\rho(\vec{r}, t)$  Coulomb/m<sup>3</sup>

$\vec{E}(\vec{r}, \omega)$  (Volt x s) /m





# Maxwell equations

## Time domain & Frequency domain

Time domain		Frequency domain
$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$	<div style="border: 1px solid black; background-color: yellow; display: inline-block; padding: 2px 10px;"> <math>t \rightarrow \omega</math> </div>	$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{array} \right.$

$\vec{e}(\vec{r}, t)$	Volt/m
$\vec{d}(\vec{r}, t)$	Coulomb/m <sup>2</sup>
$\vec{h}(\vec{r}, t)$	Ampere/m
$\vec{b}(\vec{r}, t)$	Weber/m <sup>2</sup>
$\vec{j}(\vec{r}, t)$	Ampere/m <sup>2</sup>
$\rho(\vec{r}, t)$	Coulomb/m <sup>3</sup>

$\vec{E}(\vec{r}, \omega)$	(Volt x s) /m
$\vec{D}(\vec{r}, \omega)$	(Coulomb x s)/m <sup>2</sup>
$\vec{H}(\vec{r}, \omega)$	(Ampere x s)/m
$\vec{B}(\vec{r}, \omega)$	(Weber x s)/m <sup>2</sup>
$\vec{J}(\vec{r}, \omega)$	(Ampere x s)/m <sup>2</sup>
$\rho(\vec{r}, \omega)$	(Coulomb x s)/m <sup>3</sup>



# Maxwell equations

## Time domain & Frequency domain

### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$$t \rightarrow \omega$$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}, \omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) = \rho(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) = 0 \end{array} \right.$$



# Maxwell equations

## Time domain & Frequency domain

### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}}, t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}}, t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}}, t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}}, t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}}, t) = \rho(\vec{\mathbf{r}}, t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}}, t) = 0 \end{array} \right.$$

$$t \rightarrow \omega$$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{\mathbf{E}}(\vec{\mathbf{r}}, \omega) = -j\omega \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) \\ \nabla \times \vec{\mathbf{H}}(\vec{\mathbf{r}}, \omega) = j\omega \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) + \vec{\mathbf{J}}(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{D}}(\vec{\mathbf{r}}, \omega) = \rho(\vec{\mathbf{r}}, \omega) \\ \nabla \cdot \vec{\mathbf{B}}(\vec{\mathbf{r}}, \omega) = 0 \end{array} \right.$$

$$\frac{\partial \rho(\vec{\mathbf{r}}, t)}{\partial t} + \nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}}, t) = 0$$





# Maxwell equations

## Time domain & Frequency domain

### Time domain

$$\left\{ \begin{array}{l} \nabla \times \vec{e}(\vec{r}, t) = -\frac{\partial \vec{b}(\vec{r}, t)}{\partial t} \\ \nabla \times \vec{h}(\vec{r}, t) = \frac{\partial \vec{d}(\vec{r}, t)}{\partial t} + \vec{j}(\vec{r}, t) \\ \nabla \cdot \vec{d}(\vec{r}, t) = \rho(\vec{r}, t) \\ \nabla \cdot \vec{b}(\vec{r}, t) = 0 \end{array} \right.$$

$$t \rightarrow \omega$$

### Frequency domain

$$\left\{ \begin{array}{l} \nabla \times \vec{E}(\vec{r}, \omega) = -j\omega \vec{B}(\vec{r}, \omega) \\ \nabla \times \vec{H}(\vec{r}, \omega) = j\omega \vec{D}(\vec{r}, \omega) + \vec{J}(\vec{r}, \omega) \\ \nabla \cdot \vec{D}(\vec{r}, \omega) = \rho(\vec{r}, \omega) \\ \nabla \cdot \vec{B}(\vec{r}, \omega) = 0 \end{array} \right.$$

$$\frac{\partial \rho(\vec{r}, t)}{\partial t} + \nabla \cdot \vec{j}(\vec{r}, t) = 0$$

$$j\omega \rho(\vec{r}, \omega) + \nabla \cdot \vec{J}(\vec{r}, \omega) = 0$$

# Maxwell equations

## Time domain & Phasors



# Phasors

**Time domain**

$$f(t)$$

# Phasors

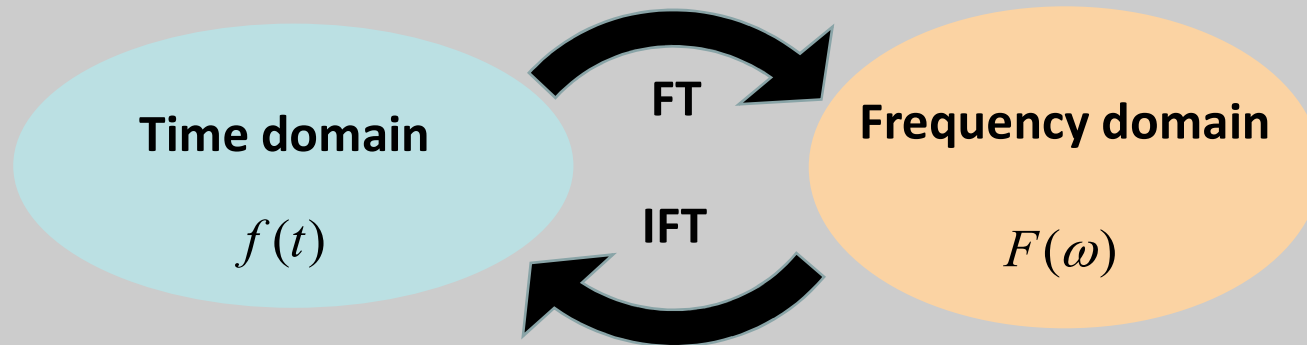
**Time domain**

$$f(t)$$

**Signals usually adopted in ICT applications**

$$f(t) = A(t) \cos(2\pi f_0 t + \alpha(t))$$

..... Memo .....



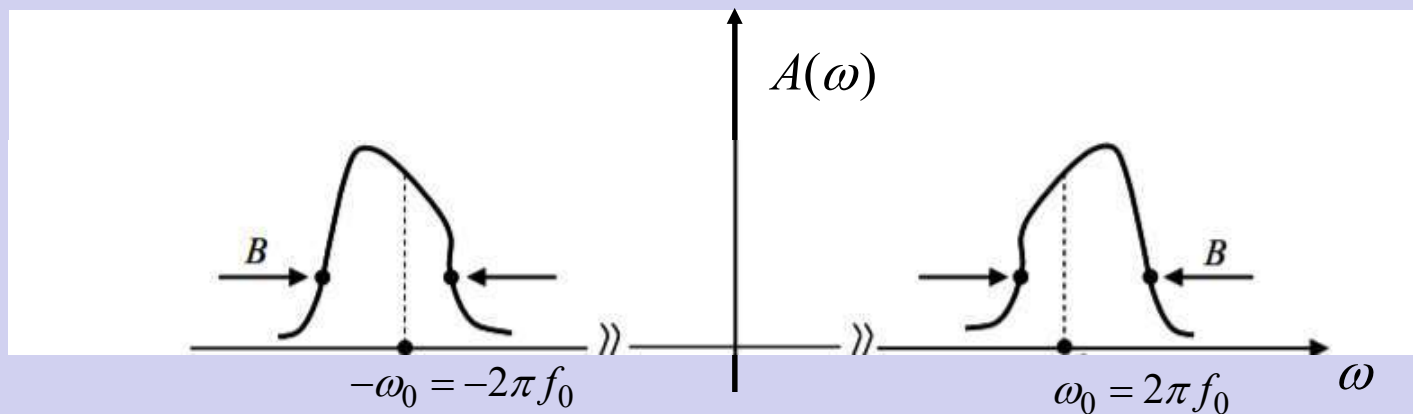
$f(t)$  → **FT** →  $F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$

$F(\omega)$  → **IFT** →  $f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{j\omega t} d\omega =$   
 $= \frac{1}{2\pi} \left[ \int_{-\infty}^{+\infty} F(\omega) \cos(\omega t) d\omega + j \int_{-\infty}^{+\infty} F(\omega) \sin(\omega t) d\omega \right]$

# Bandwidth

$$f(t) \longrightarrow \boxed{\text{FT}} \longrightarrow F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt = A(\omega) e^{j\alpha(\omega)}$$

$$f(t) = \frac{1}{2\pi} \left[ \int_{-\infty}^{+\infty} A(\omega) \cos[\omega t + \alpha(\omega)] d\omega + j \int_{-\infty}^{+\infty} A(\omega) \sin[\omega t + \alpha(\omega)] d\omega \right]$$



# Phasors

**Time domain**

$$f(t)$$

**Signals usually adopted in ICT applications**

$$f(t) = A(t) \cos(2\pi f_0 t + \alpha(t))$$

# Phasors

**Time domain**

$$f(t)$$

**Signals usually analyzed in ICT applications**

$$f(t) = A \cos(2\pi f_0 t + \alpha)$$

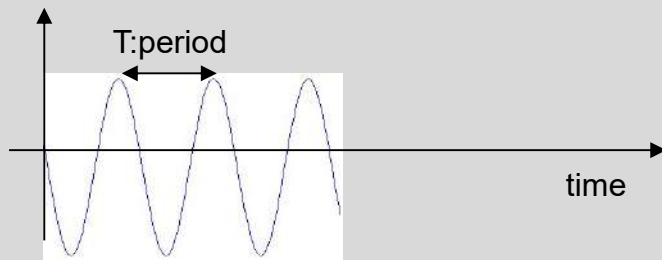


# Phasors

Time domain

$f(t)$

Signals usually adopted in ICT applications

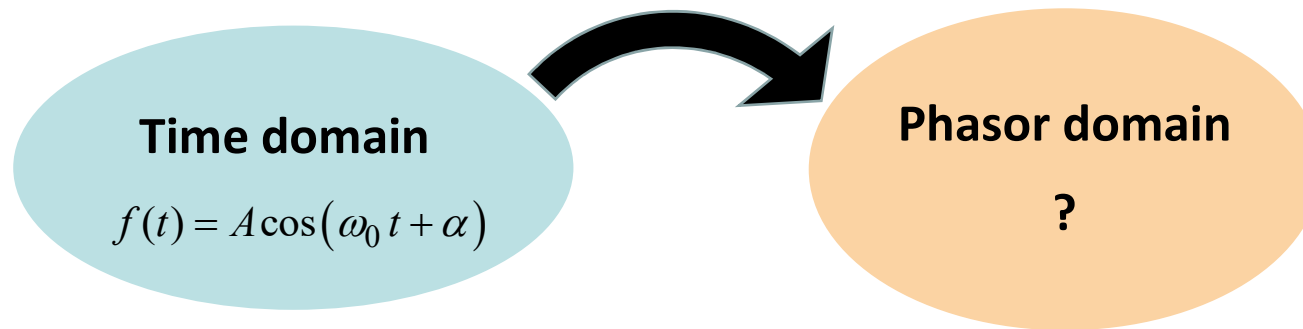


$$f(t) = A \cos(2\pi f_0 t + \alpha)$$

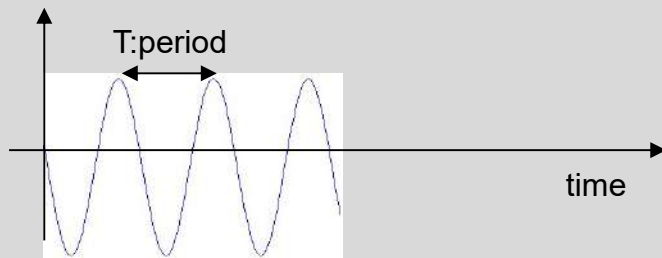
$$f_0 : \text{frequency} = \frac{1}{T}$$

$$\omega_0 : \text{angular frequency} = 2\pi f_0$$

# Phasors



## Signals usually adopted in ICT applications

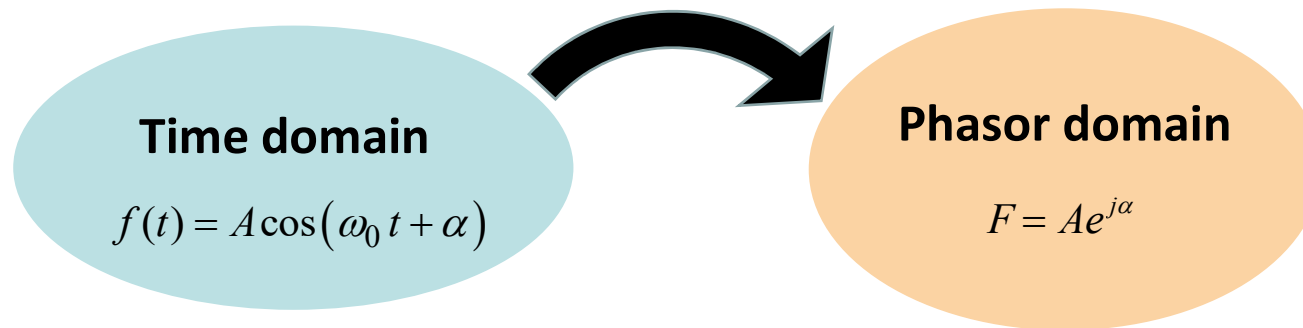


$$f(t) = A \cos(2\pi f_0 t + \alpha)$$

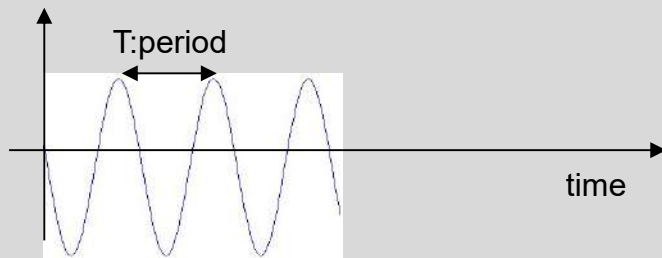
$$f_0 : \text{frequency} = \frac{1}{T}$$

$$\omega_0 : \text{angular frequency} = 2\pi f_0$$

# Phasors



## Signals usually adopted in ICT applications

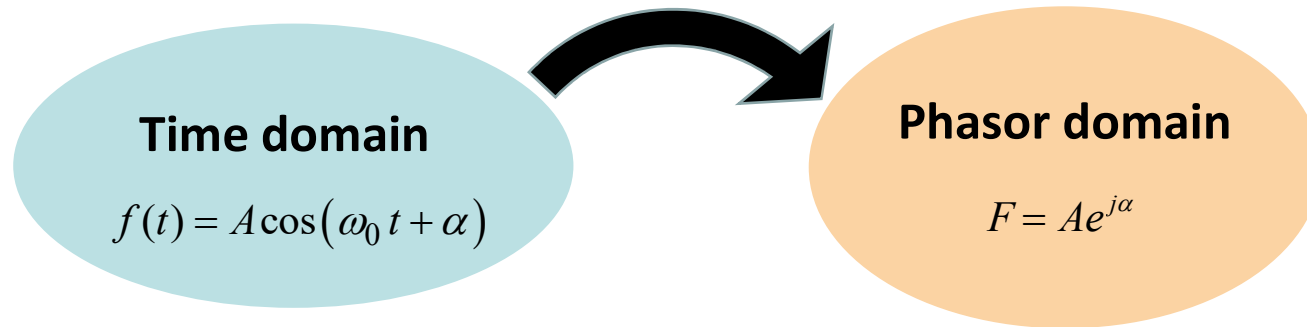


$$f(t) = A \cos(2\pi f_0 t + \alpha)$$

$$f_0 : \text{frequency} = \frac{1}{T}$$

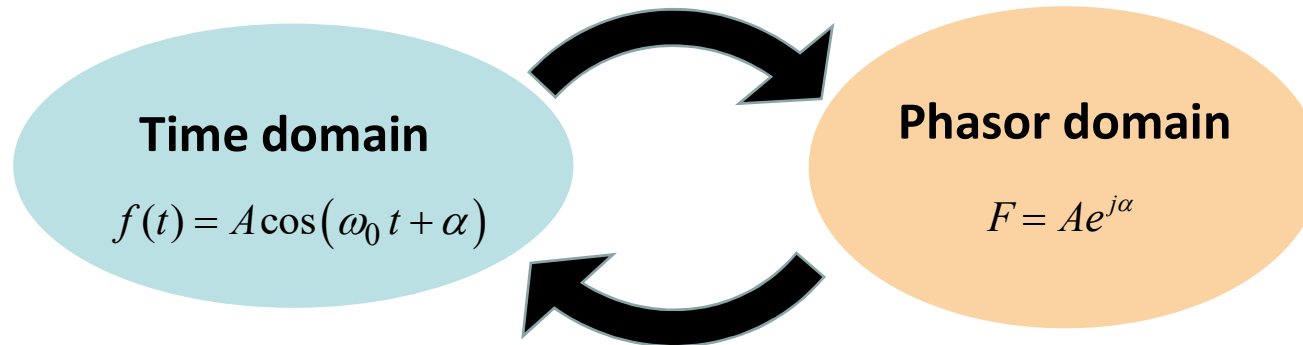
$$\omega_0 : \text{angular frequency} = 2\pi f_0$$

# Phasors



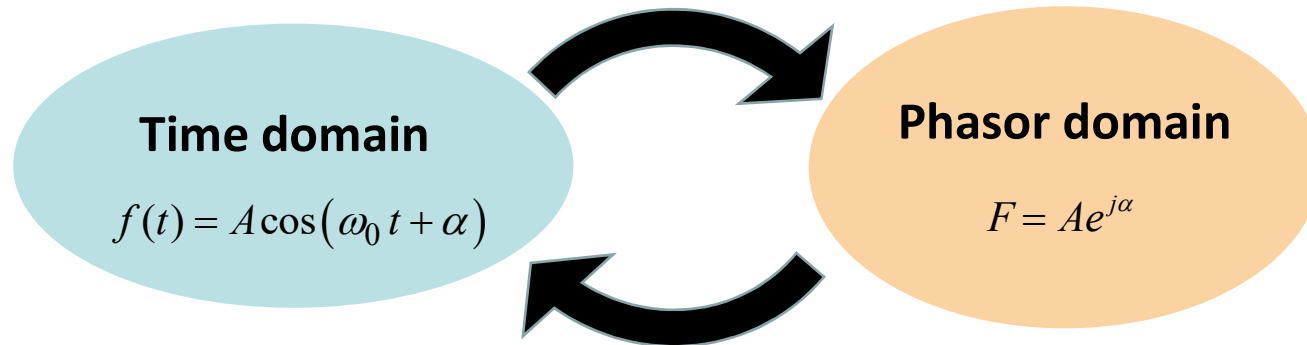
- 1) How to jump back from the Phasor domain to the Time domain
- 2) Time domain derivative and Phasors

# Phasors



**1) How to jump back from the Phasor domain to the Time domain**

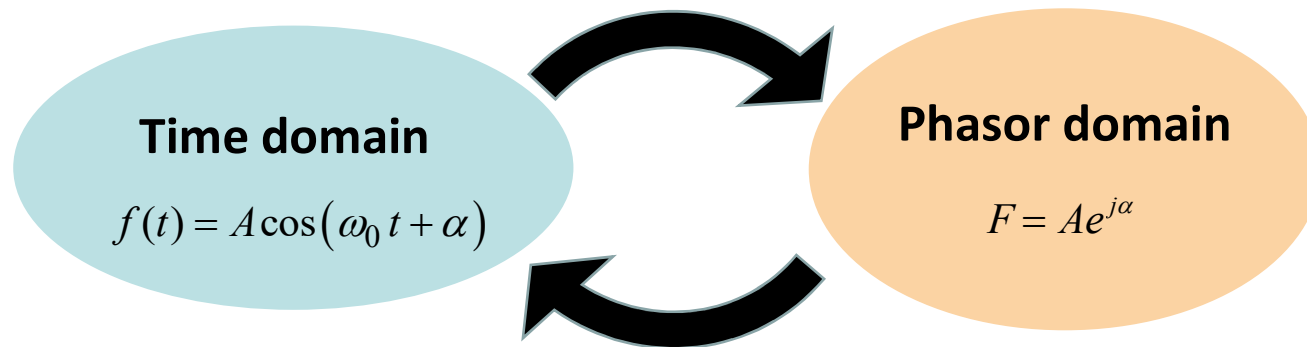
# Phasors



## 1) How to jump back from the Phasor domain to the Time domain

$$f(t) = \operatorname{Re}\{F e^{j\omega_0 t}\} = \operatorname{Re}\{A e^{j\alpha} e^{j\omega_0 t}\} = A \cos(\omega_0 t + \alpha)$$

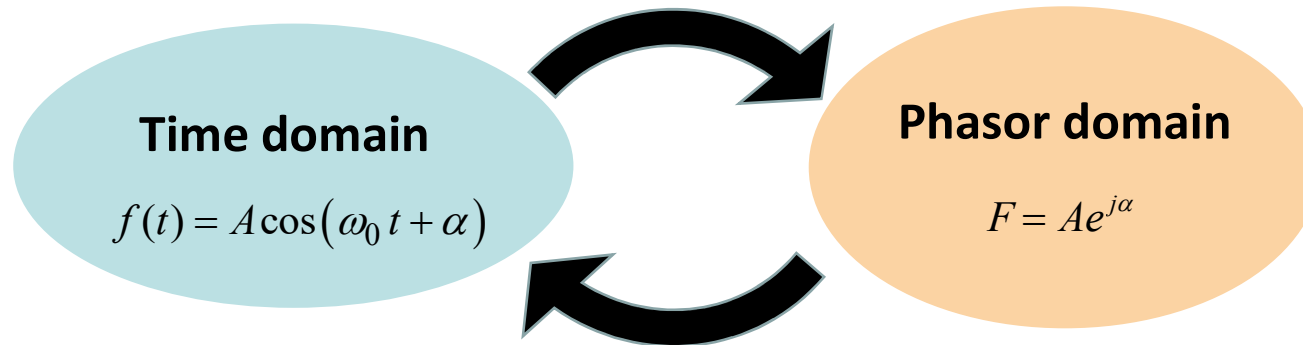
# Phasors



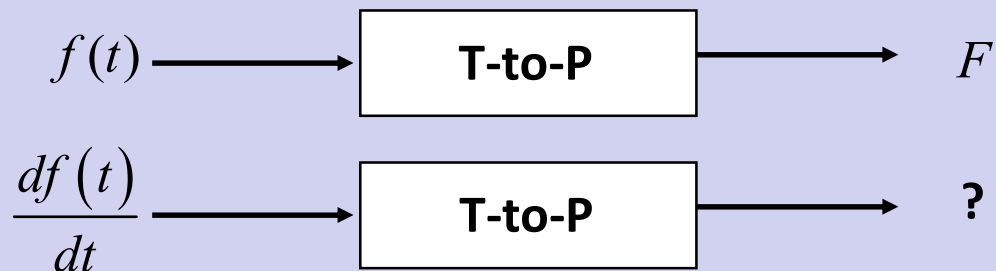
$$f(t) = A \cos(\omega_0 t + \alpha) \longrightarrow \text{T-to-P} \longrightarrow F = A e^{j\alpha}$$

$$F = A e^{j\alpha} \longrightarrow \text{P-to-T} \longrightarrow f(t) = \text{Re}\{F e^{j\omega_0 t}\} = A \cos(\omega_0 t + \alpha)$$

# Phasors

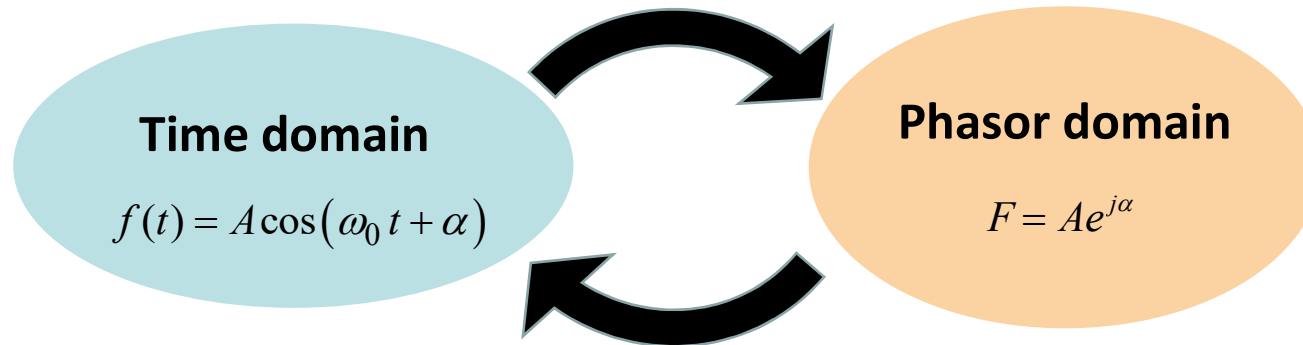


## 2) Time domain derivative and Phasors

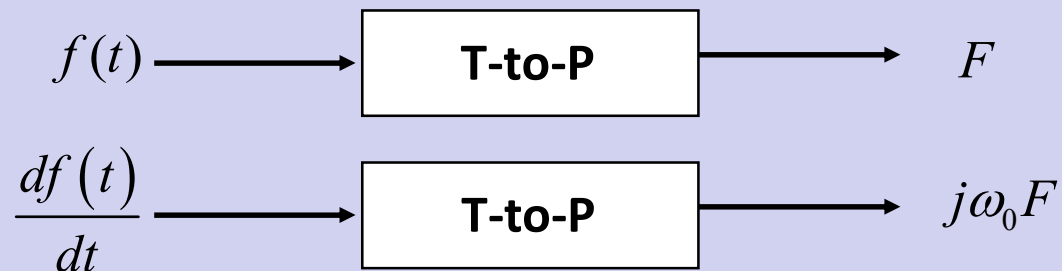




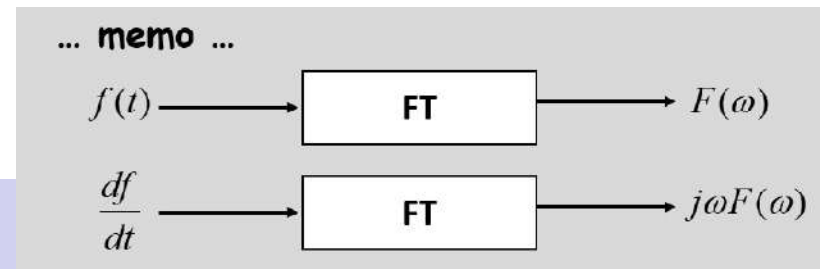
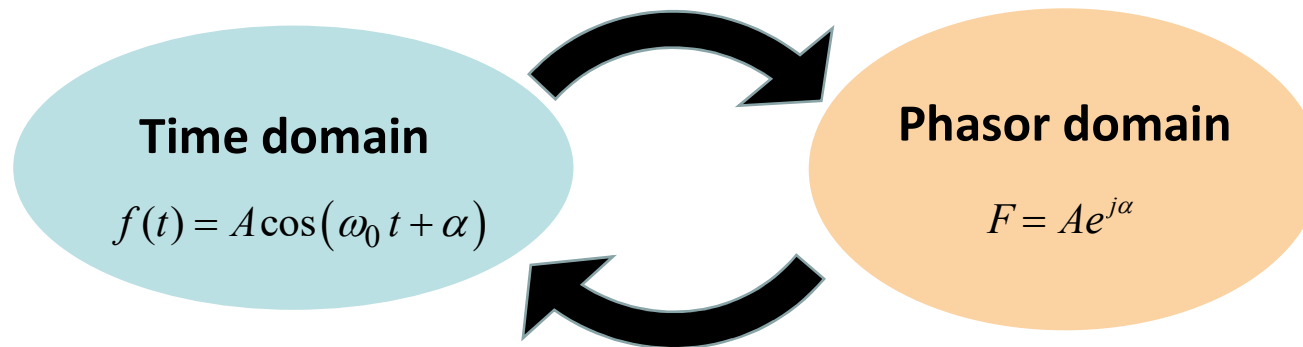
# Phasors



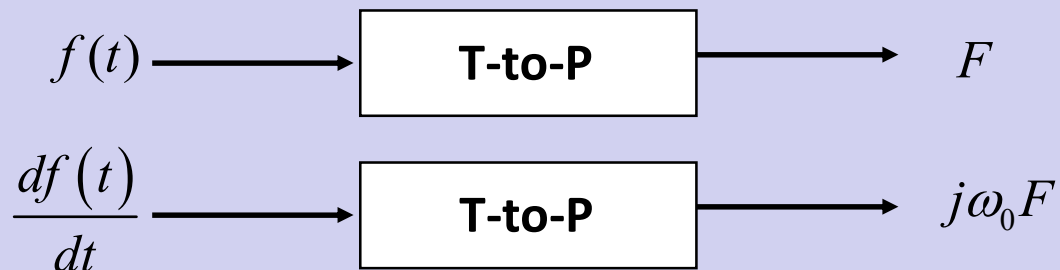
## 2) Time domain derivative and Phasors



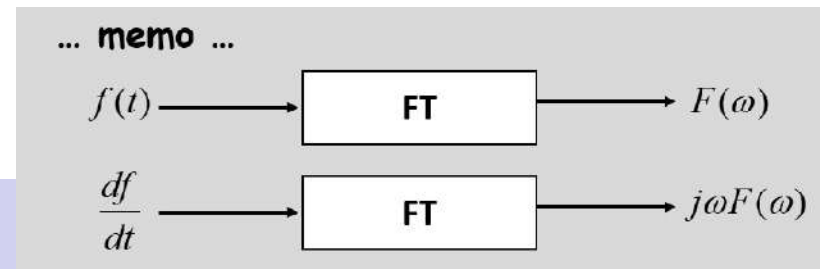
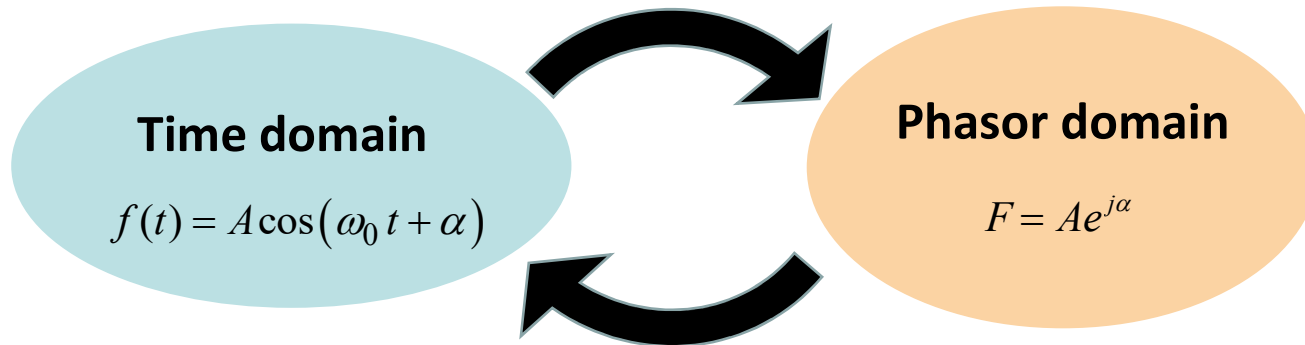
# Phasors



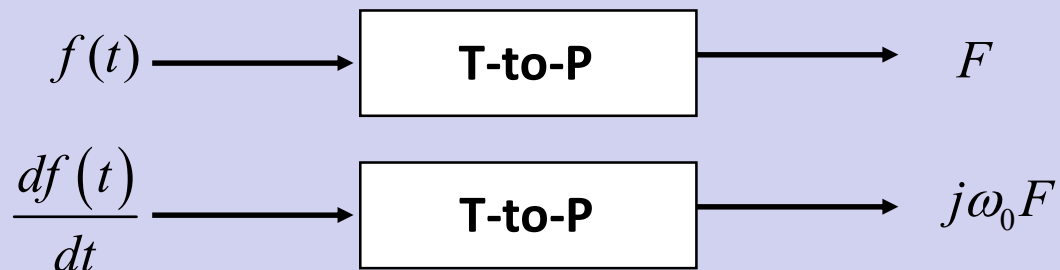
## 2) Time domain derivative and Phasors



# Phasors

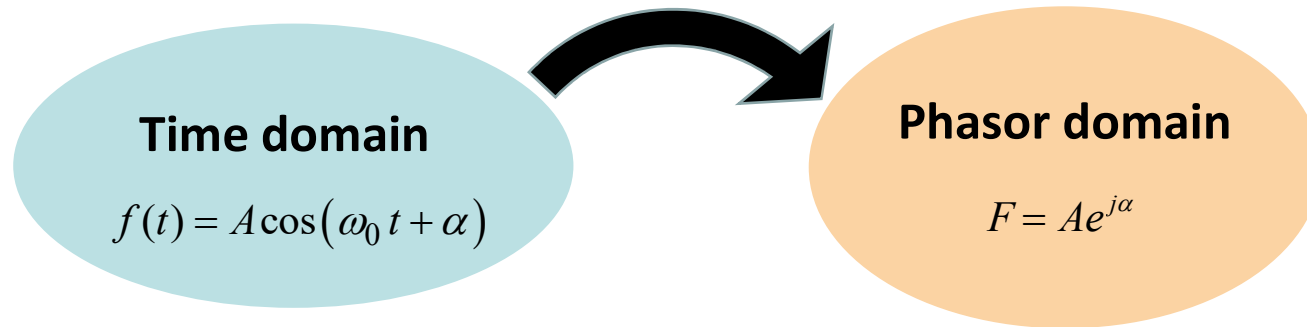


## 2) Time domain derivative and Phasors



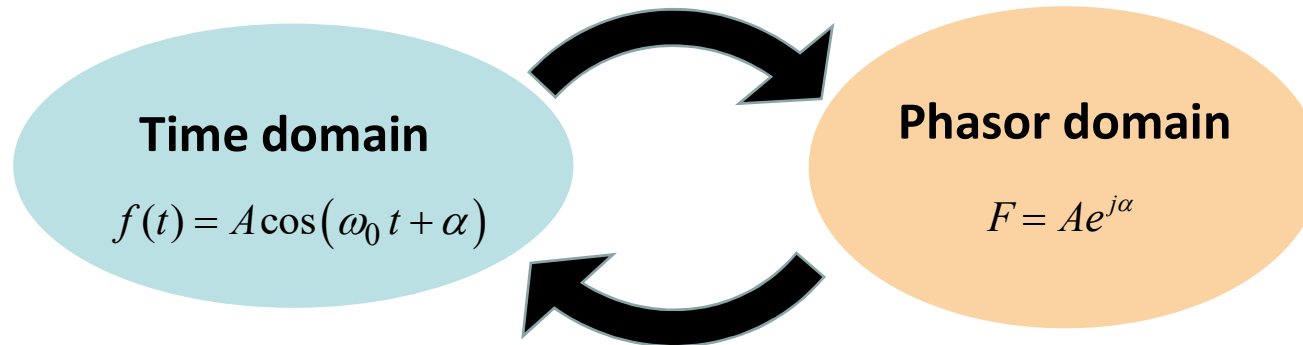
$\omega_0$  now is fixed!

# Phasors



- 1) How to jump back from the Phasor domain to the Time domain
- 2) Time domain derivative and Phasors

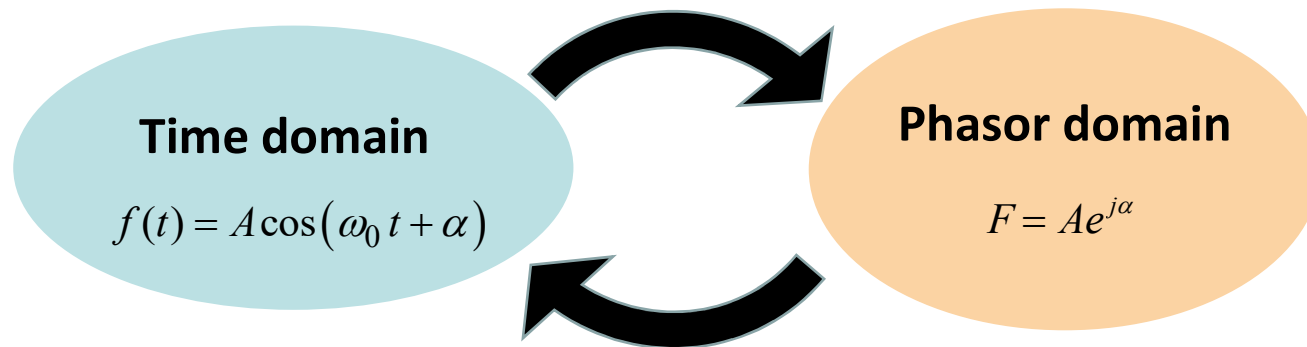
# Phasors



## 1) How to jump back from the Phasor domain to the Time domain

$$f(t) = \operatorname{Re}\{F e^{j\omega_0 t}\} = \operatorname{Re}\{A e^{j\alpha} e^{j\omega_0 t}\} = A \cos(\omega_0 t + \alpha)$$

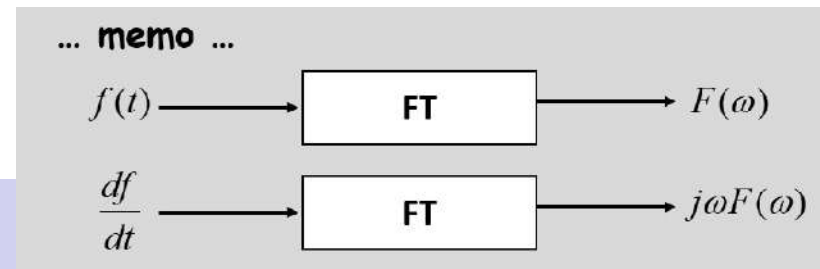
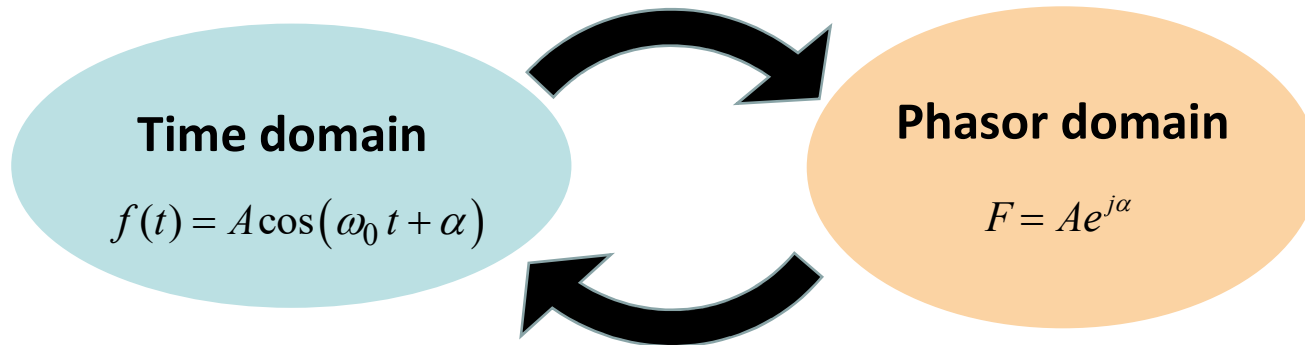
# Phasors



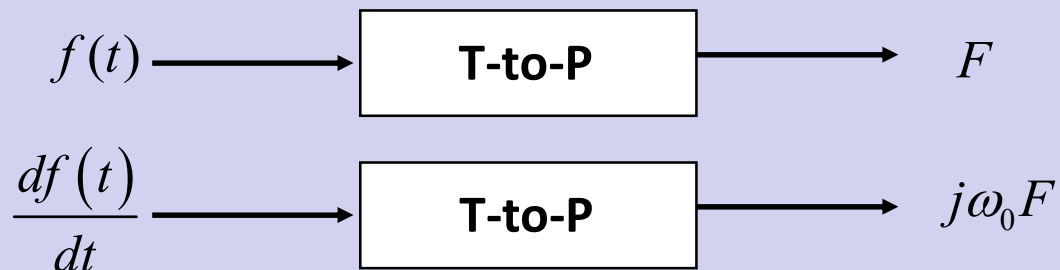
$$f(t) = A \cos(\omega_0 t + \alpha) \longrightarrow \text{T-to-P} \longrightarrow F = A e^{j\alpha}$$

$$F = A e^{j\alpha} \longrightarrow \text{P-to-T} \longrightarrow f(t) = \text{Re}\{F e^{j\omega_0 t}\} = A \cos(\omega_0 t + \alpha)$$

# Phasors



## 2) Time domain derivative and Phasors



$\omega_0$  now is fixed!

# Phasors

- Phasors and functions of  $n$  variables
- Phasors and vector functions
- Phasors and vector functions of  $n$  variables



# Phasors

- Phasors and functions of  $n$  variables
- Phasors and vector functions
- Phasors and vector functions of  $n$  variables

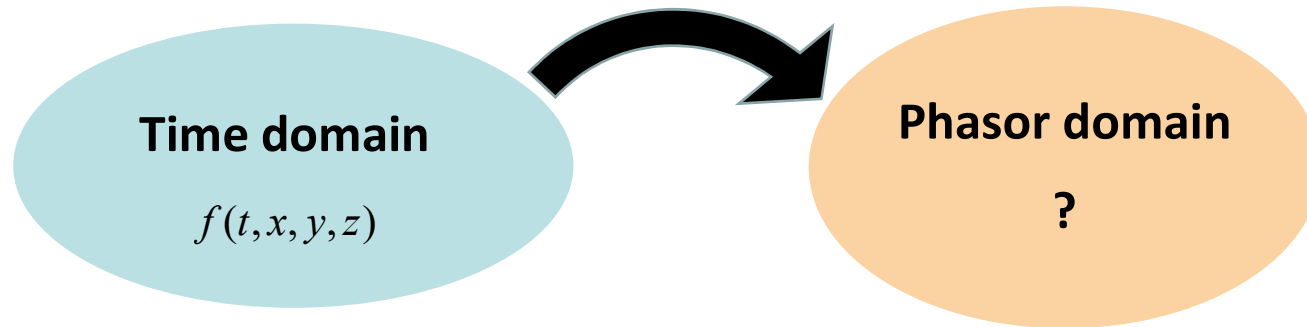
- 1) How to jump back from the Phasor domain to the Time domain
- 2) Time domain derivative and Phasors

# Phasors

- **Phasors and functions of  $n$  variables**
- Phasors and vector functions
- Phasors and vector functions of  $n$  variables

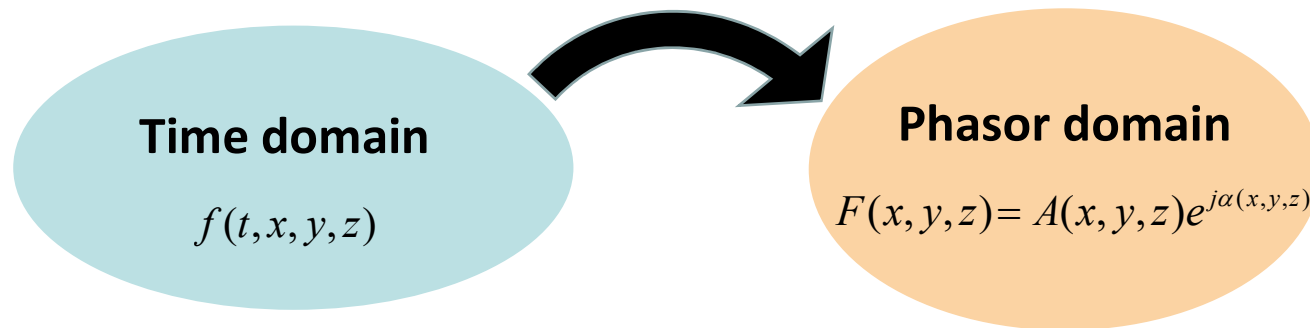
- 1) How to jump back from the Phasor domain to the Time domain**
- 2) Time domain derivative and Phasors**

# Phasors and functions of $n$ variables



$$f(t, x, y, z) = A(x, y, z) \cos(\omega_0 t + \alpha(x, y, z))$$

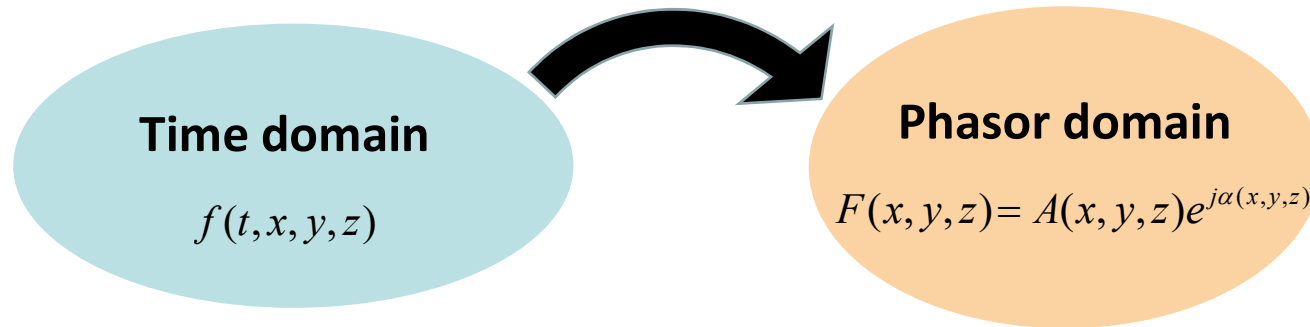
# Phasors and functions of $n$ variables



$$f(t, x, y, z) = A(x, y, z) \cos(\omega_0 t + \alpha(x, y, z))$$

$$F(x, y, z) = A(x, y, z)e^{j\alpha(x, y, z)}$$

# Phasors and functions of $n$ variables

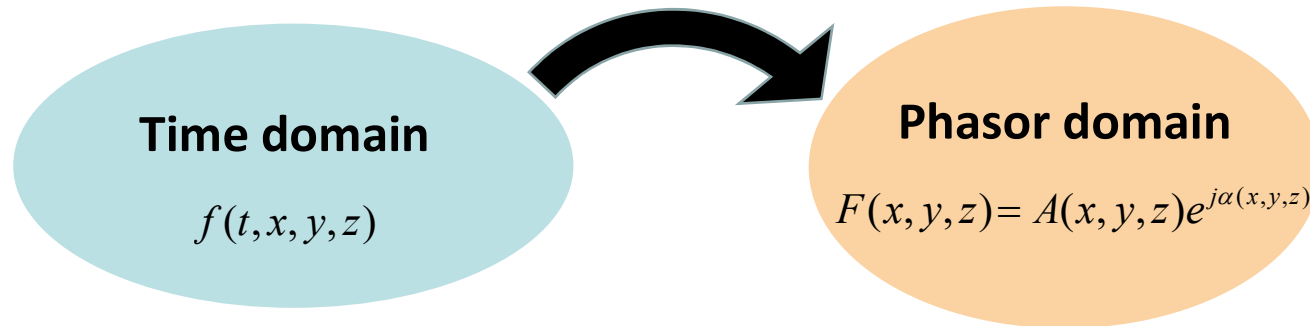


$$f(t, x, y, z) = A(x, y, z) \cos(\omega_0 t + \alpha(x, y, z))$$

$$F(x, y, z) = A(x, y, z)e^{j\alpha(x, y, z)}$$

## 1) How to jump back from the Phasor domain to the Time domain

# Phasors and functions of $n$ variables



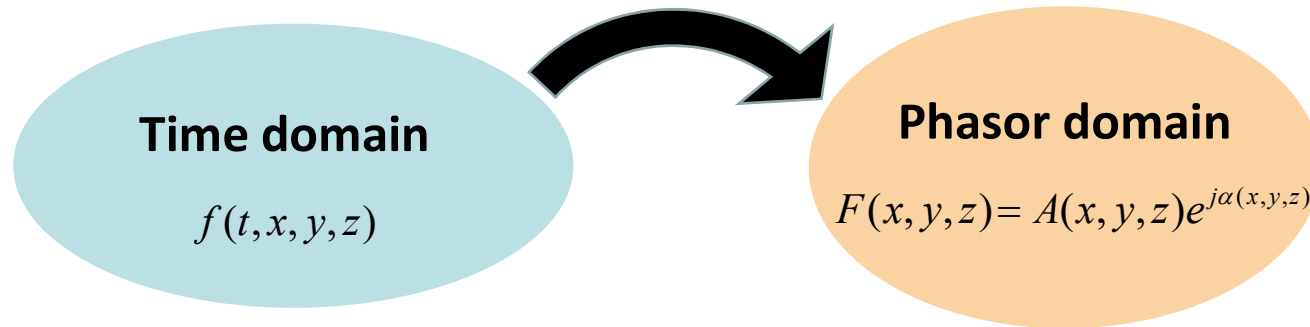
$$f(t, x, y, z) = A(x, y, z) \cos(\omega_0 t + \alpha(x, y, z))$$

$$F(x, y, z) = A(x, y, z)e^{j\alpha(x, y, z)}$$

## 1) How to jump back from the Phasor domain to the Time domain

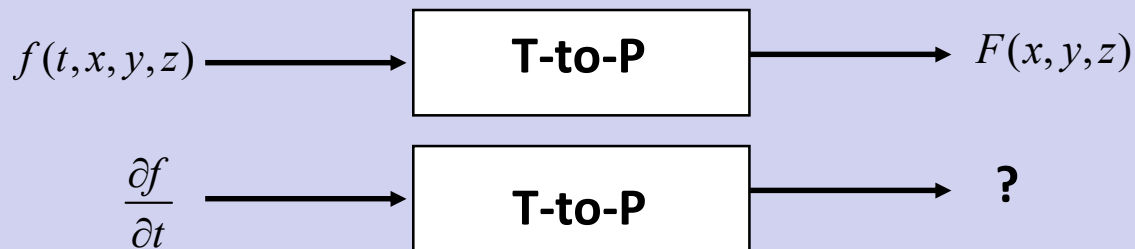
$$f(t, x, y, z) = \operatorname{Re}\{F(x, y, z)e^{j\omega_0 t}\}$$

# Phasors and functions of $n$ variables

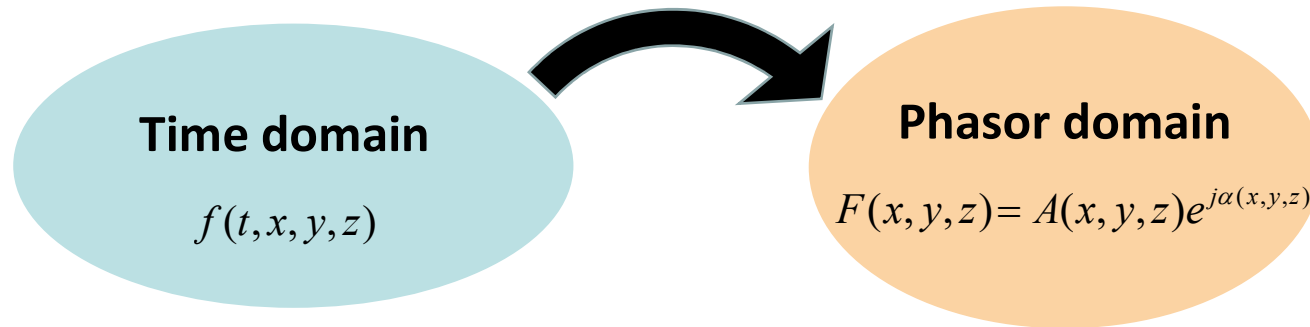


$$f(t, x, y, z) = A(x, y, z) \cos(\omega_0 t + \alpha(x, y, z))$$

## 2) Time domain derivative and Phasors

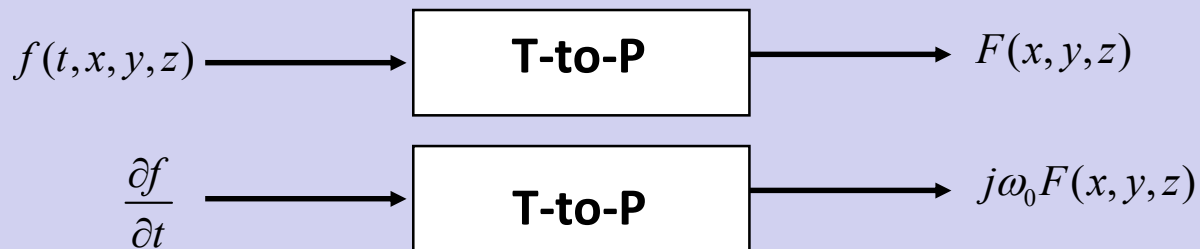


# Phasors and functions of $n$ variables



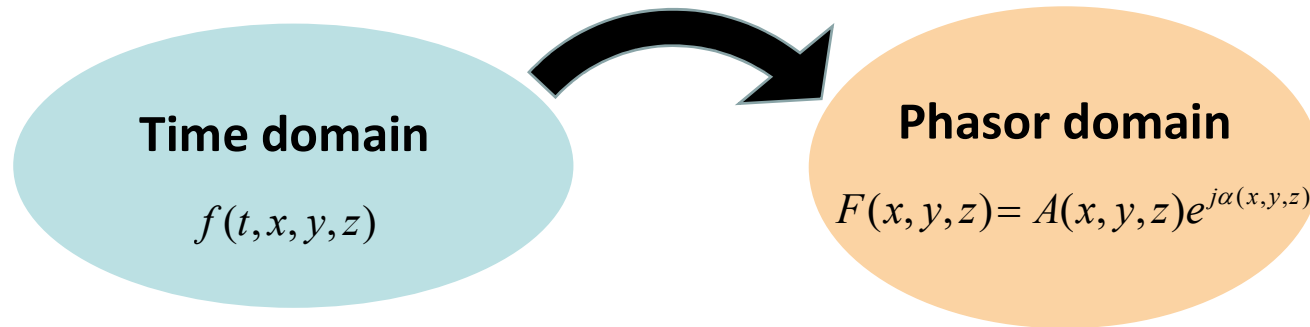
$$f(t, x, y, z) = A(x, y, z)\cos(\omega_0 t + \alpha(x, y, z))$$

## 2) Time domain derivative and Phasors



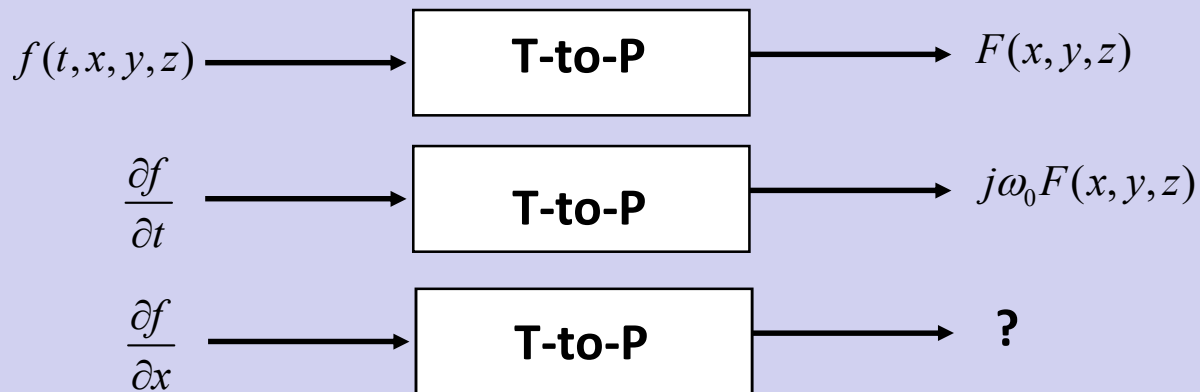


# Phasors and functions of $n$ variables

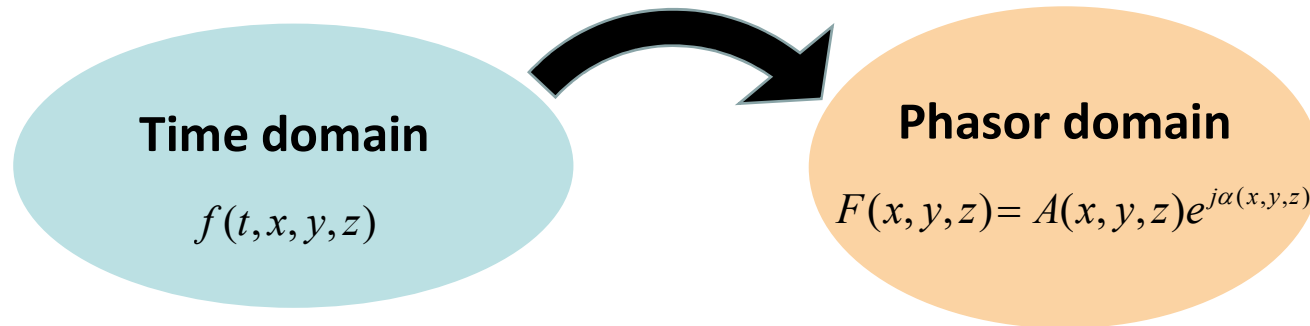


$$f(t, x, y, z) = A(x, y, z)\cos(\omega_0 t + \alpha(x, y, z))$$

## 2) Time domain derivative and Phasors

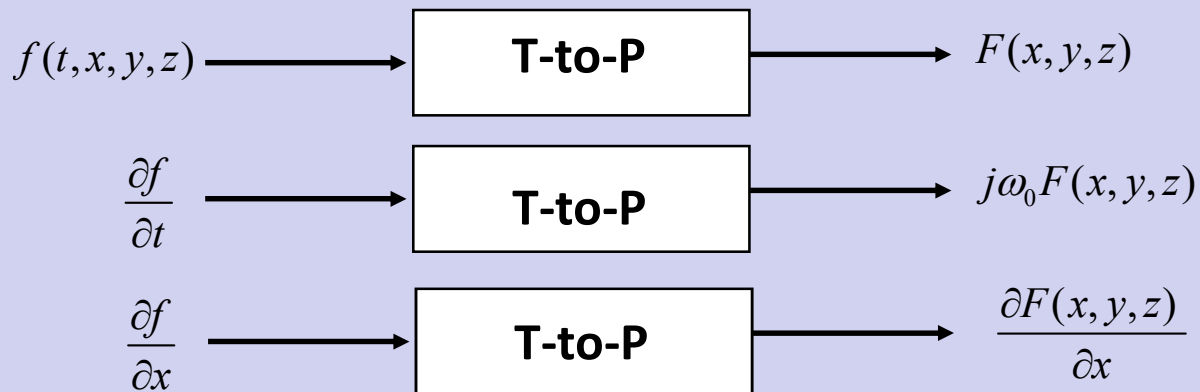


# Phasors and functions of $n$ variables



$$f(t, x, y, z) = A(x, y, z)\cos(\omega_0 t + \alpha(x, y, z))$$

## 2) Time domain derivative and Phasors

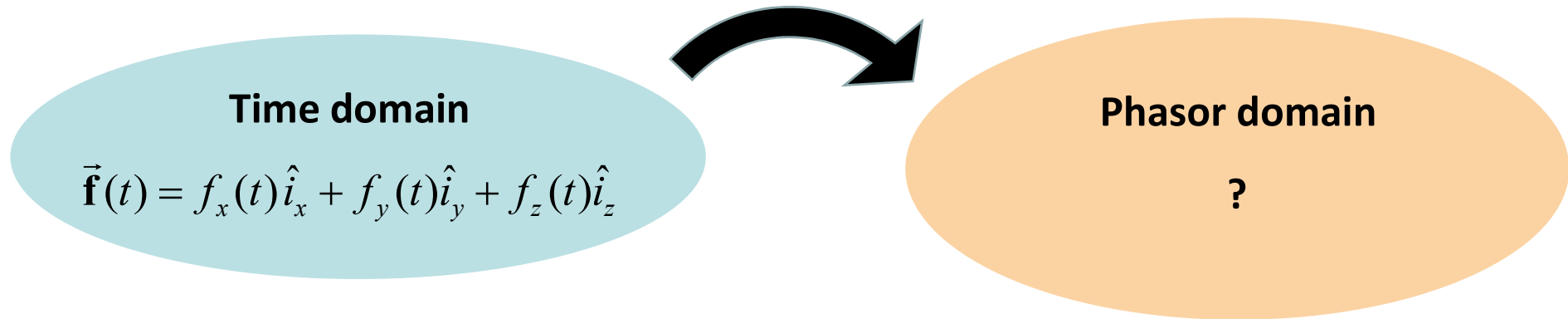


# Phasors

- Phasors and functions of  $n$  variables
- **Phasors and vector functions**
- Phasors and vector functions of  $n$  variables

- 1) How to jump back from the Phasor domain to the Time domain**
- 2) Time domain derivative and Phasors**

# Phasors and vector functions

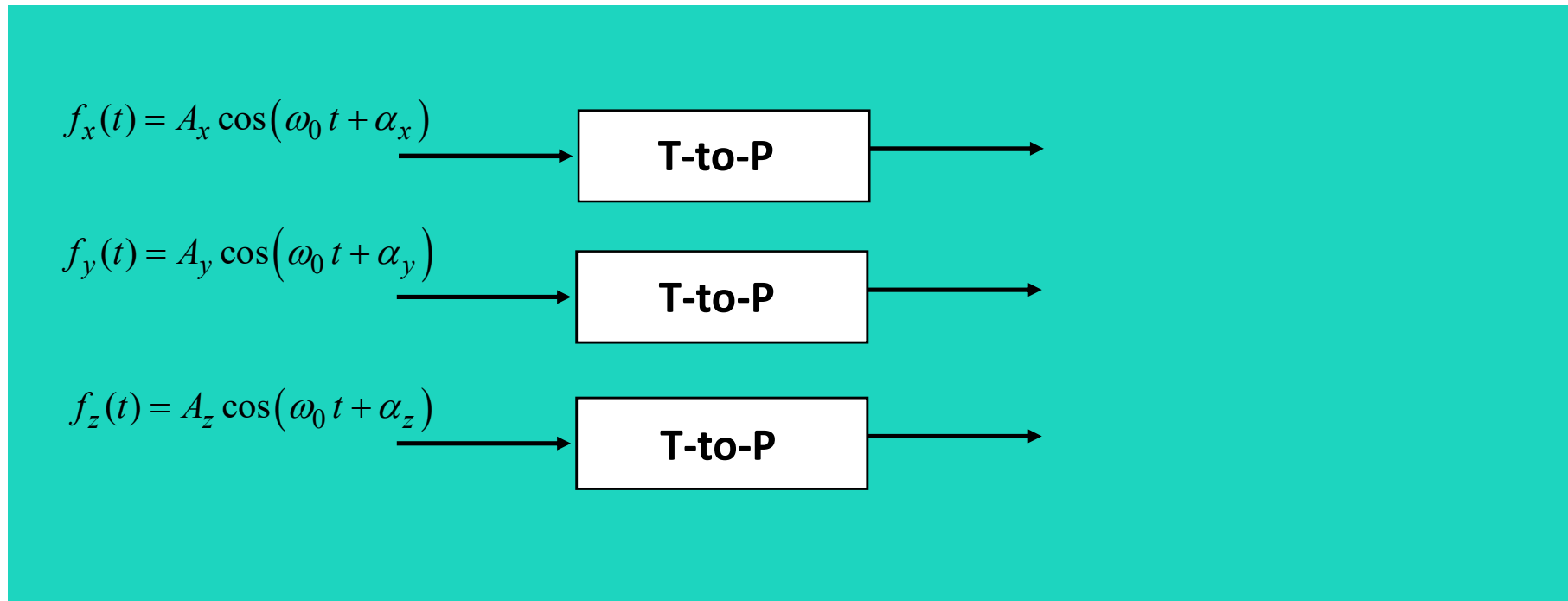
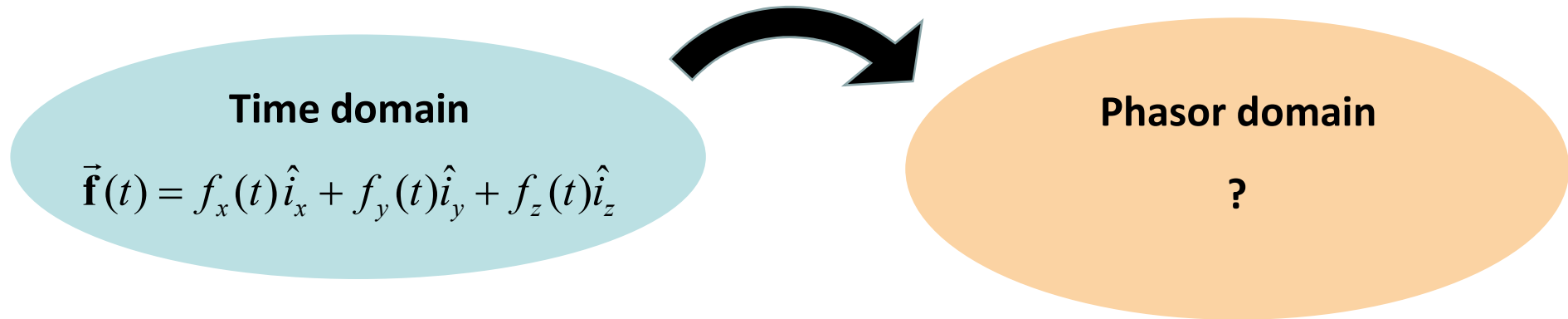


$$f_x(t) = A_x \cos(\omega_0 t + \alpha_x)$$

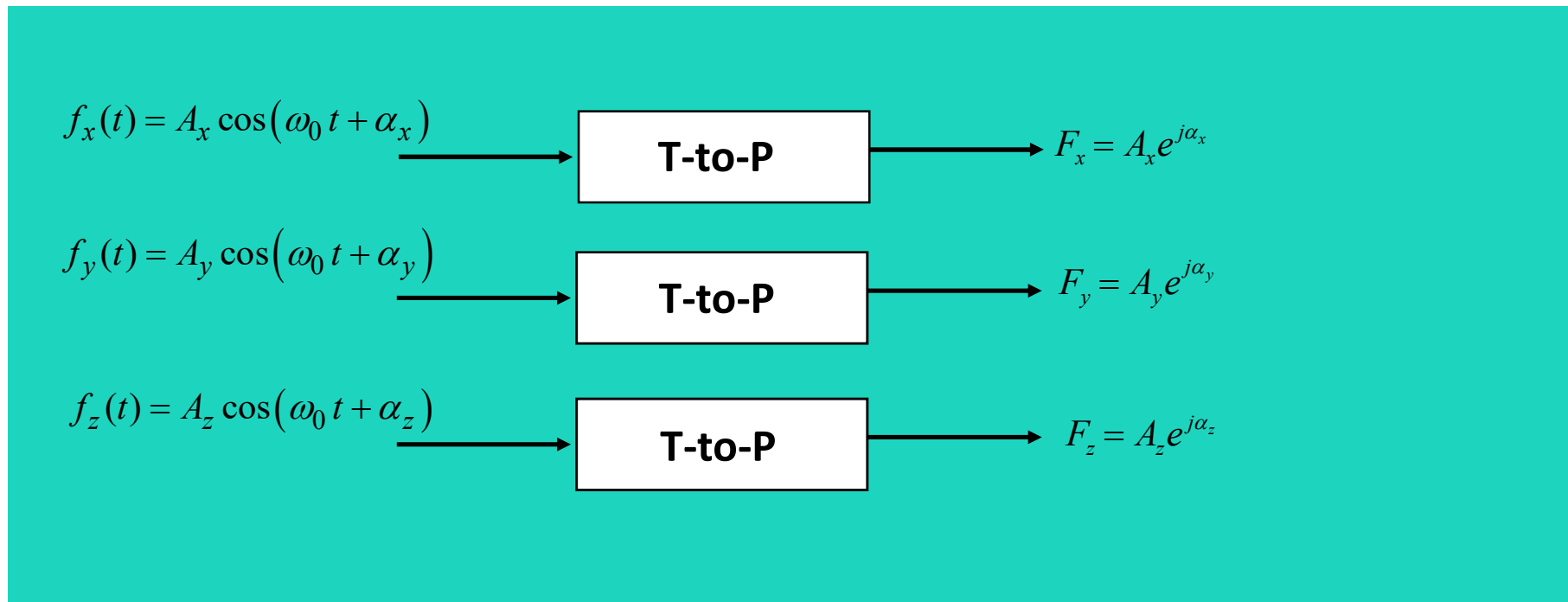
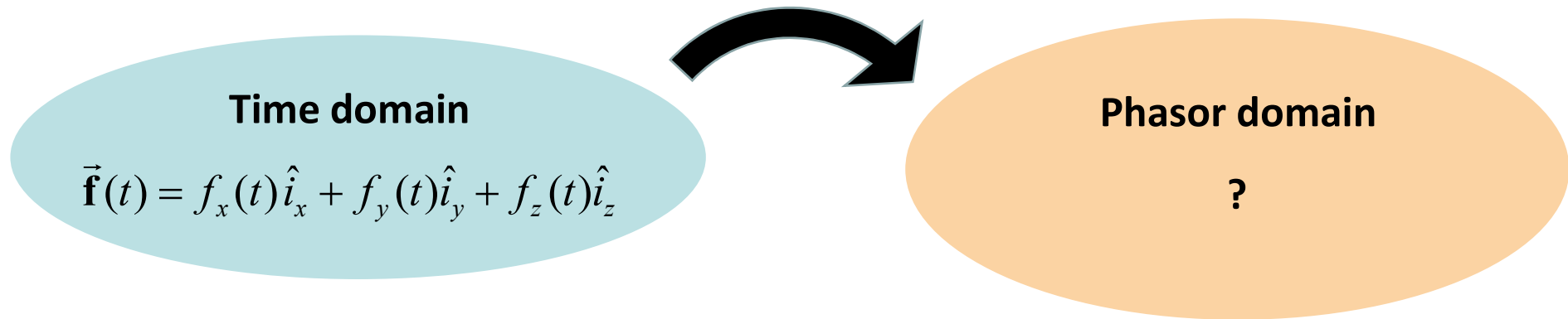
$$f_y(t) = A_y \cos(\omega_0 t + \alpha_y)$$

$$f_z(t) = A_z \cos(\omega_0 t + \alpha_z)$$

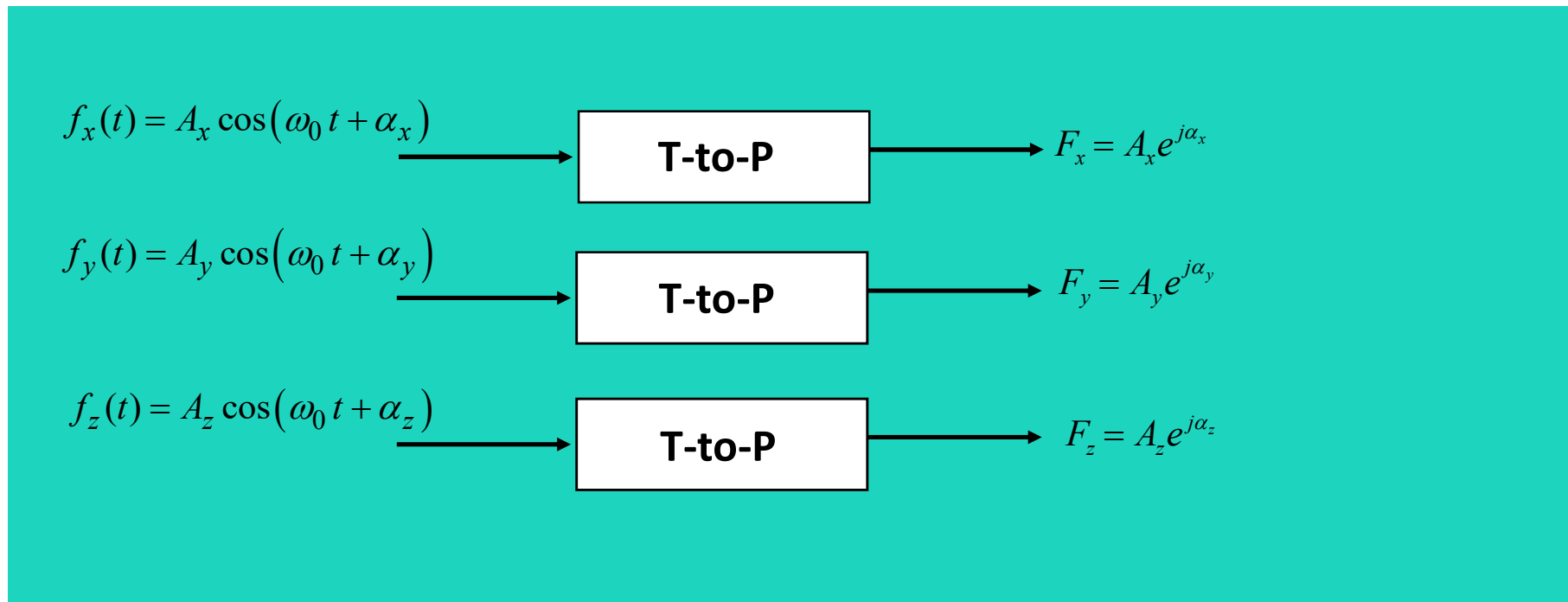
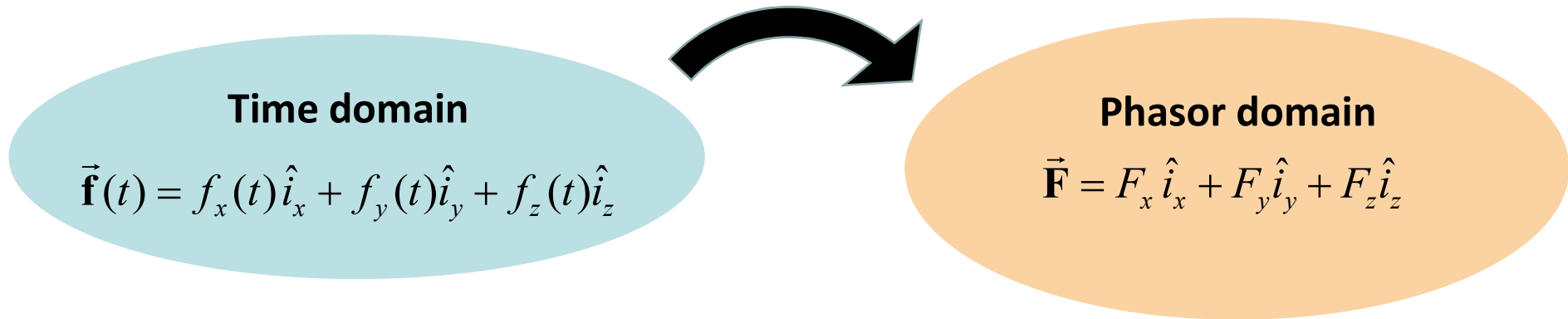
# Phasors and vector functions



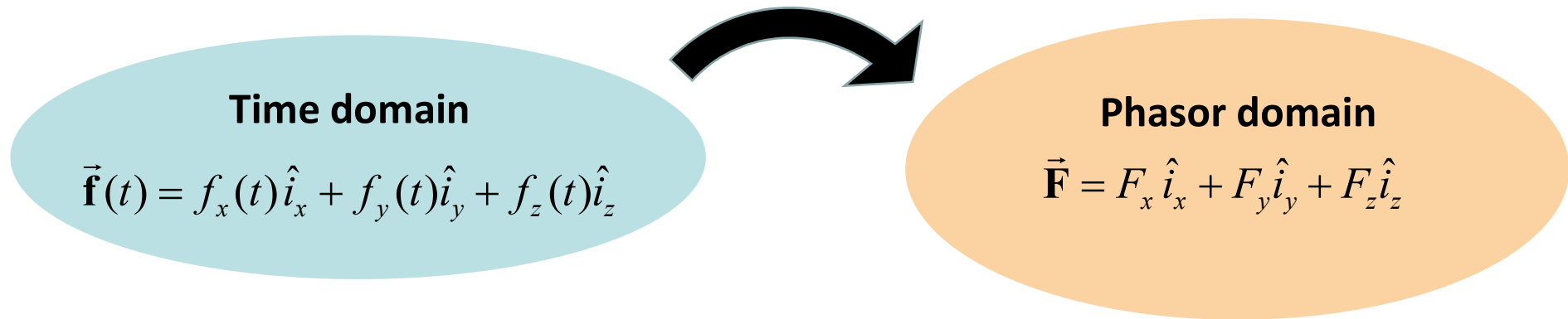
# Phasors and vector functions



# Phasors and vector functions



# Phasors and vector functions

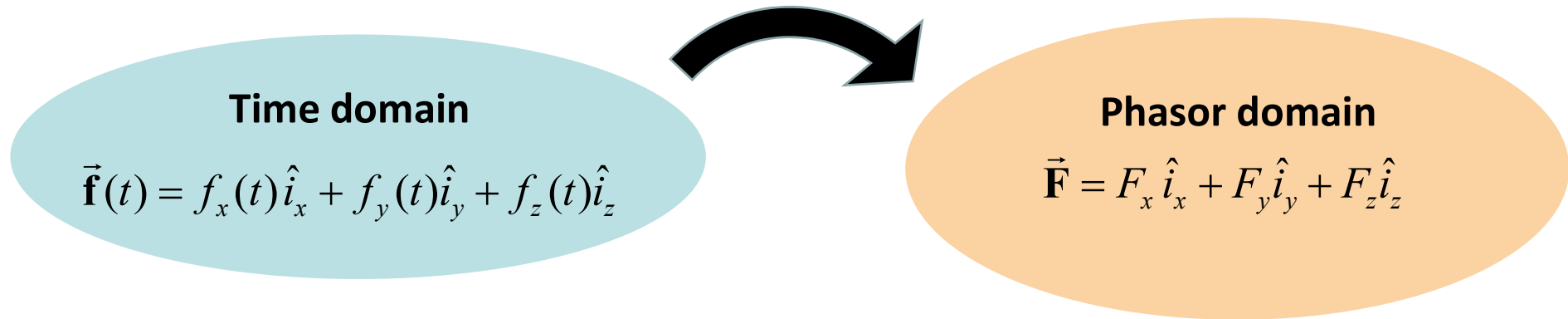


$$\vec{\mathbf{f}}(t) = f_x(t)\hat{i}_x + f_y(t)\hat{i}_y + f_z(t)\hat{i}_z = A_x \cos(\omega_0 t + \alpha_x)\hat{i}_x + A_y \cos(\omega_0 t + \alpha_y)\hat{i}_y + A_z \cos(\omega_0 t + \alpha_z)\hat{i}_z$$

$$\vec{\mathbf{F}} = F_x\hat{i}_x + F_y\hat{i}_y + F_z\hat{i}_z = A_x e^{j\alpha_x}\hat{i}_x + A_y e^{j\alpha_y}\hat{i}_y + A_z e^{j\alpha_z}\hat{i}_z$$

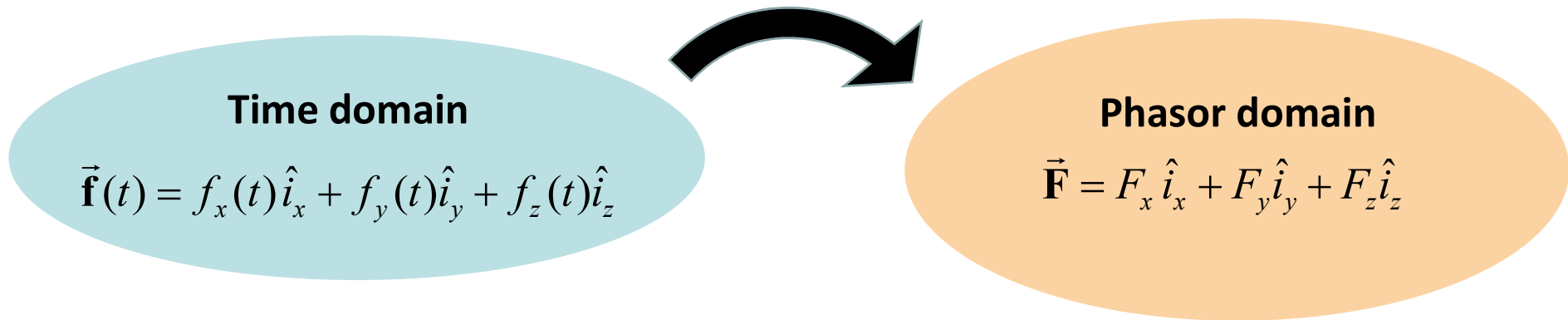


# Phasors and vector functions

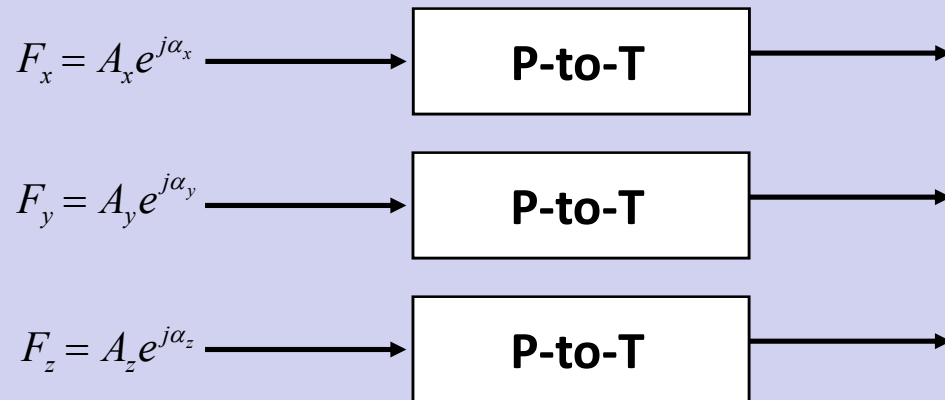


## 1) How to jump back from the Phasor domain to the Time domain

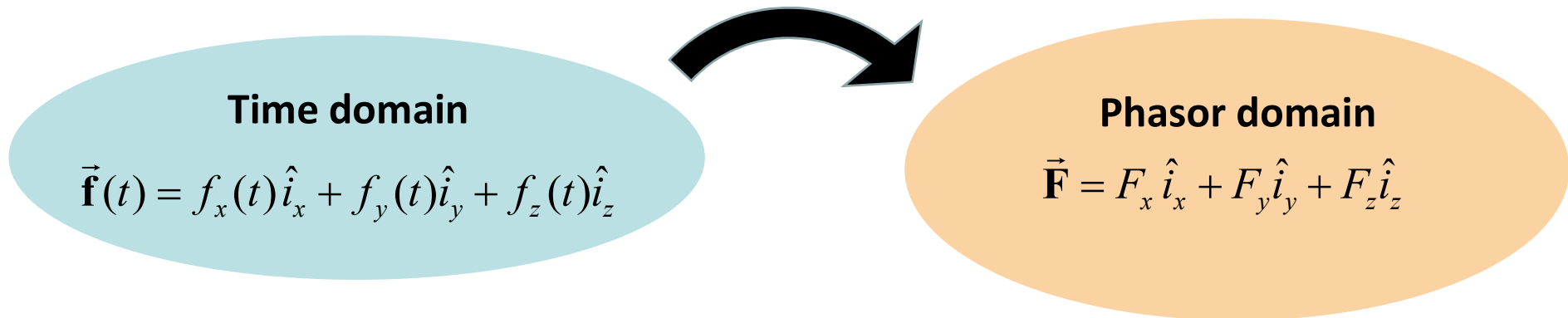
# Phasors and vector functions



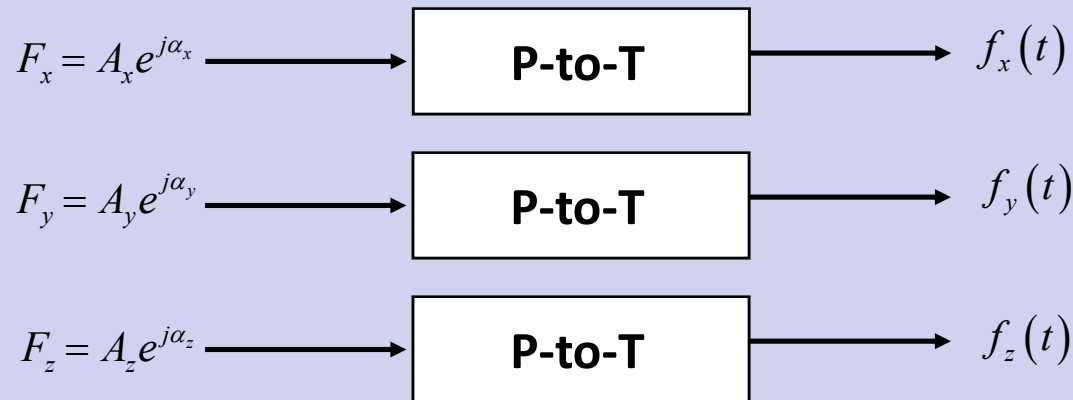
## 1) How to jump back from the Phasor domain to the Time domain



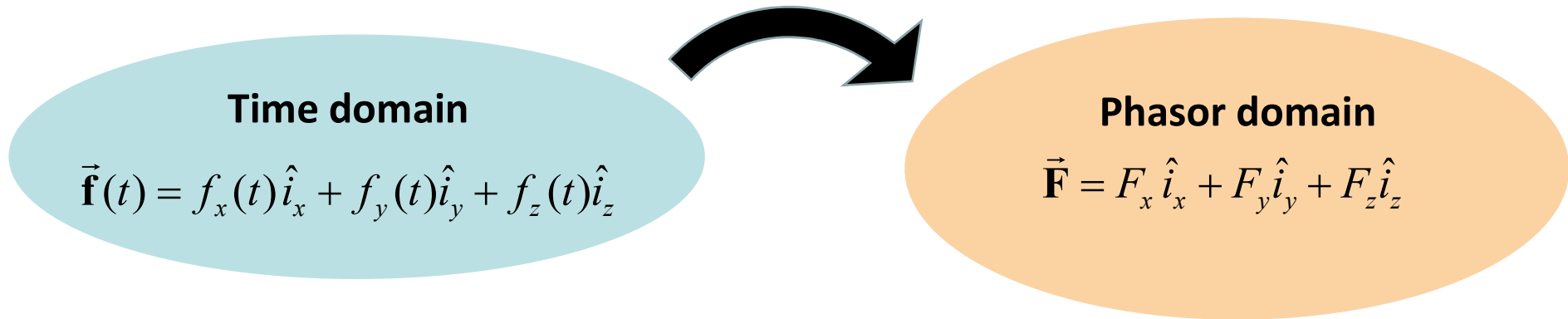
# Phasors and vector functions



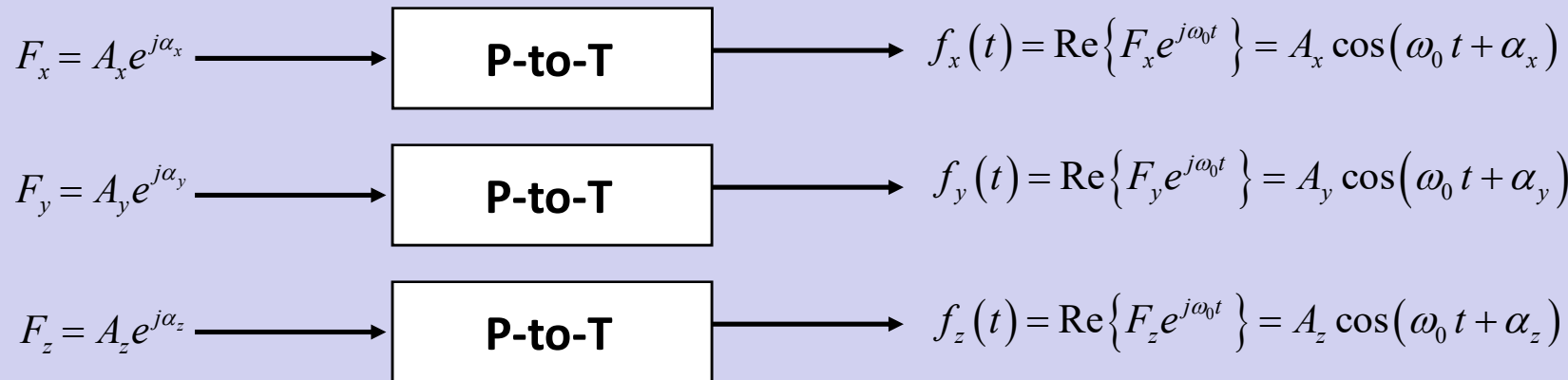
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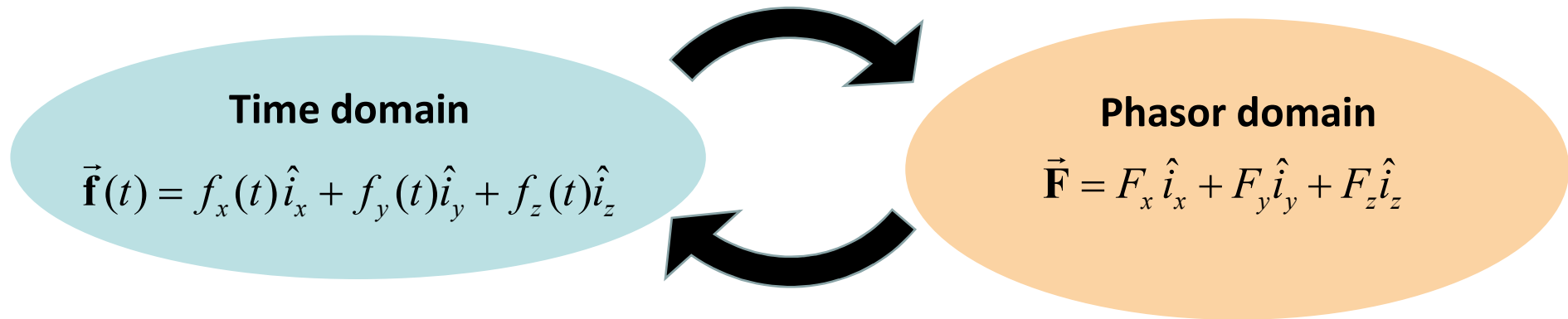
# Phasors and vector functions



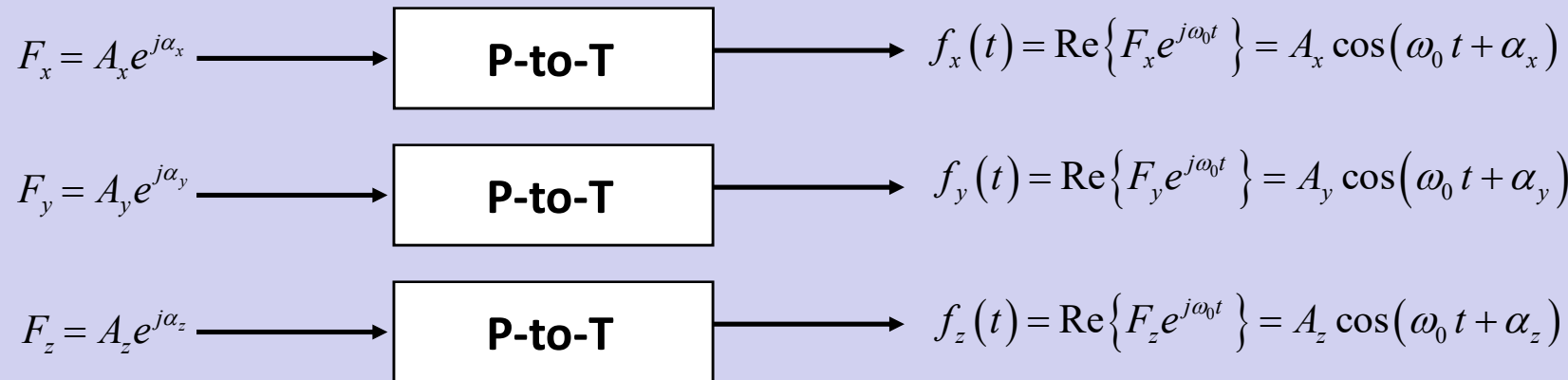
## 1) How to jump back from the Phasor domain to the Time domain



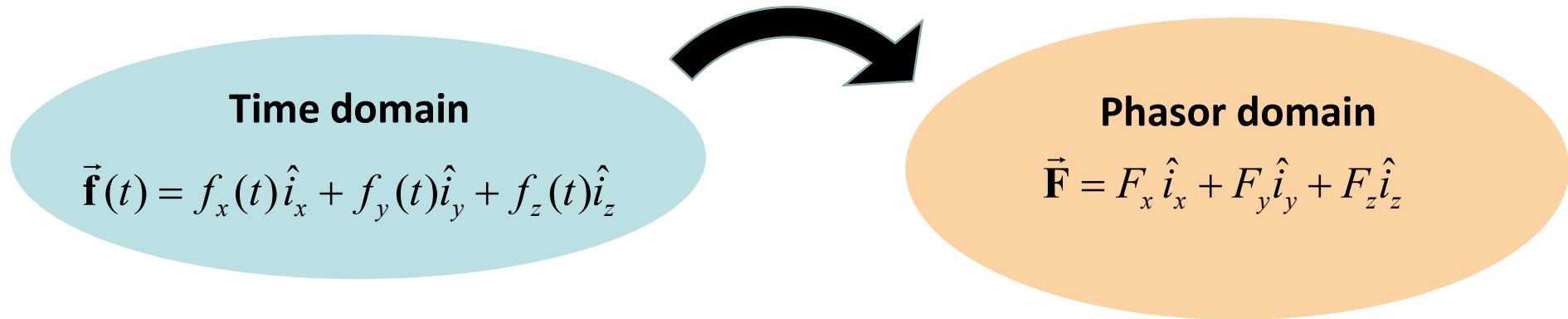
# Phasors and vector functions



## 1) How to jump back from the Phasor domain to the Time domain

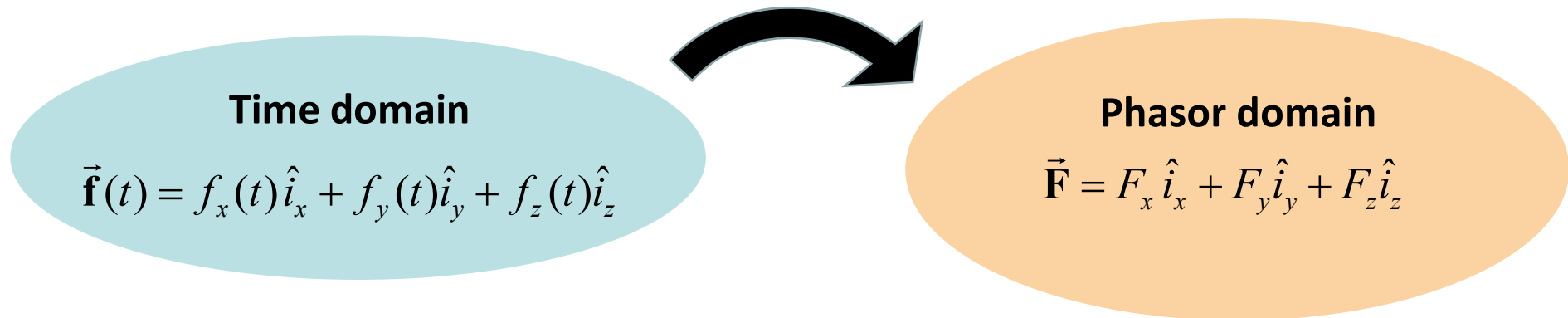


# Phasors and vector functions

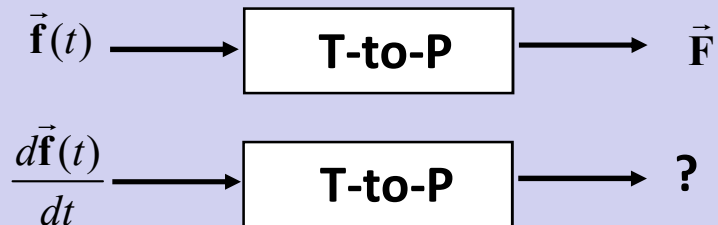


## 2) Time domain derivative and Phasors

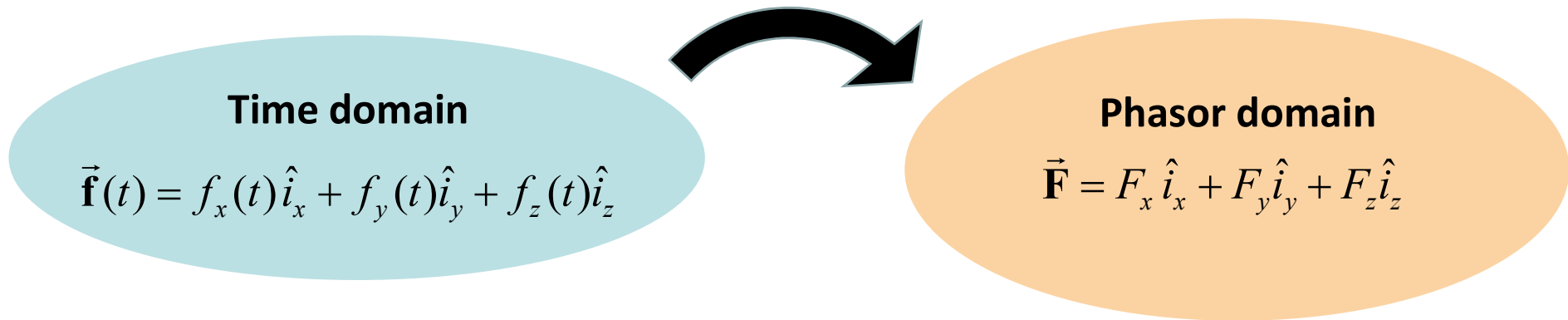
# Phasors and vector functions



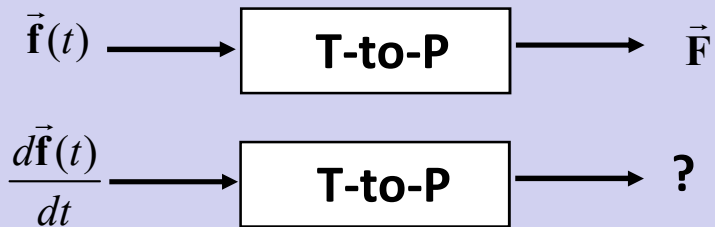
## 2) Time domain derivative and Phasors



# Phasors and vector functions



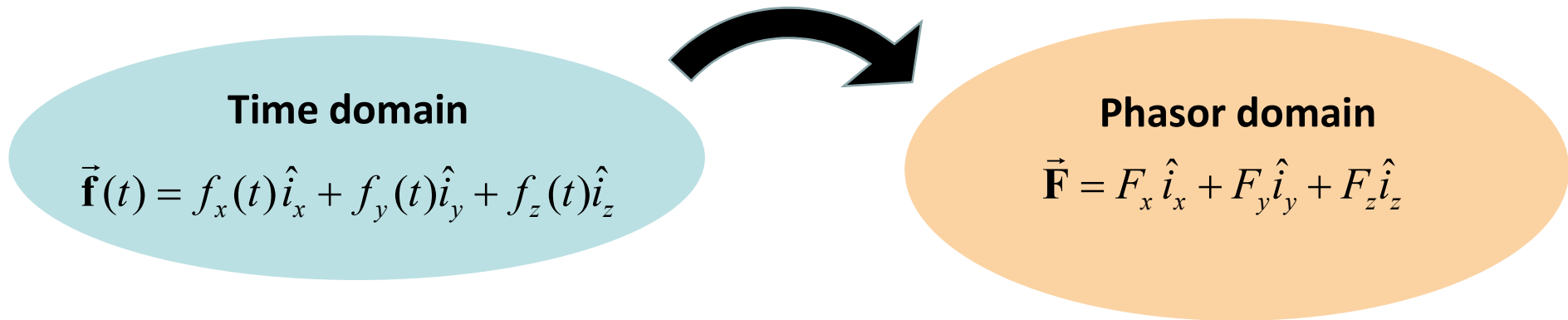
## 2) Time domain derivative and Phasors



$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$



# Phasors and vector functions



## 2) Time domain derivative and Phasors

$$\vec{\mathbf{f}}(t) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{F}}$$

$$\frac{d\vec{\mathbf{f}}(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow ?$$

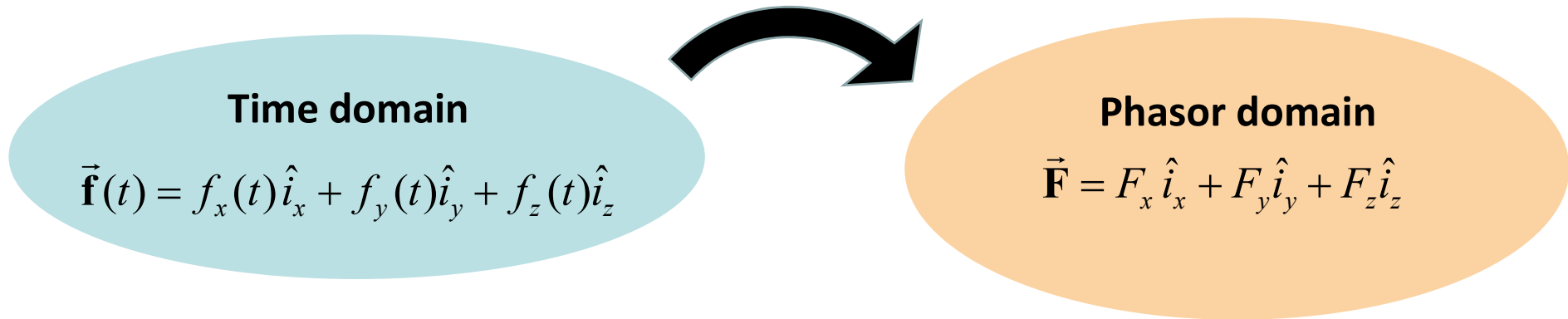
$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

$$\frac{df_x(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow ?$$

$$\frac{df_y(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow ?$$

$$\frac{df_z(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow ?$$

# Phasors and vector functions



## 2) Time domain derivative and Phasors

$$\vec{\mathbf{f}}(t) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{F}}$$

$$\frac{d\vec{\mathbf{f}}(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow ?$$

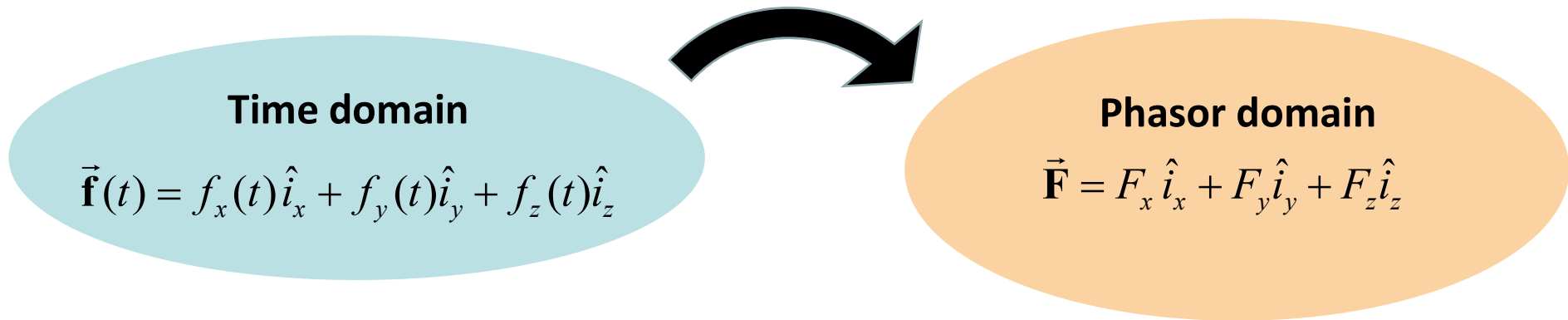
$$\frac{d\vec{\mathbf{f}}(t)}{dt} = \frac{df_x(t)}{dt}\hat{i}_x + \frac{df_y(t)}{dt}\hat{i}_y + \frac{df_z(t)}{dt}\hat{i}_z$$

$$\frac{df_x(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow j\omega_0 F_x$$

$$\frac{df_y(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow j\omega_0 F_y$$

$$\frac{df_z(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow j\omega_0 F_z$$

# Phasors and vector functions



## 2) Time domain derivative and Phasors

$$\vec{\mathbf{f}}(t) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{F}}$$

$$\frac{d\vec{\mathbf{f}}(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow j\omega_0 F_x \hat{i}_x + j\omega_0 F_y \hat{i}_y + j\omega_0 F_z \hat{i}_z$$

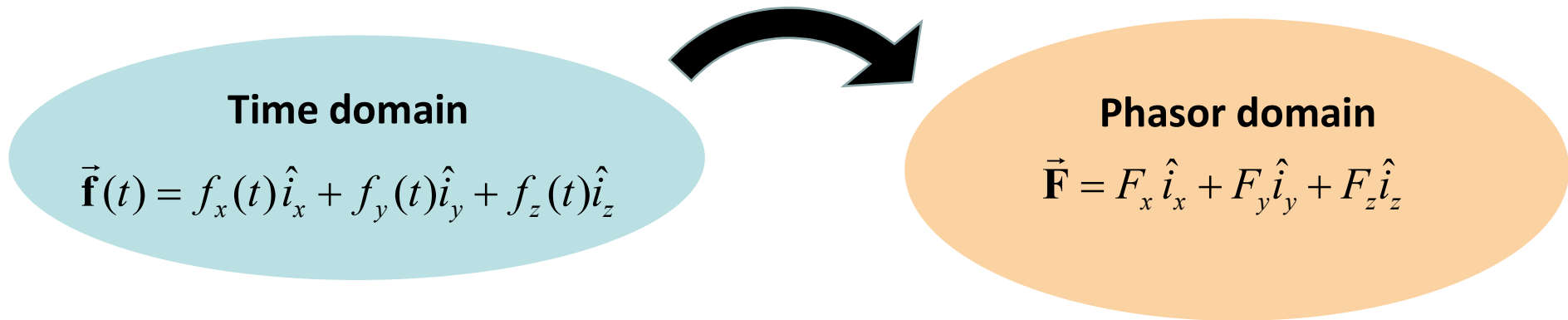
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# Phasors and vector functions



## 2) Time domain derivative and Phasors

$$\vec{\mathbf{f}}(t) \longrightarrow \boxed{\text{T-to-P}} \longrightarrow \vec{\mathbf{F}}$$

$$\frac{d\vec{\mathbf{f}}(t)}{dt} \longrightarrow \boxed{\text{T-to-P}} \longrightarrow j\omega_0\vec{\mathbf{F}} = j\omega_0F_x\hat{i}_x + j\omega_0F_y\hat{i}_y + j\omega_0F_z\hat{i}_z$$

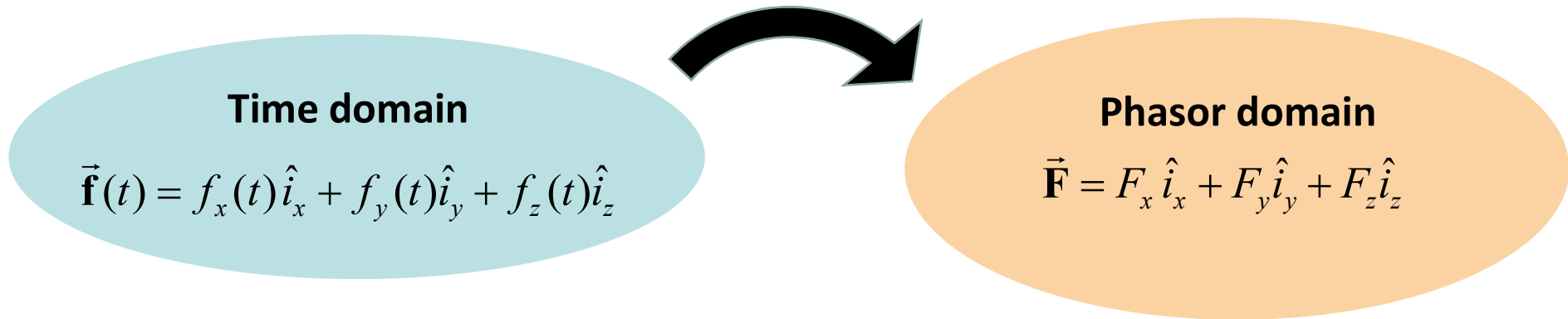
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# Phasors and vector functions



## 2) Time domain derivative and Phasors

