Campi Elettromagnetici

Corso di Laurea in Ingegneria Informatica, Biomedica e delle Telecomunicazioni

a.a. 2021-2022 - Laurea “Triennale” - Secondo semestre - Secondo anno

Università degli Studi di Napoli “Parthenope”

Stefano Perna

Riepilogo lezione precedente

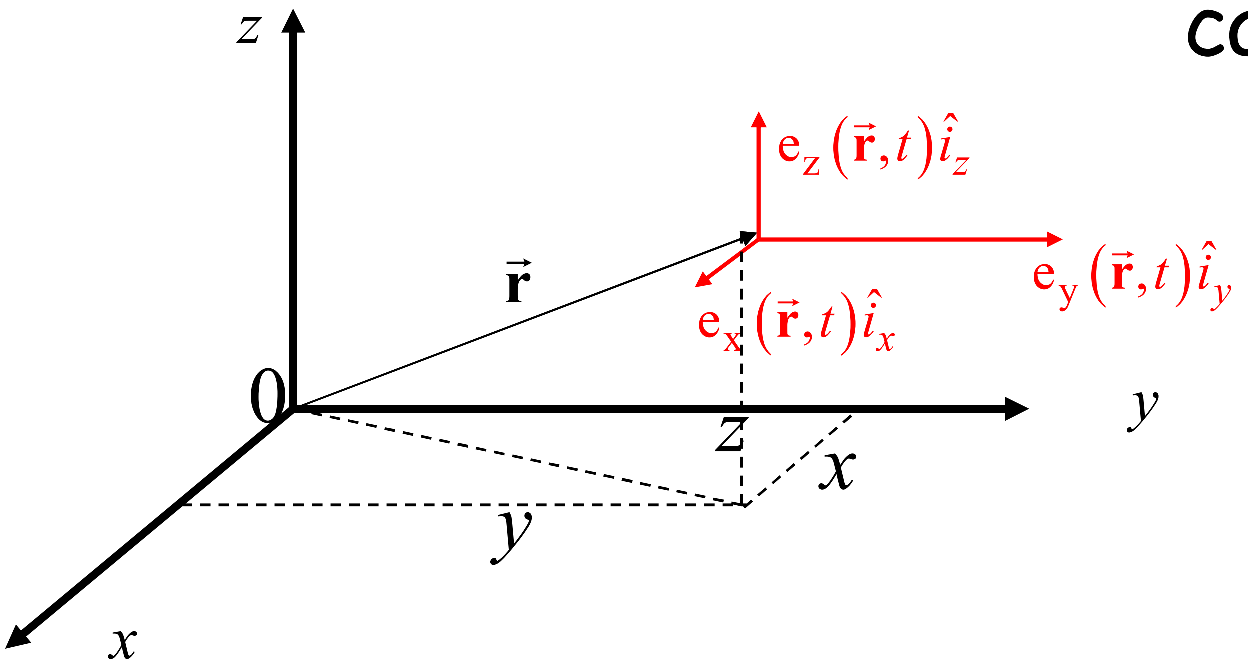
Perché si parla di campo?

Riepilogo lezione precedente

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(x,y,z,t) = \mathrm{e_x}(x,y,z,t)\hat{i}_x + \mathrm{e_y}(x,y,z,t)\hat{i}_y + \mathrm{e_z}(x,y,z,t)\hat{i}_z$$

$$\vec{\mathbf{r}} = (x,y,z)$$

Sistema di riferimento cartesiano

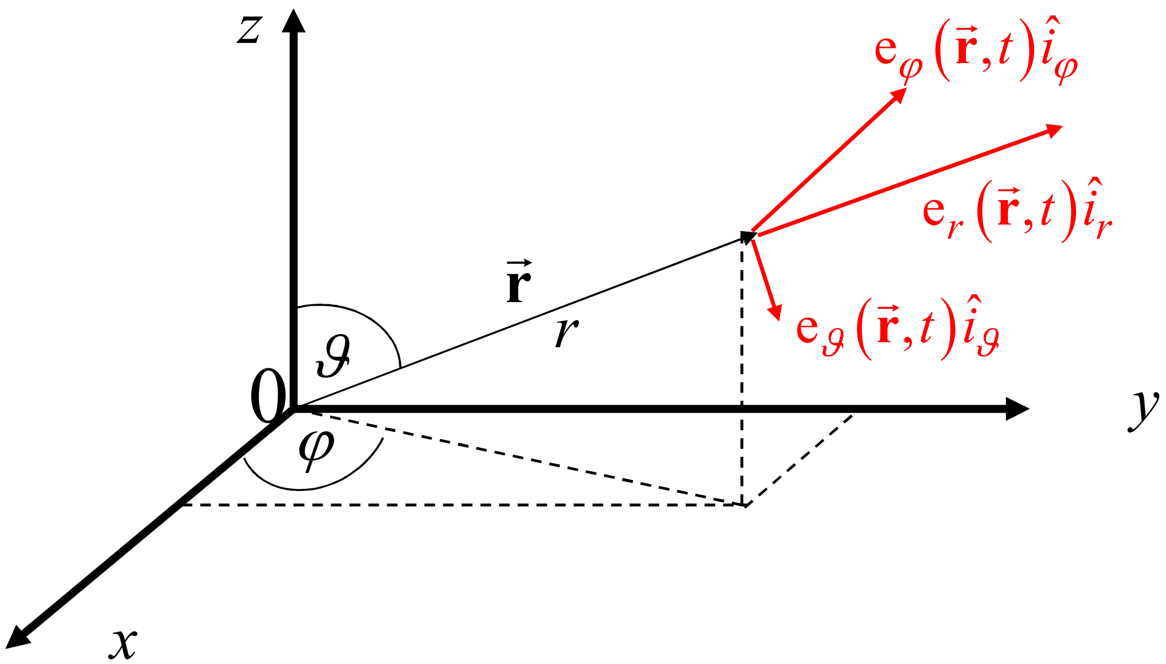


Riepilogo lezione precedente

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \vec{\mathbf{e}}(r,\vartheta,\varphi,t) = \mathrm{e}_r(r,\vartheta,\varphi,t)\hat{i}_r + \mathrm{e}_\vartheta(r,\vartheta,\varphi,t)\hat{i}_\vartheta + \mathrm{e}_\varphi(r,\vartheta,\varphi,t)\hat{i}_\varphi$$

$$\vec{\mathbf{r}} = (r,\vartheta,\varphi)$$

Sistema di riferimento sferico



Riepilogo lezione precedente

Il **<u>campo elettrico</u>** dipende dallo **<u>spazio</u>** e dal **<u>tempo</u>**

$$\vec{\mathbf{e}} = \vec{\mathbf{e}}(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \mathrm{e_x}(\vec{\mathbf{r}},t)\hat{i}_x + \mathrm{e_y}(\vec{\mathbf{r}},t)\hat{i}_y + \mathrm{e_z}(\vec{\mathbf{r}},t)\hat{i}_z$$

$$\vec{\mathbf{e}}(\vec{\mathbf{r}},t) = \mathrm{e}_r(\vec{\mathbf{r}},t)\hat{i}_r + \mathrm{e}_\vartheta(\vec{\mathbf{r}},t)\hat{i}_\vartheta + \mathrm{e}_\varphi(\vec{\mathbf{r}},t)\hat{i}_\varphi$$

Il **<u>campo magnetico</u>** dipende dallo **<u>spazio</u>** e dal **<u>tempo</u>**

$$\vec{\mathbf{h}} = \vec{\mathbf{h}}(\vec{\mathbf{r}},t)$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \mathrm{h_x}(\vec{\mathbf{r}},t)\hat{i}_x + \mathrm{h_y}(\vec{\mathbf{r}},t)\hat{i}_y + \mathrm{h_z}(\vec{\mathbf{r}},t)\hat{i}_z$$

$$\vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \mathrm{h}_r(\vec{\mathbf{r}},t)\hat{i}_r + \mathrm{h}_\vartheta(\vec{\mathbf{r}},t)\hat{i}_\vartheta + \mathrm{h}_\varphi(\vec{\mathbf{r}},t)\hat{i}_\varphi$$

Riepilogo lezione precedente

Perché si parla di campo?

Perché si parla di campo elettromagnetico?

Il campo elettrico che fine ha fatto?

Il campo magnetico che fine ha fatto?

Il campo elettrico e il campo magnetico sono legati?

Riepilogo lezione precedente

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

		Unità di misura
$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$:	Campo elettrico	Volt/m
$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)$:	Induzione elettrica	Coulomb/m^2
$\vec{\mathbf{h}}(\vec{\mathbf{r}},t)$:	Campo magnetico	Ampere/m
$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)$:	Induzione magnetica	Weber/m^2
$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$:	Densità di corrente	Ampere/m^2
$\rho(\vec{\mathbf{r}},t)$:	Densità di carica	Coulomb/m^3

Riepilogo lezione precedente

Perché si parla di campo?

Perché si parla di campo elettromagnetico?

Il campo elettrico che fine ha fatto?

Il campo magnetico che fine ha fatto?

Il campo elettrico e il campo magnetico sono legati?

Chi è la causa? Chi è l’effetto?

Riepilogo lezione precedente

..chi è la causa?

$$\begin{cases} \vec{\mathbf{j}}(\vec{\mathbf{r}},t): \text{densità di corrente della sorgente} \\ \rho(\vec{\mathbf{r}},t): \text{densità di carica della sorgente} \end{cases}$$

...chi è l'effetto?

$$\begin{cases} \vec{\mathbf{e}}(\vec{\mathbf{r}},t): \text{campo elettrico; } \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \text{ induzione elettrica} \\ \vec{\mathbf{h}}(\vec{\mathbf{r}},t): \text{campo magnetico; } \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \text{ induzione magnetica} \end{cases}$$

Riepilogo lezione precedente

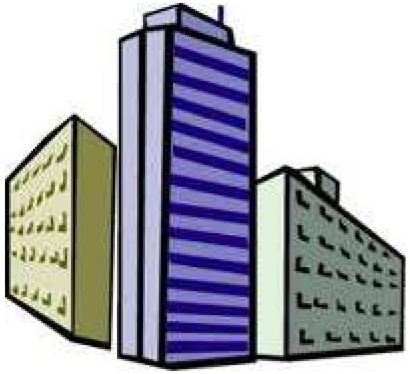
$$
\begin{cases}
\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\
\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}_0(\vec{\mathbf{r}},t) + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho_0(\vec{\mathbf{r}},t) + \rho(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0
\end{cases}
$$

$$
\begin{cases}
\vec{\mathbf{j}}_0(\vec{\mathbf{r}},t): \text{densità di corrente} \\
\rho_0(\vec{\mathbf{r}},t): \text{densità di carica}
\end{cases}
$$

Sorgenti impresse

$$
\begin{cases}
\vec{\mathbf{j}}(\vec{\mathbf{r}},t): \text{densità di corrente} \\
\rho(\vec{\mathbf{r}},t): \text{densità di carica}
\end{cases}
$$

Sorgenti indotte



… scenario più complicato …

Equazioni di Maxwell

$$
\begin{cases}
\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\
\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0
\end{cases}
$$

		Unità di misura
$\vec{\mathbf{e}}(\vec{\mathbf{r}},t)$:	Campo elettrico	Volt/m
$\vec{\mathbf{d}}(\vec{\mathbf{r}},t)$:	Induzione elettrica	Coulomb/m^2
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$\vec{\mathbf{b}}(\vec{\mathbf{r}},t)$:	Induzione magnetica	Weber/m^2
$\vec{\mathbf{j}}(\vec{\mathbf{r}},t)$:	Densità di corrente	Ampere/m^2
$\rho(\vec{\mathbf{r}},t)$:	Densità di carica	Coulomb/m^3

Color legend

New formulas, important considerations,
important formulas, important concepts

Very important for the discussion

Memo

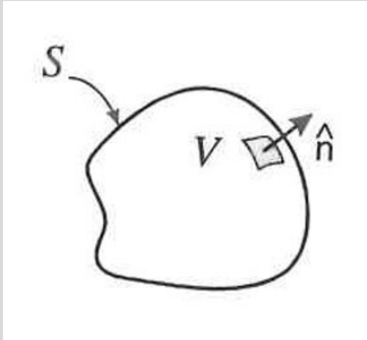
Mathematical tools to be exploited

Mathematics

… memo…

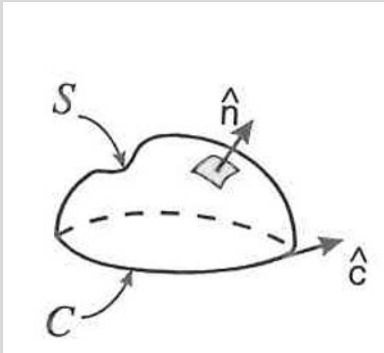
I) Divergence

$$\nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \lim_{V \to 0} \frac{1}{V} \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$



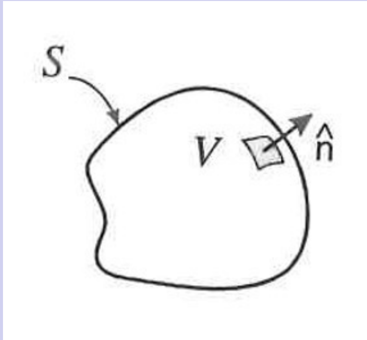
II) Curl

$$\left(\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}})\right) \cdot \hat{\mathbf{n}} = \lim_{S \to 0} \frac{1}{S} \oint_C dc\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{c}}$$



Divergence

$$\nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \lim_{V \to 0} \frac{1}{V} \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$



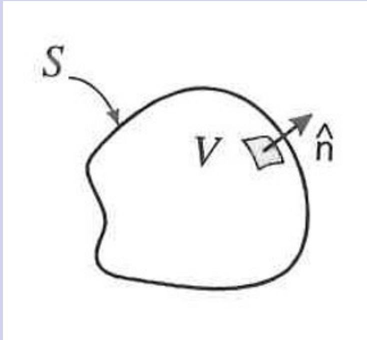
- Scalar quantity

- Its value **DOES NOT DEPEND** on the coordinate system we have chosen

- Its analytical expression **DEPENDS** on the coordinate system we have chosen

Divergence

$$\nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \lim_{V \to 0} \frac{1}{V} \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

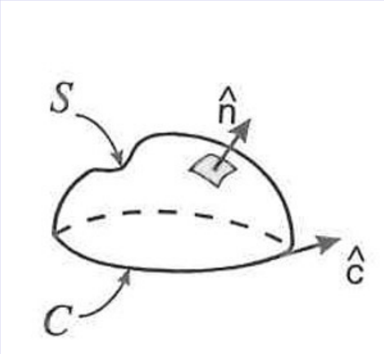


Gauss theorem

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

Curl

$$\left(\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}})\right) \cdot \hat{\mathbf{n}} = \lim_{S \to 0} \frac{1}{S} \oint_C dc\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{c}}$$



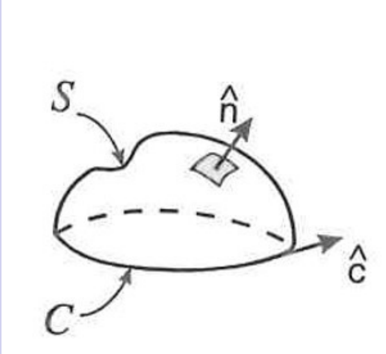
- Vector quantity

- Its value **DOES NOT DEPEND** on the coordinate system we have chosen

- Its analytical expression **DEPENDS** on the coordinate system we have chosen

Curl

$$\left(\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right)\cdot\hat{\mathbf{n}} = \lim_{S\to 0}\frac{1}{S}\oint_C dc\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{\mathbf{c}}$$



Stokes theorem

$$\iint_S dS\left(\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right)\cdot\hat{\mathbf{n}} = \oint_C dc\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{\mathbf{c}}$$

Cartesian Coordinates

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t) = A_x(x,y,z,t)\hat{i}_x + A_y(x,y,z,t)\hat{i}_y + A_z(x,y,z,t)\hat{i}_z$$

$$\nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{i}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{i}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{i}_z$$

Spherical Coordinates

$$\vec{\mathbf{A}}(\vec{\mathbf{r}},t)=A_r(r,\vartheta,\varphi,t)\hat{i}_r+A_\vartheta(r,\vartheta,\varphi,t)\hat{i}_\vartheta+A_\varphi(r,\vartheta,\varphi,t)\hat{i}_\varphi$$

$$\nabla\cdot\vec{\mathbf{A}}(\vec{\mathbf{r}})=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2A_r\right)+\frac{1}{r\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta A_\vartheta\right)+\frac{1}{r\sin\vartheta}\frac{\partial A_\varphi}{\partial\varphi}$$

$$\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})=\frac{1}{r\sin\vartheta}\left[\frac{\partial}{\partial\vartheta}\left(\sin\vartheta A_\varphi\right)-\frac{\partial A_\vartheta}{\partial\varphi}\right]\hat{i}_r+\frac{1}{r}\left[\frac{1}{\sin\vartheta}\frac{\partial A_r}{\partial\varphi}-\frac{\partial}{\partial r}\left(rA_\varphi\right)\right]\hat{i}_\vartheta+\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_\vartheta\right)-\frac{\partial A_r}{\partial\vartheta}\right]\hat{i}_\varphi$$

Color legend

New formulas, important considerations, important formulas, important concepts

Very important for the discussion

Memo

Mathematical tools to be exploited

Mathematics

Maxwell equations

James Clerk Maxwell 1831-1879

Differential form

$$
\begin{cases}
\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\
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Maxwell equations

James Clerk Maxwell 1831-1879

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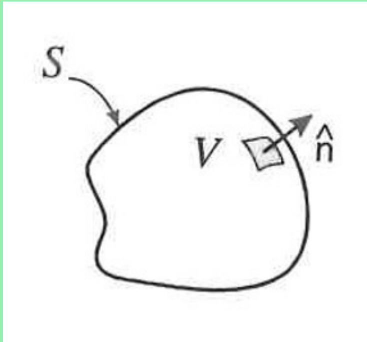
Maxwell equations: integral form

… mathematical tools that we will exploit today…

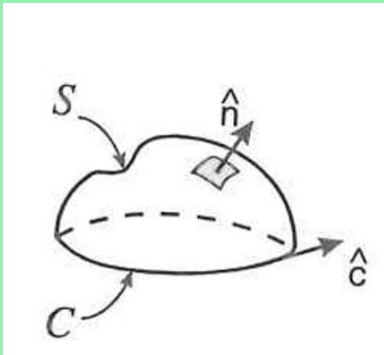
I) Gauss theorem

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$



II) Stokes theorem

$$\iint_S dS \left(\nabla \times \vec{\mathbf{A}}(\vec{\mathbf{r}})\right) \cdot \hat{\mathbf{n}} = \oint_C dc\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{c}}$$



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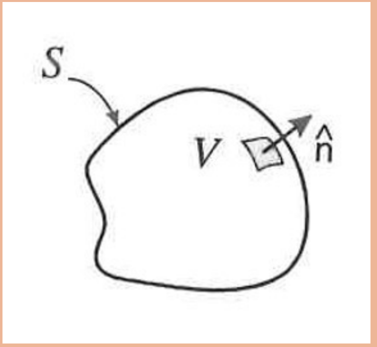

II) Stokes theorem

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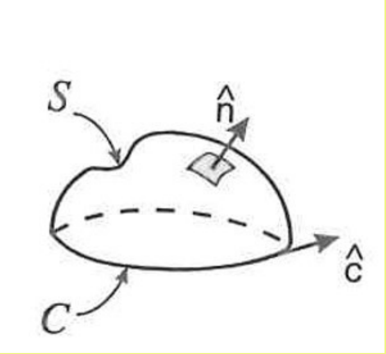
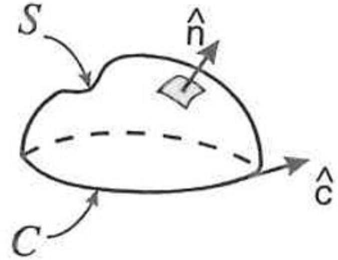


Maxwell equations: integral form

$$\nabla \times \vec{\mathbf{e}}\left(\vec{\mathbf{r}},t\right) = -\frac{\partial \vec{\mathbf{b}}\left(\vec{\mathbf{r}},t\right)}{\partial t}$$

Maxwell equations: integral form



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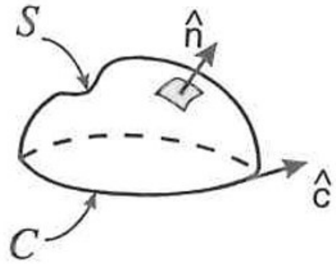
Maxwell equations: integral form

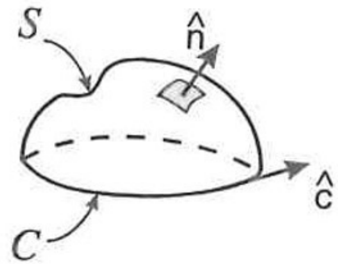


$$\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \quad \Rightarrow \quad \iint_S dS \left(\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right) \cdot \hat{\mathbf{n}} = -\iint_S dS \frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \cdot \hat{\mathbf{n}}$$

Maxwell equations: integral form

Stokes Theorem

$$\iint_S dS\left(\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right)\cdot\hat{n}=\oint_C dc\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{c}$$

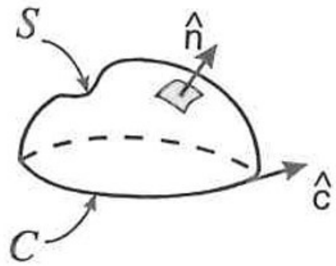


$$\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=-\frac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}\quad\Rightarrow\quad\iint_S dS\left(\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right)\cdot\hat{\mathbf{n}}=-\iint_S dS\,\frac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}\cdot\hat{\mathbf{n}}$$

Maxwell equations: integral form

Stokes Theorem

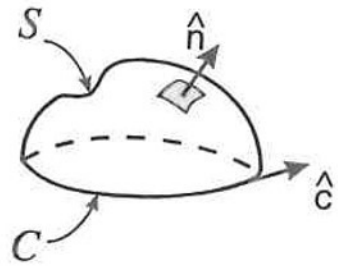
$$\iint_S dS\left(\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right)\cdot\hat{n}=\oint_C dc\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{c}$$



$$\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=-\frac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \Rightarrow \iint_S dS\left(\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right)\cdot\hat{\mathbf{n}}=-\iint_S dS\,\frac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}\cdot\hat{\mathbf{n}}$$

$$\oint_C dc\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}} \quad =$$

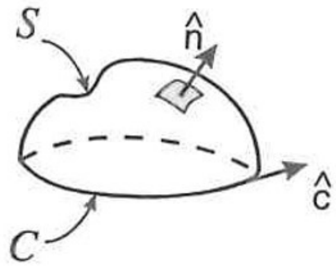
Maxwell equations: integral form



$$\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)=-\frac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \quad\Rightarrow\quad \iint_S dS\left(\nabla\times\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\right)\cdot\hat{\mathbf{n}} = -\iint_S dS\,\frac{\partial\vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}\cdot\hat{\mathbf{n}}$$

$$\oint_C dc\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}} = -\frac{d}{dt}\iint_S dS\,\vec{\mathbf{b}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}$$

Maxwell equations: integral form



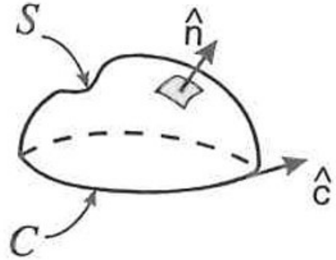
$$\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}$$

➡

Lenz-Neumann law

$$\oint_C dc\,\vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}} = -\frac{d}{dt}\iint_S dS\,\vec{\mathbf{b}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}$$

Maxwell equations: integral form



$$\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}$$

➡

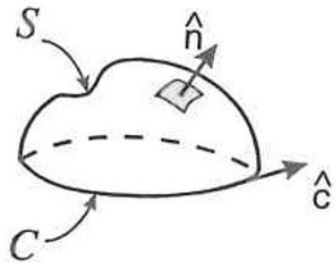
Lenz-Neumann law

$$\oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot \hat{\mathbf{c}} = -\frac{d}{dt}\iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t)\cdot \hat{\mathbf{n}}$$

….considerations

Stationary fields $\left(\frac{d}{dt}=0\right)$ ➡ $\oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot \hat{\mathbf{c}} = 0$

Maxwell equations: integral form



$$\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\frac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t}$$

➡

Lenz-Neumann law

$$\oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot \hat{\mathbf{c}} = -\frac{d}{dt}\iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t)\cdot \hat{\mathbf{n}}$$

….considerations

Stationary fields $\left(\frac{d}{dt}=0\right)$ ➡ $\oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot \hat{\mathbf{c}} = 0$ **Kirchhoff's second law**

Maxwell equations

Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

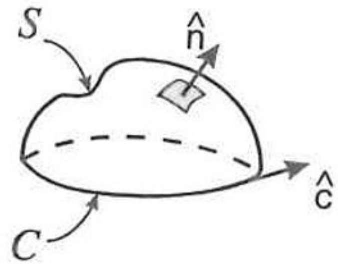
Integral form

$$\begin{cases} \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \end{cases}$$

Maxwell equations: integral form

$$\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t)$$

Maxwell equations: integral form

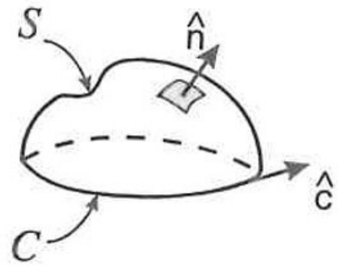


$$\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iint_S dS \left(\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right) \cdot \hat{\mathbf{n}} = \iint_S dS \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} \cdot \hat{\mathbf{n}} + \iint_S dS \, \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}}$$

Maxwell equations: integral form

Stokes Theorem

$$\iint_S dS\left(\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right)\cdot\hat{n}=\oint_C dc\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{c}$$



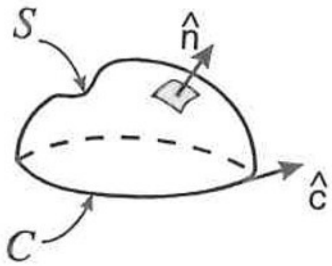
$$\nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\frac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t}+\vec{\mathbf{j}}(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iint_S dS\left(\nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right)\cdot\hat{\mathbf{n}}=\iint_S dS\,\frac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t}\cdot\hat{\mathbf{n}}+\iint_S dS\,\vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}$$

$\downarrow$

Maxwell equations: integral form

Stokes Theorem

$$\iint_S dS\left(\nabla\times\vec{\mathbf{A}}(\vec{\mathbf{r}})\right)\cdot\hat{n}=\oint_C dc\,\vec{\mathbf{A}}(\vec{\mathbf{r}})\cdot\hat{c}$$

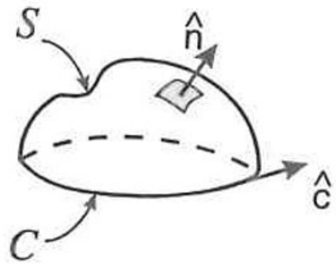


$$\nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)=\frac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t}+\vec{\mathbf{j}}(\vec{\mathbf{r}},t)\quad\Rightarrow\quad\iint_S dS\left(\nabla\times\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right)\cdot\hat{\mathbf{n}}=\iint_S dS\,\frac{\partial\vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t}\cdot\hat{\mathbf{n}}+\iint_S dS\,\vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}}$$

$$\downarrow$$

$$\oint_C dc\,\vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}}\;=$$

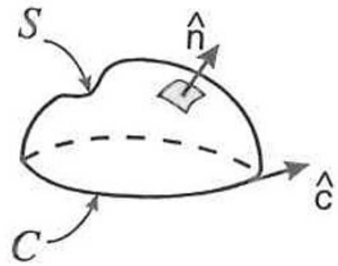
Maxwell equations: integral form



$$\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iint_S dS \left(\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right) \cdot \hat{\mathbf{n}} = \iint_S dS \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} \cdot \hat{\mathbf{n}} + \iint_S dS\, \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}}$$

$$\oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = \frac{d}{dt} \iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}}$$

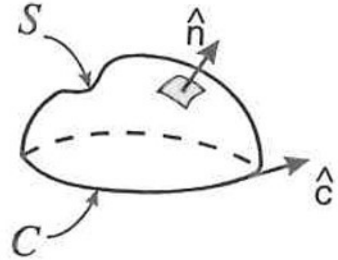
Maxwell equations: integral form



$$\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \Rightarrow \iint_S dS \left(\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t)\right) \cdot \hat{\mathbf{n}} = \iint_S dS \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} \cdot \hat{\mathbf{n}} + \iint_S dS\, \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}}$$

$$\oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = \frac{d}{dt} \iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + i(t)$$

Maxwell equations: integral form



$$\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \frac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t)$$

➡

Ampere-Faraday law

$$\oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = \frac{d}{dt} \iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + i(t)$$

Maxwell equations

Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

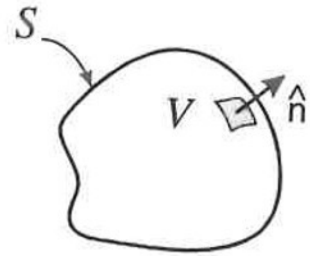
Integral form

$$\begin{cases} \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \\ \oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = \dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + i(t) \end{cases}$$

Maxwell equations: integral form

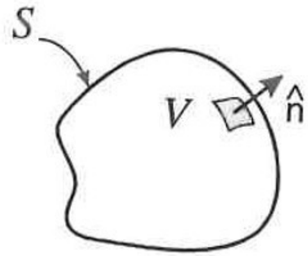
$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t)$$

Maxwell equations: integral form



$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t)$$

Maxwell equations: integral form

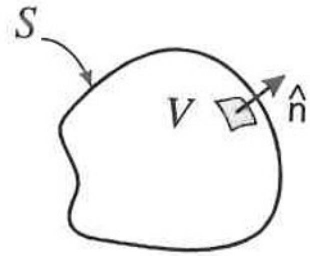


$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iiint_V dV\, \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \iiint_V dV\, \rho(\vec{\mathbf{r}},t)$$

Maxwell equations: integral form

Gauss theorem

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

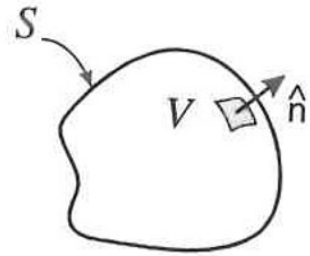


$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iiint_V dV\, \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \iiint_V dV\, \rho(\vec{\mathbf{r}},t)$$

Maxwell equations: integral form

Gauss theorem

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$

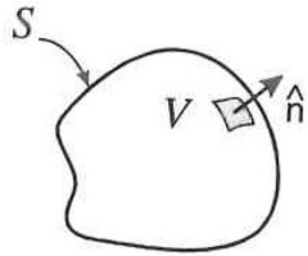


$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iiint_V dV\, \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \iiint_V dV\, \rho(\vec{\mathbf{r}},t)$$

$$\downarrow$$

$$\oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \;=$$

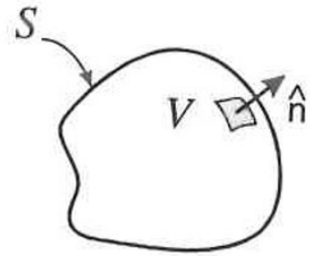
Maxwell equations: integral form



$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \quad \Rightarrow \quad \iiint_V dV\, \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \iiint_V dV\, \rho(\vec{\mathbf{r}},t)$$

$$\oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = q(t)$$

Maxwell equations: integral form



$$\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t)$$

➡

Coulomb law

$$\oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = q(t)$$

Maxwell equations

Differential form

$$
\begin{cases}
\nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\
\nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\
\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0
\end{cases}
$$

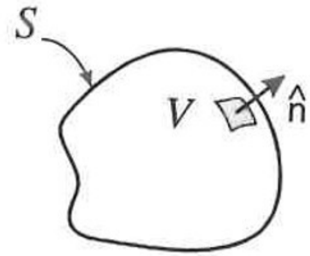
Integral form

$$
\begin{cases}
\oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \\
\oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = \dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + i(t) \\
\oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = q(t)
\end{cases}
$$

Maxwell equations: integral form

$$\nabla \cdot \vec{\mathbf{b}}\left(\vec{\mathbf{r}},t\right)=0$$

Maxwell equations: integral form

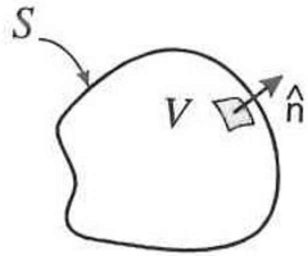


$$\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \quad \Rightarrow \quad \iiint_V dV\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0$$

Maxwell equations: integral form

Gauss theorem

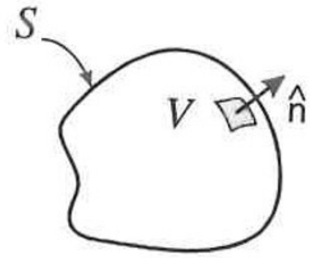
$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$



$$\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \quad \Rightarrow \quad \iiint_V dV\, \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0$$

$$\oiint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = 0$$

Maxwell equations: integral form



$$\nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \quad \Rightarrow \quad \oiint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = 0$$

Maxwell equations

Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

Integral form

$$\begin{cases} \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = -\dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} \\ \oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{c}} = \dfrac{d}{dt} \iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + i(t) \\ \oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = q(t) \\ \oiint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} = 0 \end{cases}$$

… mathematical tools …

$$\nabla \cdot \left(\nabla \times \vec{\mathbf{A}}\left(\vec{\mathbf{r}} \right) \right) = 0$$

Maxwell equations

Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

$$\Rightarrow \quad \frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} + \nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) = 0$$

Maxwell equations

Differential form

$$\nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} = 0$$

Current density equation

Integral form

Current density equation

Maxwell equations

Differential form

$$\nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} = 0$$

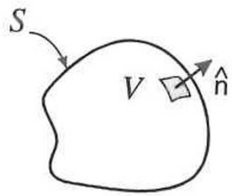
Current density equation

Integral form

Current density equation

Gauss theorem

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oiint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$



Maxwell equations

Differential form

$$\nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} = 0$$

Current density equation

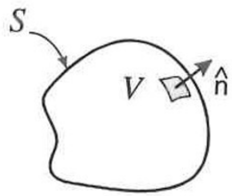
Integral form

$$\oint\!\!\!\oint_S dS\, \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \cdot \hat{\mathbf{n}} + \frac{dq(t)}{dt} = 0$$

Current density equation

Gauss theorem

$$\iiint_V dV\, \nabla \cdot \vec{\mathbf{A}}(\vec{\mathbf{r}}) = \oint\!\!\!\oint_S dS\, \vec{\mathbf{A}}(\vec{\mathbf{r}}) \cdot \hat{\mathbf{n}}$$



Maxwell equations

Integral form

$$\oiint_S dS\, \vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} + \frac{dq(t)}{dt} = 0$$

Current density equation

….considerations

Stationary fields $\left(\frac{d}{dt}=0\right) \Rightarrow \oiint_S dS\, \vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} = 0$ **Kirchhoff's first law**

Maxwell equations

Differential form

$$\begin{cases} \nabla \times \vec{\mathbf{e}}(\vec{\mathbf{r}},t) = -\dfrac{\partial \vec{\mathbf{b}}(\vec{\mathbf{r}},t)}{\partial t} \\ \nabla \times \vec{\mathbf{h}}(\vec{\mathbf{r}},t) = \dfrac{\partial \vec{\mathbf{d}}(\vec{\mathbf{r}},t)}{\partial t} + \vec{\mathbf{j}}(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{d}}(\vec{\mathbf{r}},t) = \rho(\vec{\mathbf{r}},t) \\ \nabla \cdot \vec{\mathbf{b}}(\vec{\mathbf{r}},t) = 0 \end{cases}$$

$$\nabla \cdot \vec{\mathbf{j}}(\vec{\mathbf{r}},t) + \frac{\partial \rho(\vec{\mathbf{r}},t)}{\partial t} = 0$$

Integral form

$$\begin{cases} \oint_C dc\, \vec{\mathbf{e}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}} = -\dfrac{d}{dt}\iint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} \\ \oint_C dc\, \vec{\mathbf{h}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{c}} = \dfrac{d}{dt}\iint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} + i(t) \\ \oiint_S dS\, \vec{\mathbf{d}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} = q(t) \\ \oiint_S dS\, \vec{\mathbf{b}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} = 0 \end{cases}$$

$$\oiint_S dS\, \vec{\mathbf{j}}(\vec{\mathbf{r}},t)\cdot\hat{\mathbf{n}} + \frac{dq(t)}{dt} = 0$$