Transmission lines

Electromagnetics
and
Remote Sensing Lab
(ERSLab)

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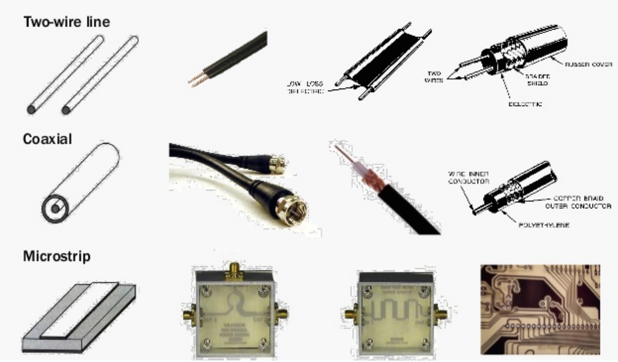
Outline

Introduction

ERSLab

F. Nunziata

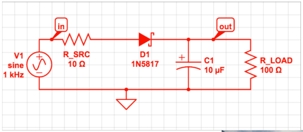


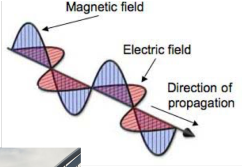
Transmission line

A transmission line is a two-port network connecting a generator circuit to a load.

Transmission line theory

Transmission line theory bridges the gap between field analysis and basic circuit theory.





It is of paramount importance to analyze:

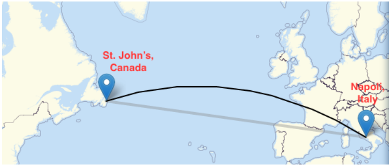
- microwave circuits;
- microwave devices.

Outline

When must wire be considered a T-Line?

Electricity supplied to households: $f = 50\text{Hz}$

$$\lambda = \frac{c}{f} \approx \frac{3 \times 10^8}{50} = 6000km$$

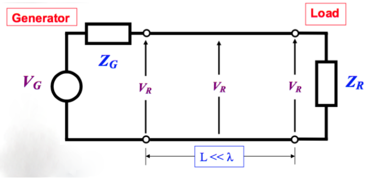


X-band network system

$$\lambda = \frac{c}{f} \approx \frac{3 \times 10^8}{10 \times 10^9} = 3cm$$

From lumped elements to distributed parameters

In circuit theory, lines connecting the various circuit elements are considered as perfect wires, with no voltage drop and no impedance associated to them: lumped impedance circuits



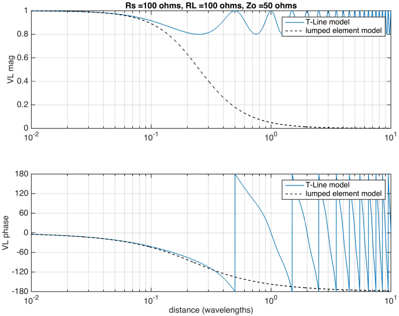
- The length of the wires is much smaller than λ.

Lumped-circuit

At any given time, the measured voltage and current are the same for each location on the same wire.

When must wire be considered a T-Line?

For an ideal connecting wire, the magnitude of V_L would be constant at 1 V and the phase would be constant at 0°.

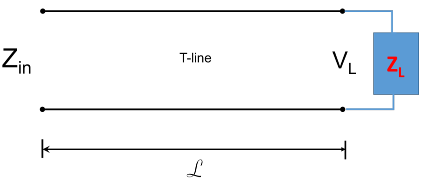


- The length of the wire impacts the load voltage at distances less than 0.01λ.
- Lumped and distributed element models exhibit appreciable differences at about 0.10λ

A first issue to be dealt with

The simplest problem consists of a voltage generator connected to a load through a uniform T-line.

Is the impedance seen by the generator the same as the impedance of the load ?



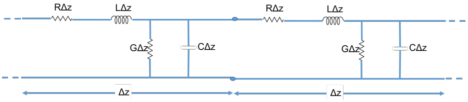
- $Z_{in} \neq Z_L$.
- Except when $\mathcal{L} = n\frac{\lambda}{2}$

Evaluating the equivalent impedance seen by the generator

How evaluating Z_{in}, i.e., the input impedance of a T-line terminated by a load.

Equivalent lumped-element circuit model

A uniform T-line is a distributed circuit that can be described as a cascade of identical cells with infinitesimal length (Δz).

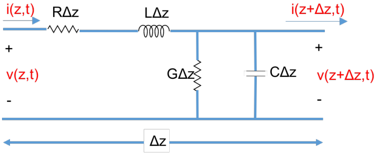


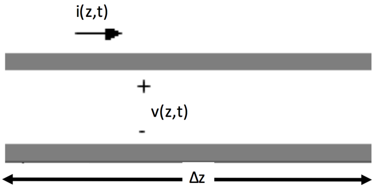
Theoretical rationale

Under the assumption of T-line uniform along its length, once the differential behavior of an elementary cell of the distributed circuit is determined in terms of voltage and current, we can find a global differential equation describing the entire T-line.

Equivalent lumped-element circuit model

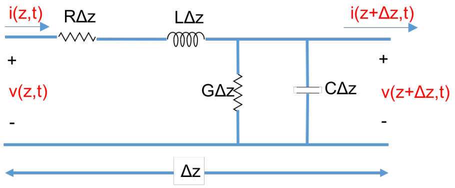
A T-line that propagates a transverse electromagnetic (TEM) wave is schematically represented as a two-wire line.





The piece of line of infinitesimal length Δz can be modeled as a lumped-element circuit.

Equivalent lumped-element circuit model



- R $[\frac{\Omega}{m}]$: is the series resistance per unit length and it accounts for the finite conductivity of the individual conductors.
- L $[\frac{H}{m}]$: is the series inductance per unit length and it accounts for the total self-inductance of the two conductors.
- G $[\frac{S}{m}]$: is the shunt conductance due to dielectric loss in the material within the two conductors.
- C $[\frac{F}{m}]$: is the shunt capacitance due to the close proximity of the two conductors.

Outline

Telegrapher equations - time domain

The lumped-element circuit can be analyzed using Kirchhoff's current and voltage law:

$$i(z,t) - i(z+\Delta z,t) = G\Delta z v(z+\Delta z,t) + C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} \quad (1)$$

$$v(z,t) - v(z+\Delta z,t) = R\Delta z i(z,t) + L\Delta z \frac{\partial i(z,t)}{\partial t}. \quad (2)$$

- Dividing by Δz and taking the limit as $\Delta z \to 0$, the following differential equations are obtained

Telegrapher equations

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t} \quad (3)$$

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t} \quad (4)$$

Telegrapher equations - time domain

Oliver Heaviside

Born	18 May 1850 Camden Town, Middlesex, England
Died	3 February 1925 (aged 74) Torquay, Devon, England
Nationality	British
Fields	Electrical engineering, mathematics and physics

- They come from Oliver Heaviside who developed the transmission line model.
- They are a pair of coupled, linear differential equations.
- They describe the voltage and current on an electrical transmission line with distance and time.

Telegrapher equations - phasor domain

- In the case of sinusoidal steady-state conditions, the voltage and current take the form of phasors:

$$v(z,t) = \Re\{V(z)e^{j\omega t}\} \quad (5)$$

$$i(z,t) = \Re\{I(z)e^{j\omega t}\} \quad (6)$$

- Hence, the telegrapher equations can be written as:

Telegrapher equations - phasor domain

$$\frac{dV(z)}{dz} = -(R + j\omega L)I(z) \quad (7)$$

$$\frac{dI(z)}{dz} = -(G + j\omega C)V(z) \quad (8)$$

Note the similarity with Maxwell's equations!

Wave equations for T-lines

Telegrapher equations can be decoupled by solving them simultaneously to give wave equations for $V(z)$ and $I(z)$.

Wave equations - Telephonists' equations

$$\frac{d^2 V(z)}{dz^2} - \gamma^2 V(z) = 0 \tag{9}$$

$$\frac{d^2 I(z)}{dz^2} - \gamma^2 I(z) = 0 \tag{10}$$

with γ being the frequency-dependent propagation constant:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \tag{11}$$

Outline

Traveling wave solution

- To solve wave equations, one can start from either (9) or (10) to obtain $V(z)$ or $I(z)$, respectively. Then, the remaining variable ($I(z)$ or $V(z)$) can be obtained using (7).
- Solving the wave equation in the $V(z)$ variable (9), one obtains:

$$V(z) = V^{+}e^{-\gamma z} + V^{-}e^{\gamma z} \tag{12}$$

where:

- V^{+} and V^{-} are two complex constants to be determined imposing boundary conditions;
- $e^{-\gamma z}$, $e^{\gamma z}$ stand for waves traveling in the positive (progressive wave), negative (regressive wave) z direction, respectively.

Traveling wave solution

The real part of the propagation constant

The real part α of the propagation constant describes the attenuation of the signal due to resistive losses.

The imaginary part of the propagation constant

The imaginary part β of the propagation constant describes the propagation properties of the signal as in lossless lines.

In a nutshell

Substituting $\gamma = \alpha + j\beta$ in (12) one can note that: the exponential term including α only affects magnitude of the voltage phasor; the exponential term including β affects only the phase of the waves in space.

Traveling wave solution

- To obtain the $I(z)$ wave, the $V(z)$ solution (see eq.(12)), must be inserted into eq.(7):

$$\begin{aligned} I(z) &= -\frac{1}{R+j\omega L}\frac{dV(z)}{dz} \\ &= \frac{\gamma}{R+j\omega L}\left(V^{+}e^{-\gamma z} - V^{-}e^{\gamma z}\right) \end{aligned} \tag{13}$$

- The ratio:

$$Z_o = \frac{R+j\omega L}{\gamma} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \tag{14}$$

has the physical dimension of an impedance and it is termed as characteristic impedance.

Using (13)-(14), the traveling wave solution can be written as:

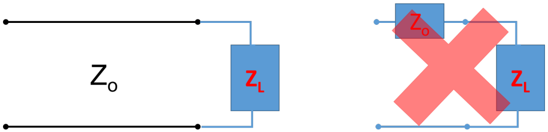
Traveling wave solution

$$V(z) = V^+ e^{-\gamma z} + V^- e^{\gamma z} \quad (15)$$

$$I(z) = \frac{V^+}{Z_o} e^{-\gamma z} - \frac{V^-}{Z_o} e^{\gamma z} \quad (16)$$

A common mistake!

Z_o does not depend on the length of T-line



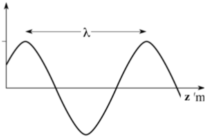
Z_o cannot be replaced by a lumped impedance in an equivalent circuit

Note that Z_o depends only on the characteristics of the conductors, the dielectric medium and the cross-section geometry of the T-line.

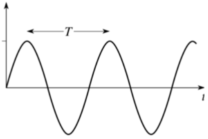
Traveling wave solution - time domain

The time-domain solution can be obtained as follows:

$$v(z,t) = \Re\left(V(z)e^{j\omega t}\right) = \quad |V^+|\cos\left(\omega t - \beta z + \phi^+\right) e^{-\alpha z} + |V^-|\cos\left(\omega t + \beta z + \phi^-\right) e^{\alpha z} \quad (17)$$



for a fixed value of z



- $\alpha = 0$, i.e. lossless case.
- The wavelength is: $\lambda = \frac{2\pi}{\beta}$.
- The phase velocity is: $v_f = \frac{\omega}{\beta} = \lambda f$.

The lossless T-line

In many practical cases the loss of the line is so small that can be neglected.

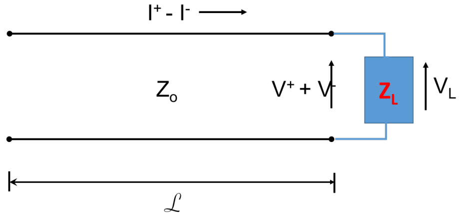
Lossless T-line: R=G=0

- The propagation constant (11) becomes an imaginary number $\gamma = \alpha + j\beta = j\beta = j\omega\sqrt{LC}$.
- The characteristic impedance (14) becomes a real number: $Z_o = \sqrt{\frac{L}{C}}$.
- The wavelength is: $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}}$.
- The phase velocity is: $v_f = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$.

Outline

Terminated T-line

The properties of a T-line terminated in an arbitrary load impedance Z_L are examined



BCs

This analysis illustrates how positive and negative traveling waves combine to satisfy the boundary conditions at a termination.

Terminated T-line

Traveling wave solution for a lossless T-line

$$V(z) = V^{+}e^{-j\beta z} + V^{-}e^{j\beta z} \quad (18)$$

$$I(z) = \frac{V^{+}}{Z_o}e^{-j\beta z} - \frac{V^{-}}{Z_o}e^{j\beta z} \quad (19)$$

BCs

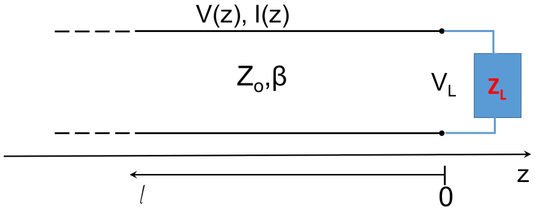
$V(z)$ and $I(z)$ are the solutions of the 2nd order wave equation; hence, two arbitrary constants V^{+} and V^{-} must be specified imposing BCs related to load and generator.

V^{+} and V^{-} represent the amplitudes of steady-state voltage waves, traveling in the positive and in the negative direction, respectively.

Coordinate reference system

A new reference system centered into the load

A reference system such that the zero reference is at the location of the load (instead of the generator) is more convenient, since T-line analysis starts from the load itself.



Note that the positive direction of the space coordinate is reversed: it increases when moving from load to generator along the T-line.

BC imposed by the load - Reflection coefficient

- A new coordinate $l = -z$ is adopted; hence, substituting in (18) and considering the load coordinate, i.e. $l = 0$, (18) becomes:

$$V(0) = V^+ + V^- \tag{20}$$

$$I(0) = \frac{1}{Z_o}\left(V^+ - V^-\right) \tag{21}$$

BC

The BC imposed by the load, whose impedance is Z_L, is:

$$V(0) = Z_L I(0) \tag{22}$$

Reflection coefficient

- From eqs.(20)-(22) one obtains:

$$V^+ + V^- = \frac{Z_L}{Z_o}\left(V^+ - V^-\right), \tag{23}$$

- Solving for V^- gives:

Reflection coefficient

$$\Gamma = \frac{V^-}{V^+} = \frac{Z_L - Z_o}{Z_L + Z_o} \tag{24}$$

Note that both the direct ($-z$) and the reflected ($+z$) waves are needed to satisfy BCs.

Reflection coefficient

Using (20-21) and considering the reflection coefficient (24), one can write the V and I waves at the load ($z = 0$) as follows:

$$\begin{aligned} V(0) = V^{+} + V^{-} = & \quad V^{+}(1+\Gamma) \\ Z_o I(0) = V^{+} - V^{-} = & \quad V^{+}(1-\Gamma) \end{aligned} \tag{25}$$

- Since $\frac{V(0)}{I(0)} = Z_L$,
- defining $\bar{Z}_L = \frac{Z_L}{Z_o}$ as the normalized load impedance:

$$\bar{Z}_L = \frac{1+\Gamma}{1-\Gamma} \tag{26}$$

Note also that (26) can be solved for Γ to obtain:

$$\Gamma = \frac{Z_L - Z_o}{Z_l + Z_o} = \frac{\bar{Z}_L - 1}{\bar{Z}_L + 1} \tag{27}$$

Standing waves

$V(z)$ and $I(z)$ waves (18-19) can be written in terms of Γ:

Traveling wave solution - standing waves

$$V(z) = V^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right) \tag{28}$$

$$I(z) = \frac{V^+}{Z_o}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right) \tag{29}$$

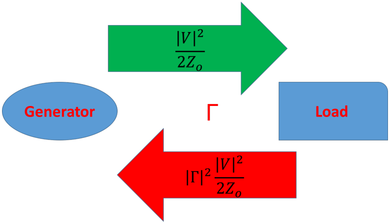
Standing waves: The voltage and current waves consist of the superposition of an incident and reflected wave.
A special case occurs under the matched load condition: $\Gamma = 0$, i.e. $Z_L = Z_o$. No reflected wave!

A constant average power flow applies

- Using (28), the time-average power flow along T-line can be evaluated:

$$\begin{aligned} P &= \frac{1}{2}\Re\left(V(z)I^*(z)\right) \\ &= \frac{1}{2}\frac{|V^+|^2}{Z_o}\Re\left(1-\Gamma^* e^{-2j\beta z}+\Gamma e^{2j\beta z}-|\Gamma|^2\right) \\ &= \frac{1}{2}\frac{|V^+|^2}{Z_o}\left(1-|\Gamma|^2\right) \end{aligned} \tag{30}$$

A constant average power flow applies



Total power delivered to the load

It is equal to the incident power ($\frac{|V^+|^2}{2Z_o}$) minus the reflected power ($\frac{|V^+|^2|\Gamma|^2}{2Z_o}$).

- $|\Gamma| = 0$ ($|\Gamma| = 1$) implies maximum (no) power is delivered to the load.

Return loss

Return loss - RL (dB)

In telecommunications, return loss is the loss of power in the signal returned/reflected by a discontinuity in a T-line.

$$RL = -20\log|\Gamma| \quad (dB) \tag{31}$$

- Matched load: RL=∞ dB - no reflected power.
- Total reflection: RL=0 dB - all incident power is reflected.
- $RL = -10$dB: $1/10^{th}$ of the energy is reflected. Usually this is the threshold when most devices are considered to be tuned.
- $RL = -20$dB: $1/100^{th}$ of the energy is reflected. This is a very good matching.

The magnitude of the standing wave

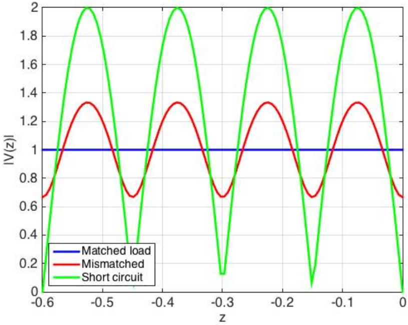
The magnitude of the standing wave depends on the load.

$$
\begin{aligned}
|V(z)| &= \left| V^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z} \right) \right| \\
&= |V^+| \left| 1 + \Gamma e^{2j\beta z} \right| \\
&= |V^+| \left| 1 + \Gamma e^{-2j\beta l} \right| \\
&= |V^+| \left| 1 + |\Gamma| e^{j(\theta - 2\beta l)} \right|
\end{aligned}
$$

- where $l = -z$ has been considered and the reflection coefficient is expressed in polar format $\Gamma = |\Gamma| e^{j\theta}$.

The magnitude of the standing wave

The magnitude is constant (flat line) when $\Gamma = 0$; otherwise, it oscillates with position z along the line.



- Matched load: $|V(z)| = |V^+|$
- Mismatch - max: $e^{\theta - 2\beta l} = 1$
  $V_{max} = |V^+|(1 + |\Gamma|)$
- Mismatch - min: $e^{\theta - 2\beta l} = -1$
  $V_{min} = |V^+|(1 - |\Gamma|)$

Do it yourself

- Let's try with different Z_L to simulate: matched ($Z_L = Z_o$), short circuit ($Z_L = 0$) and partially matched ($Z_l \neq Z_o$) loads.

```
f      = 1*10^9;
c      = 3*10^8;
L      = 100; %number of points
Vmax   = 1;
Zo     = 50;
lambda = c/f;
beta   = 2*pi/lambda;
gamma  = (ZL - Z0) / (ZL + Z0);
teta   = angle(gamma);
z      = linspace(-2*lambda,0,L);
V      = Vmax*sqrt(1+abs(gamma^2)+2*abs(gamma)*...
         cos(2*beta.*z+teta));
```

Standing wave

It is clear that $|V|$ oscillates back and forth between maximum and minimum values.

Maxima

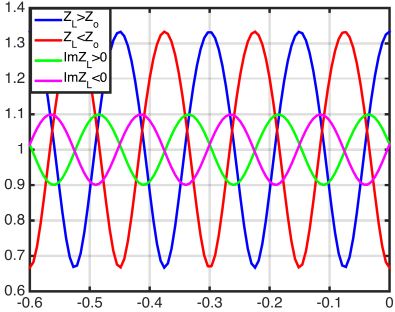
- The voltage maxima occur when there is constructive interference between the incident and reflected waves.
  - The pattern of maxima repeats with a period given by: $2\beta l = 2\pi \rightarrow d = \frac{\lambda}{2}$.

Minima

- The voltage minima occur when there is destructive interference between the incident and reflected waves.
  - The pattern of minima repeats with a period given by: $2\beta l = 2\pi \rightarrow d = \frac{\lambda}{2}$.

Standing wave

The voltage standing wave pattern provides immediate info on the T-line circuit



- $Z_L > Z_o$: starts with a maximum at load;
- $Z_L < Z_o$: starts with a minimum at load;
- $\Im(Z_L) > 0$ (inductive): initially increases;
- $\Im(Z_L) < 0$ (capacitive): initially decreases.

```
ZL                  = 100;
[V,ROS,gamma,z] = standingw(L,ZL,ZO,Vp,lambda);
figure(1), plot(z,V,'b','LineWidth',2.5), grid on;

ZL                  = 25;
[V,ROS,gamma,z] = standingw(L,ZL,ZO,Vp,lambda);
plot(z,V,'r','LineWidth',2.5)

ZL                  = complex(50,10);
[V,ROS,gamma,z] = standingw(L,ZL,ZO,Vp,lambda);
plot(z,V,'g','LineWidth',2.5)

ZL                  = complex(50,-10);
[V,ROS,gamma,z] = standingw(L,ZL,ZO,Vp,lambda);
plot(z,V,'m','LineWidth',2.5)
legend('Z_L>Z_o','Z_L<Z_o','Im{Z_L}>0','Im{Z_L}<0')
```

Standing Wave Ratio (SWR)

SWR (a.k.a. Voltage SWR (VSWR) or in Italian "Rapporto d'onda stazionaria (ROS)")

It is a real number that measures the impedance matching of loads to the characteristic impedance of a T-line.

SWR is defined as the ratio of the partial standing wave's amplitude at an antinode (maximum) to the amplitude at a node (minimum) along the line.

SWR

$$SWR = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|} \tag{32}$$

- $|\Gamma| = 0 \quad \rightarrow \quad SWR = 1$
- $|\Gamma| = 1 \quad \rightarrow \quad SWR = \infty$

Input impedance

Γ, defined as the reflected-to-incident wave ratio measured at the load (24), can be easily defined at any point, i.e., $z = -l$ on the T-line:

$$\Gamma(l) = \frac{V^- e^{-j\beta l}}{V^+ e^{j\beta l}} = \frac{V^-}{V^+} e^{-2j\beta l} = \Gamma e^{-2j\beta l} \tag{33}$$

Hence, the normalized impedance seen looking toward the load at $z = -l$:

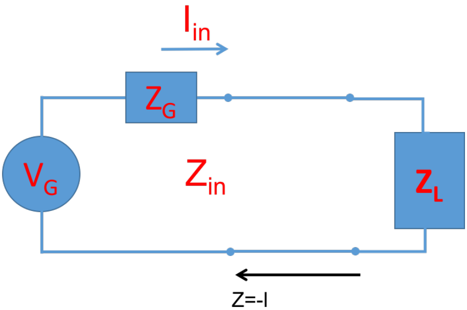
$$\begin{aligned} \bar{Z}_{in} &= \frac{Z_{in}}{Z_o} = \frac{V}{IZ_o} = \frac{V^+ e^{j\beta l} + V^- e^{-j\beta l}}{V^+ e^{j\beta l} - V^- e^{-j\beta l}} \\ &= \frac{1+\Gamma(l)}{1-\Gamma(l)} = \frac{1+\Gamma e^{-2j\beta l}}{1-\Gamma e^{-2j\beta l}} \end{aligned} \tag{34}$$

Replacing Γ with (27) and considering that $e^{\pm\beta l} = cos\beta l \pm jsin\beta l$:

$$\bar{Z}_{in} = \frac{Z_{in}}{Z_o} = \frac{Z_L + jZ_o \tan\beta l}{Z_o + jZ_L \tan\beta l} \tag{35}$$

BC imposed by the generator end

The BC condition at the generator end can be obtained using (35) to evaluate the input impedance seen looking towards the load at the the generator end



- V_g is the open-circuit voltage;
- Z_G is the internal impedance of the generator.
- The total voltage V at $z = -l$ is given by: $V = V_G \frac{Z_{in}}{Z_{in}+Z_G}$

BC imposed by the generator end

- V is given by the sum of the progressive and reflected waves; hence:

$$V_G \frac{Z_{in}}{Z_{in} + Z_G} = V^+ e^{j\beta l}\left(1 + \Gamma e^{-2j\beta l}\right) \tag{36}$$

This expression can be solved for V^+ :

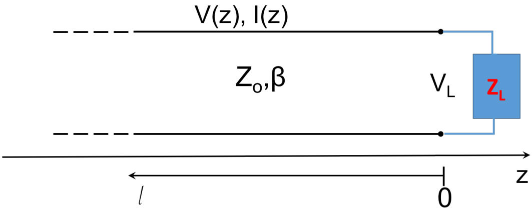
BC imposed by the generator

$$V^+ = \frac{Z_{in}(Z_L + Z_o)V_G}{2(Z_{in} + Z_G)(Z_L \cos\beta l + jZ_o \sin\beta l)} \tag{37}$$

Outline

Special cases of lossless terminated lines

A number of special cases of lossless terminated T-lines frequently apply in practical cases.



Now let us look at Z_{in} for some "special" load impedance and T-line lengths.

T-line with special length

1. T-line electrically small. This means that the T-line is much smaller than λ: $l \ll \lambda$:

$$\beta l = 2\pi \frac{l}{\lambda} \approx 0 \tag{38}$$

- This implies that, according to eq.(35), $\tan \beta l = 0$ and, hence, $Z_{in} = Z_L$

Circuit theory approximation

If the T-line length is much smaller than the wavelength, the input impedance Z_{in} will always be equal to the load impedance Z_L. Hence, voltage and current do not vary appreciably over the elements that can be approximated as lumped elements.

T-line with special length

2. $l = n\frac{\lambda}{2}$

$$Z_{in}\left(l = \frac{\lambda}{2}\right) = Z_L \tag{39}$$

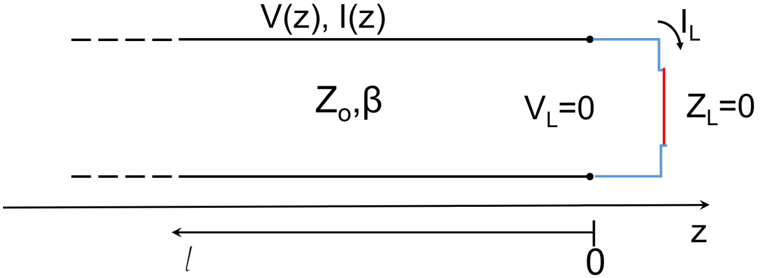
This is an ideal one-to-one impedance transformer that does not alter the load impedance, independently of Z_o.

3. $l = \frac{\lambda}{4} + n\frac{\lambda}{2}$

$$Z_{in}\left(l = \frac{\lambda}{4}\right) = \frac{Z_o^2}{Z_L} \tag{40}$$

This T-line is also termed as *quarter-wave transformer* since it is equivalent to invert Z_L depending on the characteristic impedance.

4. $Z_L = 0$. The T-line is terminated in a short circuit:



T-line terminated in a short circuit

- According to eq.(27): $\Gamma = -1$.
- According to eq.(28):

$$V(z) = V^{+}\left(e^{-j\beta z} - e^{j\beta z}\right) = -2jV^{+}\sin\beta z \quad (41)$$

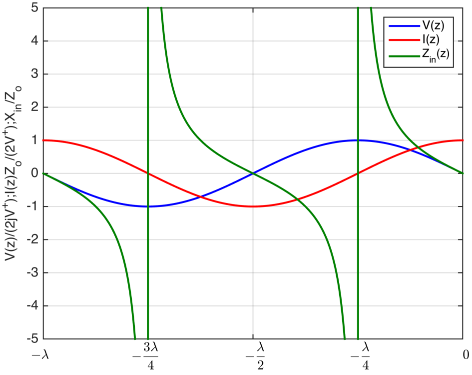
$$I(z) = \frac{V^{+}}{Z_o}\left(e^{-j\beta z} + e^{j\beta z}\right) = \frac{2V^{+}}{Z_o}\cos\beta z \quad (42)$$

- According to eq.(35): $Z_{in} = jZ_o \tan\beta l$.

Reactive impedance

The input impedance is purely imaginary for any length l (reactive impedance) and can take all the values between $+j\infty$ and $-j\infty$. Note that, when $l = 0$ $Z_{in} = 0$, i.e.; SC; while, when $l = \frac{\lambda}{4}$ $Z_{in} = \infty$, i.e.; OC. In addition, Z_{in} is periodic ($\frac{\lambda}{2}$) in l.

T-line terminated in a short circuit

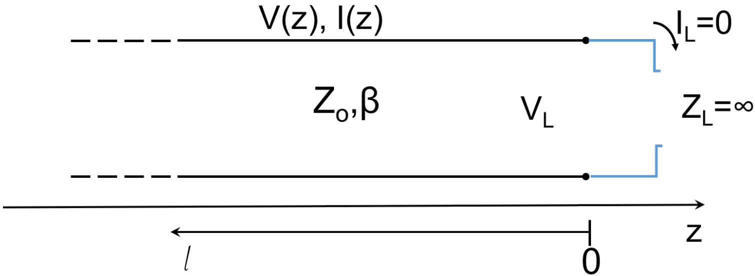


Reactance

The input reactance can be either inductive (when $X > 0$) or capacitive (when $X < 0$). Hence, a proper choice of l can make the SC T-line equivalent to a capacitor or an inductor.

T-line terminated in an open circuit

5. $Z_L = \infty$. The T-line is terminated in an open circuit:



T-line terminated in an open circuit

- According to eq.(27): $\Gamma = 1$.
- According to eq.(28):

$$V(z) = V^+ \left(e^{-j\beta z} + e^{j\beta z} \right) = 2V^+ \cos \beta z \quad (43)$$

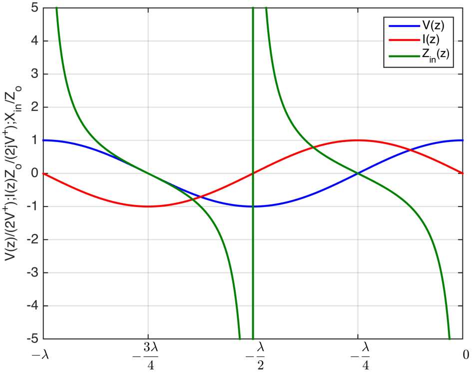
$$I(z) = \frac{V^+}{Z_o} \left(e^{-j\beta z} - e^{j\beta z} \right) = \frac{-2jV^+}{Z_o} \sin \beta z \quad (44)$$

- According to eq.(35): $Z_{in} = -jZ_o \cot \beta l$.

Reactive impedance

The input impedance is purely imaginary for any length l (reactive impedance) and can take all the values between $-j\infty$ and $+j\infty$. Note that, when $l = 0$ $Z_{in} = \infty$, i.e.; OC; while, when $l = \frac{\lambda}{4}$ $Z_{in} = 0$, i.e.; SC. In addition, Z_{in} is periodic ($\frac{\lambda}{2}$) in l.

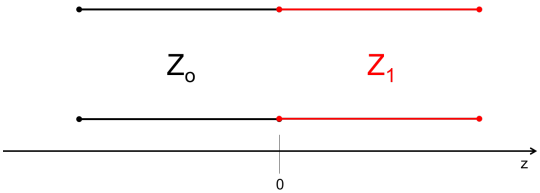
T-line terminated in an open circuit



Reactance

The input reactance can be either inductive (when $X > 0$) or capacitive (when $X < 0$). Hence, a proper choice of l can make the OC T-line equivalent to a capacitor or an inductor.

Junction of two T-lines



A T-line of characteristic impedance Z_o feeds a T-line whose characteristic impedance is Z_1. The latter T-line is assumed to be infinitely long or terminated in a load $Z_L = Z_1$, i.e.; no reflected wave occurs from its far end.

Junction of two T-lines - Insertion loss

- The reflection coefficient at $z = 0$ is given by:

$$\Gamma = \frac{Z_1 - Z_o}{Z_1 + Z_o} \tag{45}$$

- Hence, the voltage for $z < 0$ is given by:

$$V(z) = V^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z} \right), \qquad z < 0 \tag{46}$$

- Part of the incident wave is transmitted onto the second T-line with a voltage amplitude weighted by the transmission coefficient T:

$$V(z) = V^+ T e^{-j\beta z}, \qquad z > 0 \tag{47}$$

- Equating the above equations at $z = 0$ one obtains T:

$$T = 1 + \Gamma = \frac{2Z_1}{Z_1 + Z_o} \tag{48}$$

$|T|$ in dB, i.e.; $-20 log|T|$, is termed as *Insertion loss*.

Lossy T-line

Actual T-lines have losses due to finite conductivity and/or lossy dielectric.
However, those losses are small.

- All the above equations hold except that $j\beta$ must be replaced with $\alpha + j\beta$.

Low-loss T-line, i.e. $R \ll \omega L$ and $G \ll \omega C$

- Z_o can be still considered real, i.e. $Z_o \simeq \sqrt{\frac{L}{C}}$;
- $\alpha \neq 0$;
- $\beta \simeq \omega\sqrt{LC}$.

Outline

Reflection coefficient

α does not have any effect on Γ at the load position, i.e. $z = 0$

$\alpha \neq 0$ does affect $\Gamma(l)$

$$\Gamma(l) = \Gamma e^{-2\alpha l} e^{-2j\beta l} \tag{49}$$

For increasing l, $\Gamma(l)$ decreases exponentially and it essentially vanishes for large l.

Any load appears matched to the T-line when viewed through a long section of lossy line

Input impedance

Input impedance for a lossy line

$$
\begin{aligned}
Z_{in} &= Z_o \frac{1 + \Gamma e^{-2j\beta l - 2\alpha l}}{1 - \Gamma e^{-2j\beta l - 2\alpha l}} \\
&= Z_o \frac{Z_L + Z_o \tanh(j\beta l + \alpha l)}{Z_o + Z_L \tanh(j\beta l + \alpha l)} \\
&\approx Z_L \qquad (50)
\end{aligned}
$$

- The last simplification holds for large l, since tanh approaches 1.
- Note also that SWR approaches 1 as one moves away from the load toward the generator.

Outline

Not only a nice picture

What it is

Nomogram

It is a graphical calculation device, aka nomogram, proposed by P.H. Smith in 1939 to solve transmission line problems quickly and with enough accuracy.

Smith Chart

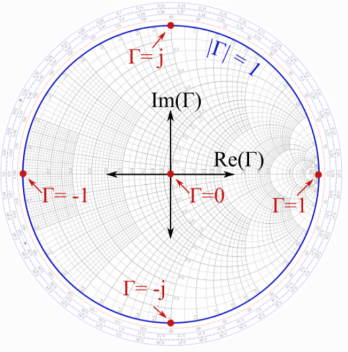
Nomography stands for the graphical representation of a mathematical relationship or law. Smith Chart provides a quick and effective way to visualize T-line phenomenon without the need of detailed numerical calculations.

https://www.microwaves101.com/smith-chart/smith-chart-tool-v1

Outline

Unit circle

It is based on a polar plot of the reflection coefficient Γ.



The magnitude $|\Gamma|$ is plotted as a $|\Gamma| \leq 1$ radius from the center of the chart and the angle $-\pi \leq \theta \leq \pi$ is measured counterclockwise from the right-hand side of the horizontal diameter

The real benefit of the Smith chart consists of representing any normalized impedance (admittance) using the circles printed on the chart.

Constant resistance/reactance loci

At the very root the Smith chart can be considered as the graphical way to represent eq.(27):

$$\Gamma = \frac{Z_L - Z_o}{Z_l + Z_o}$$

This is a bi-linear transformation that maps every impedance into the complex Γ plane.

- The first step is to deal with normalized impedance:

$$\bar{Z}_L = \frac{Z_L}{Z_o}$$

- Then, eq.(27) can be written as:

$$\bar{Z}_L = r + jx = \frac{1+\Gamma}{1-\Gamma} = \frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r - j\Gamma_i} \tag{51}$$

Constant resistance/reactance loci

- Arranging eq.(51) to single out r and x one obtains:

$$r = \frac{1 - |\Gamma|^2}{1 - 2\Gamma_r + |\Gamma|^2} \tag{52}$$

and

$$x = \frac{2\Gamma_i}{1 - 2\Gamma_r + |\Gamma|^2} \tag{53}$$

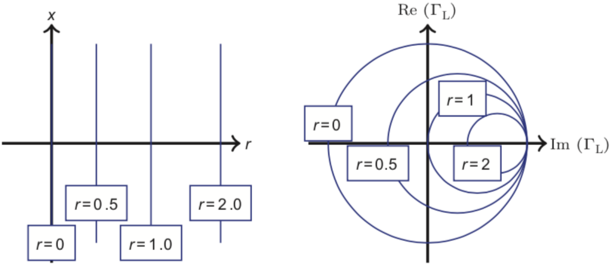
These two equations can be rearranged according to a parametric equation:

$$(x - a)^2 + (y - b)^2 = R^2$$

that represents a circle in the complex Γ plane.

Constant resistance circles

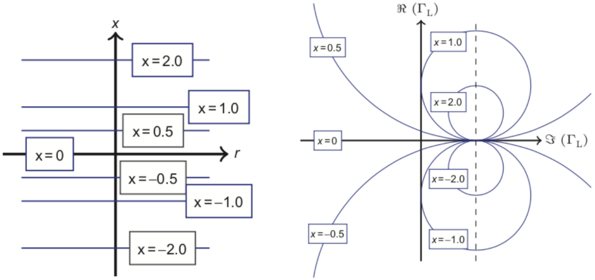
Constant resistance contours in the normalized impedance plane are transformed according to the bi-linear transformation into constant resistance circles.



Center: $\left(\Gamma_r = \frac{r}{1+r}, \Gamma_i = 0\right)$; Radius: $\frac{1}{1+r}$.

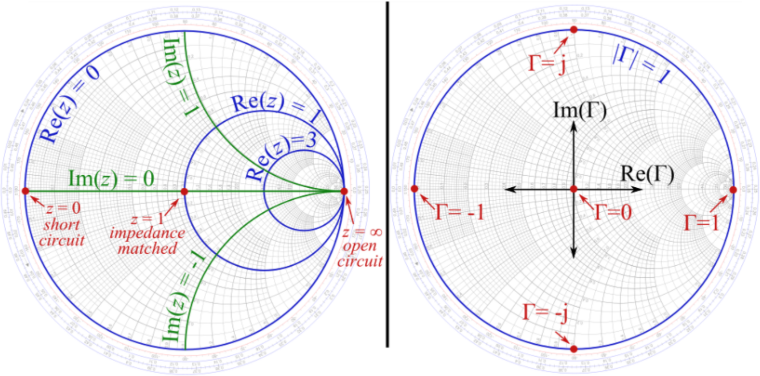
Constant reactance circles

Constant reactance contours in the normalized impedance plane are transformed according to the bi-linear transformation into constant reactance circles.

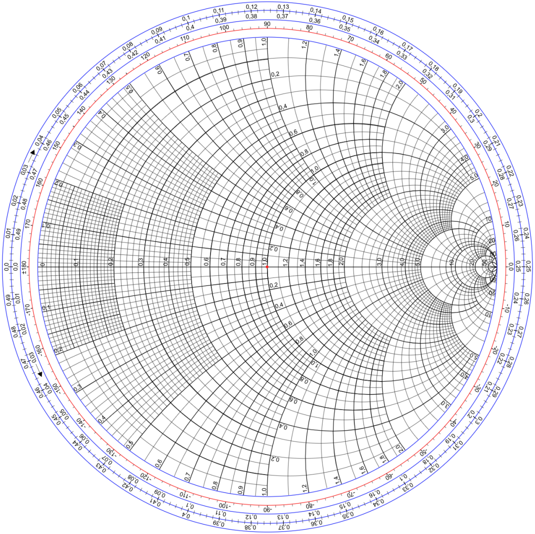


Center: $\left(\Gamma_r = 1, \Gamma_i = \frac{1}{x}\right)$; Radius: $\frac{1}{|x|}$.

Smith Chart: Γ vs $\bar{Z}_L$



The complete Smith Chart



Smith Chart: scales along the perimeter

Angular scale

It indicates the angle θ of the reflection coefficient (in degrees).

Wavelength scale

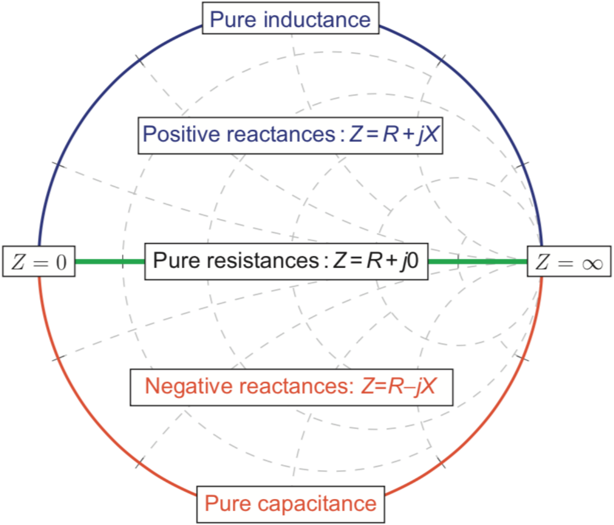
It indicates electrical lengths in fraction of wavelength in the range $(0 - 0.5\lambda)$ and includes a twofold labeling:

- the outer scale is calibrated clockwise and stands for wavelengths toward the generator;
- the inner scale is calibrated counter-clockwise and stands for wavelengths toward the load.

The two labeling are complementary, i.e.; 0.1λ on the outer scale stands for $0.5 - 0.1 = 0.4\lambda$ on the inner scale.

Those scales allow plotting Γ and determining the length of the t-line,

Smith Chart regions

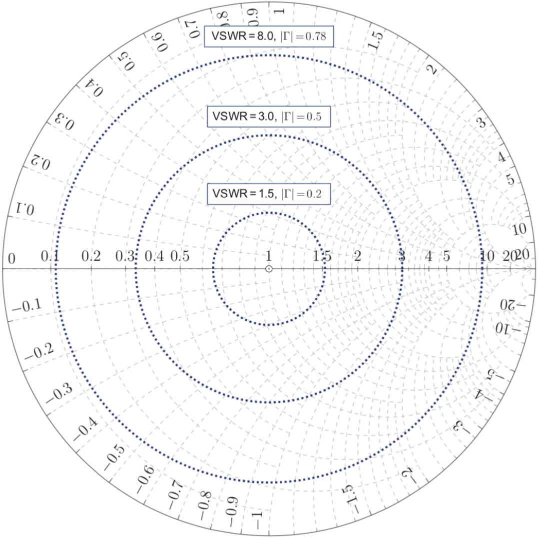


Smith Chart regions

A number of regions can be identified:

1. The upper half part above the horizontal axis includes all impedances with a positive reactive part (i.e., inductive impedances).
2. The lower half part includes all impedances with a negative reactive part (i.e., capacitive impedances).
3. The horizontal axis includes all pure resistances.
4. The outer perimeter includes all purely reactive impedances (i.e., zero resistance): pure inductances/capacitance on the upper/lower semicircle.
5. The rightmost point on the horizontal axis stands for an infinite impedance (a perfect open circuit).
6. The leftmost point on the horizontal axis represents zero impedance (a perfect short circuit).

Smith Chart VSWR regions



Smith Chart VSWR regions

Constant VSWR circles

Circles of various radii on the Smith Chart, with centers at the origin, represent a constant SWR, which is equivalent to a constant magnitude of reflection coefficient

T-line

Any point on one of these circles, therefore, represents a point on a lossless transmission line at some distance from the load, since, as we travel away from the load on a lossless line, the reflection coefficient magnitude remains constant but the angle of the reflection coefficient changes.

For Further Reading I

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