Fresnel reflection coefficients
Examples

Electromagnetics
and
Remote Sensing Lab
(ERSLab)

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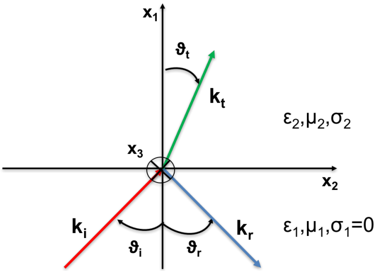
Outline

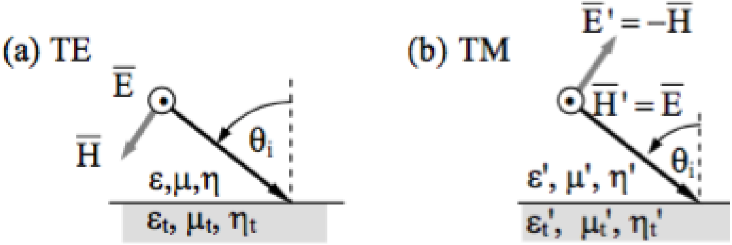
Reflection coefficient

The general formula for the reflection coefficient is given by:



$$\rho = \frac{Z_2(\hat{x}_1) - Z_1(\hat{x}_1)}{Z_2(\hat{x}_1) + Z_1(\hat{x}_1)} \qquad (1)$$

Reflection coefficient



$$Z_{TE} = \frac{Z}{\cos\vartheta} \qquad Z_{TM} = Z\cos\vartheta \tag{2}$$

Reflection coefficient - TE

$$\rho_{TE} = \frac{\frac{Z_2}{\cos\vartheta_2} - \frac{Z_1}{\cos\vartheta_1}}{\frac{Z_2}{\cos\vartheta_2} + \frac{Z_1}{\cos\vartheta_1}} = \frac{\frac{Z_2\cos\vartheta_1 - Z_1\cos\vartheta_2}{\cos\vartheta_1\cos\vartheta_1}}{\frac{Z_2\cos\vartheta_1 + Z_1\cos\vartheta_2}{\cos\vartheta_1\cos\vartheta_1}} \tag{3}$$

Reflection coefficient - TE

$$\rho_{TE} = \frac{Z_2\cos\vartheta_1 - Z_1\cos\vartheta_2}{Z_2\cos\vartheta_1 + Z_1\cos\vartheta_2} \tag{4}$$

Reflection coefficient - TM

Reflection coefficient - TM

$$\rho_{TM} = \frac{Z_2 \cos\vartheta_2 - Z_1 \cos\vartheta_1}{Z_2 \cos\vartheta_2 + Z_1 \cos\vartheta_1} \tag{5}$$

- $Z = \sqrt{\frac{\mu}{\epsilon}}$
- When dealing with a non-magnetic medium $\mu = \mu_o$. Hence:

$$Z = \sqrt{\frac{\mu_o}{\epsilon_o \epsilon_r}} = \frac{Z_o}{\sqrt{\epsilon_r}} = \frac{Z_o}{n} \tag{6}$$

- where n is the refractive index.

Reflection coefficients as function of incidence angle only

- According to eq.(6), reflection coefficients can be rewritten as:

$$\rho_{TE} = \frac{n_1 \cos\vartheta_1 - n_2 \cos\vartheta_2}{n_1 \cos\vartheta_1 + n_2 \cos\vartheta_2} \quad (7)$$

$$\rho_{TM} = \frac{n_1 \cos\vartheta_2 - n_2 \cos\vartheta_1}{n_1 \cos\vartheta_2 + n_2 \cos\vartheta_1} \quad (8)$$

- Using Snell's law $n_1 \sin\vartheta_1 = n_2 \sin\vartheta_2$ and trigonometric formulas, one can write:

$$n_2 \cos\vartheta_2 = n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\vartheta_1\right)^2} \quad (9)$$

$$n_1 \cos\vartheta_2 = n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\vartheta_1\right)^2} \quad (10)$$

Reflection coefficients as function of incidence angle only

Reflection coefficients

$$\rho_{TE} = \frac{n_1 \cos\vartheta_1 - n_2\sqrt{1 - \left(\frac{n_1}{n_2}\sin\vartheta_1\right)^2}}{n_1 \cos\vartheta_1 + n_2\sqrt{1 - \left(\frac{n_1}{n_2}\sin\vartheta_1\right)^2}} \quad (11)$$

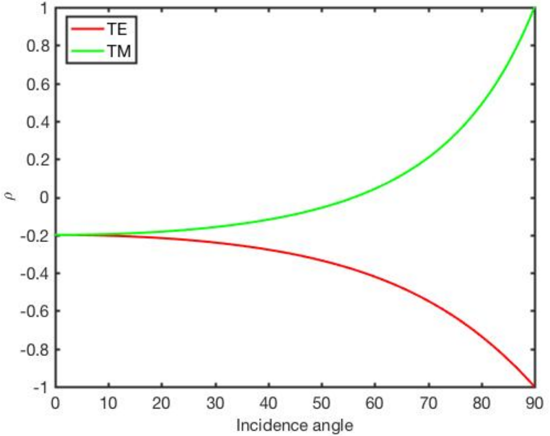
$$\rho_{TM} = \frac{n_1\sqrt{1 - \left(\frac{n_1}{n_2}\sin\vartheta_1\right)^2} - n_2 \cos\vartheta_1}{n_1\sqrt{1 - \left(\frac{n_1}{n_2}\sin\vartheta_1\right)^2} + n_2 \cos\vartheta_1} \quad (12)$$

Outline

1 Introduction

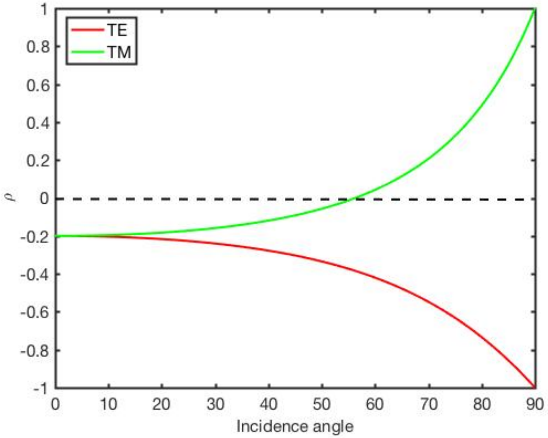
2 Examples
- Lossless media
- Lossy media

TE and TM reflection coefficients



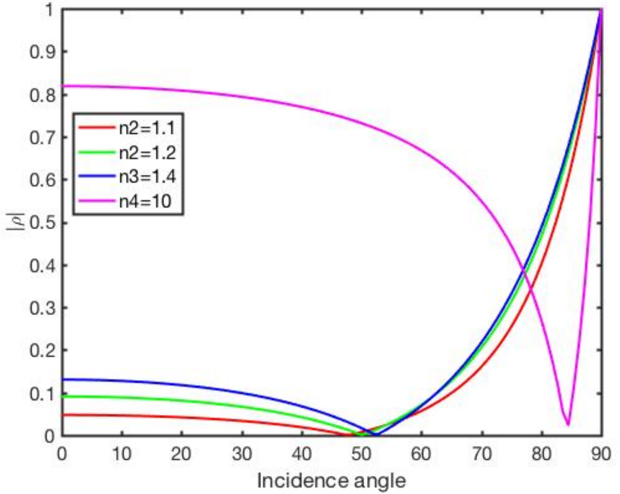
n1=1, n2=1.5

Brewster angle



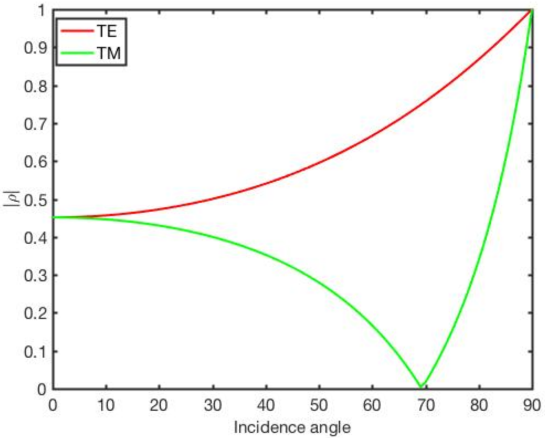
$$\vartheta_B = tan^{-1} \frac{n_2}{n_1} \tag{13}$$

TM reflection coefficient for varying n_2



Outline

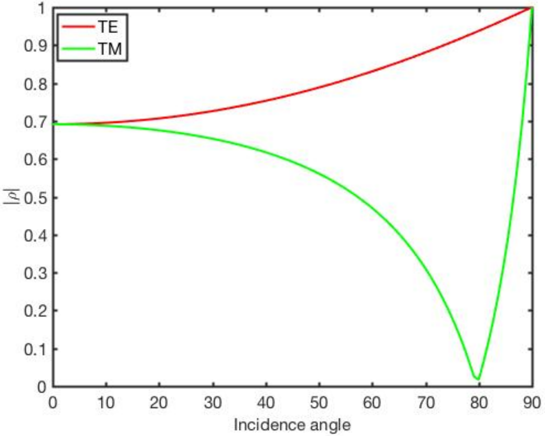
TE and TM reflection coefficients



$$n = \sqrt{\epsilon_c} = \sqrt{\left(\epsilon - j\frac{\sigma}{\omega\epsilon}\right)} \tag{14}$$

$\omega = 100MHz, \epsilon = 7, \sigma = 0.001$

TE and TM reflection coefficients



$\omega = 100MHz$, $\epsilon = 30$, $\sigma = 0.02$