



ERSLab

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Introduction

Shadowing

Theoretical facts

Coverage

Edge of the cell

Whole cell coverage

Correlated shadowing

Autocorrelation

Cross-correlation

Shadowing

Electromagnetics and Remote Sensing Lab (ERSLab)

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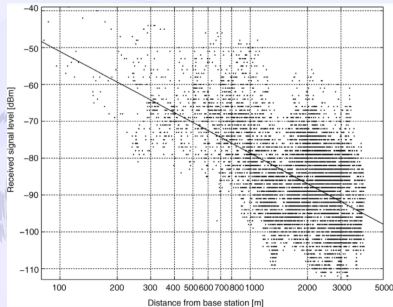
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Path loss models developed to deal with macrocells include only dependencies on parameters such as antenna heights, environment and distance.



Those parameters alone do not explain the large scatter of measurements



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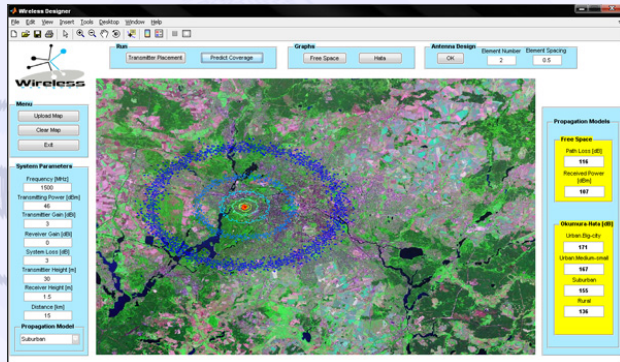
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The key problem relies on the fact that, for a given environment, the path-loss is constant for an assigned TX-RX distance.





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A mobile receiver that is driven at a constant distance around a transmitting station will measure a signal that fluctuates around its median level.

- The variation occurs over distances comparable to the widths of buildings or hills that are located within the region of the mobile (i.e.; tens or hundreds of meters).

Shadowing - Lognormal

Those fluctuations can be statistically described by a log-normal distribution, i.e.; the signal measured in dB is normally distributed.

The underlying physical phenomenon is termed as shadowing



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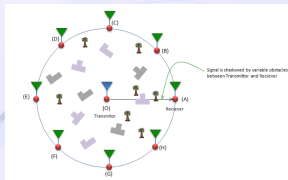
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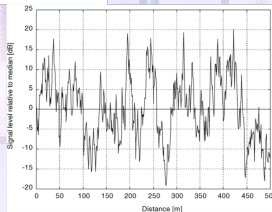
Cross-correlation

The clutter at a given distance will change from a path to another generating changes with respect to the path loss predicted by the reference model.



Some path suffer increased loss; while others are less obstructed

This results in an received signal that fluctuates wrt its median value



This phenomenon is termed as shadowing or slow fading



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Log-normal distribution

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It is assumed that the contributions to signal attenuation act independently from each other.

- The total loss (dB) is given by the sum of individual losses:

$$L = L_1 + L_2 + \dots + L_N \quad (1)$$

- Since each loss is a random variable, according to the central limit theorem, L would be Gaussian distributed.
- This implies that the attenuation in natural values will be Log-normal distributed.



Shadowing statistical distribution

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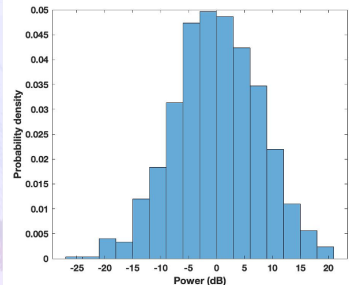
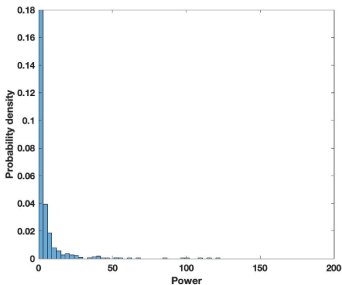
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Why we do care

The effect of shadowing is to transform the coverage radius of a cell from a fixed, predictable value into a statistical quantity affecting the coverage and capacity of a system in ways which can be predicted.

The shadowing affects:

- The dynamic of signal variation at the mobile
- The percentage of locations which receive sufficient power.
- The percentage of locations which receive sufficient signal-to-interference ratio.



Coverage

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Due to shadowing, the total path loss at a given distance is no longer a deterministic value. This impacts significantly the area coverage.

- The path loss is a random variable given by:

$$L = L_{50} + L_S \quad (2)$$

- L_{50} is the path loss predicted by any path loss model and it represents the path loss not exceeded at 50% of the locations at a given distance, it is often termed as **local median** path loss.
- L_S is the shadowing component, i.e.; a zero-mean Gaussian random variable with standard deviation σ_L . The latter is termed as **location variability**.



Shadowing statistical distribution

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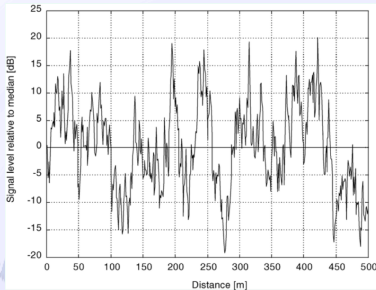
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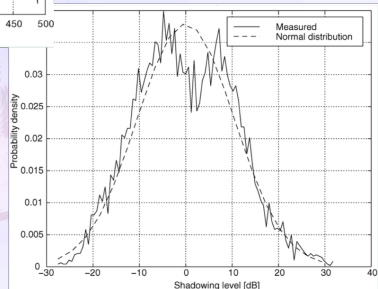
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Typical signal variations due to shadowing. Note that the median level [dB] has been subtracted

Empirical distribution of the shadowing component, i.e.; total path loss measurement – path loss predicted by a reference model, versus theoretical normal distribution





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Margin of the cell: L_{50}

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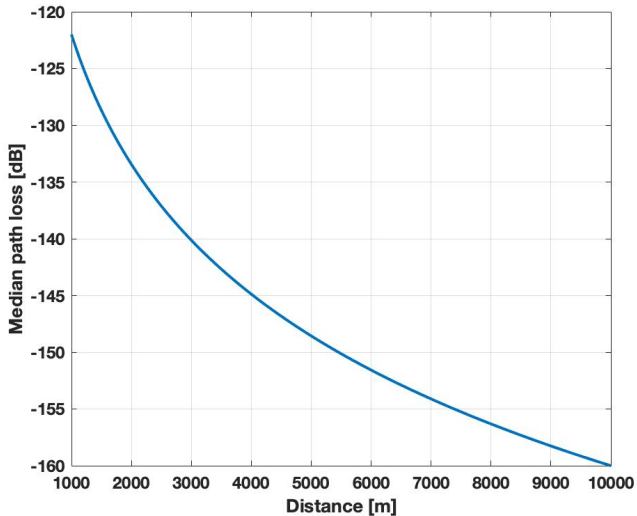
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Margin of the cell: $L_S + L_{50}$ and fade margin

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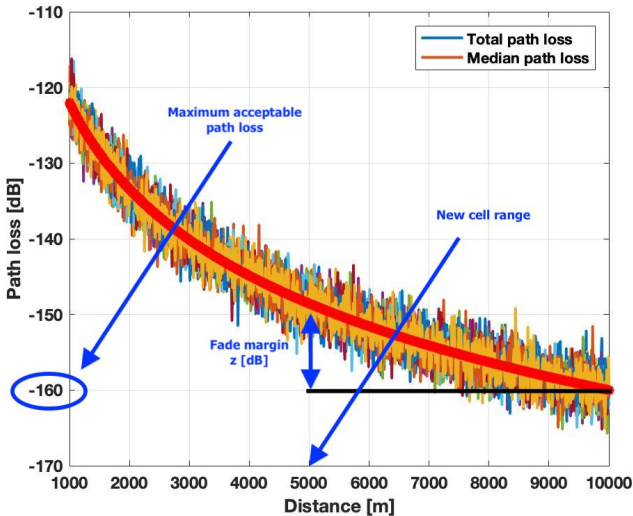
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Margin of the cell range: shadowing

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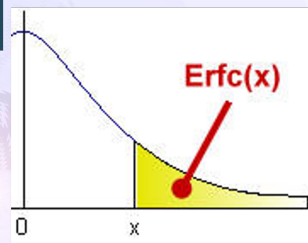
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- Let's assume that the reference path loss model predicts a cell range of around 10km with a maximum acceptable path loss of 160dB.
- Due to shadowing, this median path loss results in 50% of the location at the cell's edge properly covered. L_S must be included!
- Including L_S corresponds to adding a fade margin z that increases the reliability at the expense of the cell's radius.
- The previous picture shows that the inclusion of the fade margin let the cell range reducing from around 10km to around 5km.



How can fade margin be calculated ?



The fade margin shall be related to the reliability required by the QoS



How can fade margin be calculated ?

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- The probability that, due to shadowing, the median path loss is increased by the fade margin, i.e.; by z dB is given by:

$$Pr[L_S > z] = \int_{L_S=z}^{\infty} p(L_S) dL_S = \int_{L_S=z}^{\infty} \frac{1}{\sqrt{2\pi\sigma_L^2}} e^{-\frac{L_S^2}{2\sigma_L^2}} dL_S \quad (3)$$

- which, normalizing z to the location variability, leads to ($t = z/\sigma_L$):

$$Pr[L_S > z] = \int_t^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = Q(z/\sigma_L) \quad (4)$$

- where $Q(t)$ is the cumulative normal distribution with $t = \frac{z}{\sigma_L}$.



$Q(t)$

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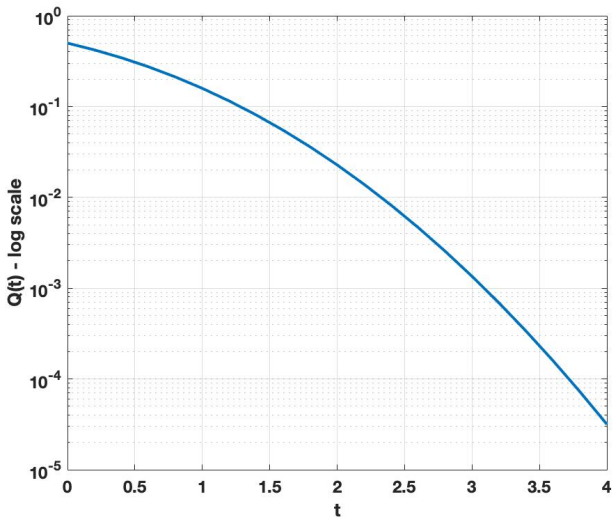
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Coverage fraction

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Reliability in terms of coverage fraction

According to (4), a system can be designed to meet a given level of reliability.

The latter is usually provided in terms of a user-defined percentage of locations at the edge of the macrocell that result in an acceptable coverage.

Fractional coverage

To evaluate the fractional coverage, i.e., the probability that at a given range r a user-defined probability of coverage is met, one can proceed evaluating the probability of outage assuming the maximum acceptable path loss is L_{max} dB.



Coverage fraction

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- The outage probability can be evaluated as follows:

$$\begin{aligned} p_{out} &= Pr[L_T > L_{max}] = Pr[L_{50} + L_S > L_{max}] \\ &= Pr[L_S > L_{max} - L_{50}] \\ &= Q\left(\frac{L_{max} - L_{50}}{\sigma_L}\right) \end{aligned} \quad (5)$$

(6)

- Hence, the fraction of locations covered at a given range r is simply given by:

$$p_e(r) = 1 - p_{out} = 1 - Q\left(\frac{L_{max} - L(r)}{\sigma_L}\right) = 1 - Q\left(\frac{z}{\sigma_L}\right) \quad (7)$$

- L_{max} is the maximum acceptable path loss.
- $L(r)$ is the median path loss at a range r .
- z is the fade margin chosen for this system.



Coverage fraction vs σ_L

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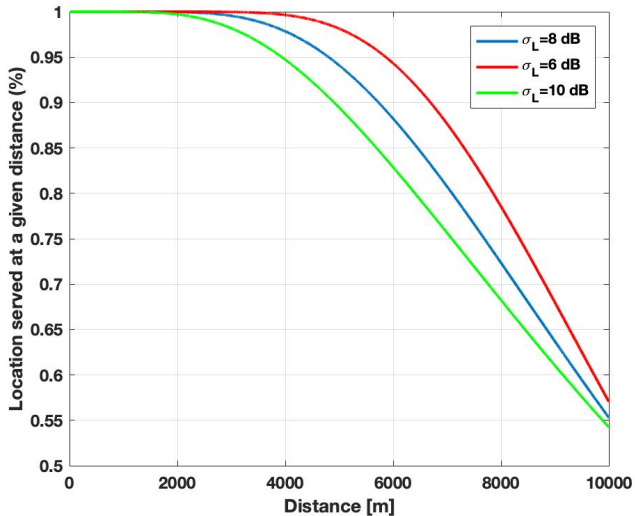
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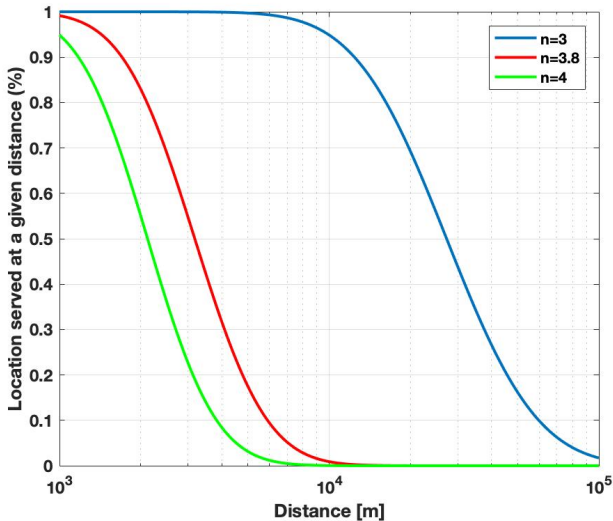
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Coverage fraction vs power law exponent





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Whole cell

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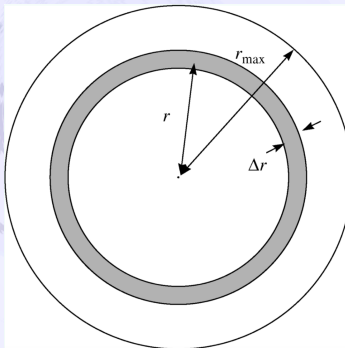
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In general, the closer is the mobile to the base station, the better will be its coverage probability.

- This implies that it is by far better designing the system considering the coverage probability experienced by the whole cell (instead of the edge only).





Probability of coverage

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The coverage probability for the whole cell can be evaluated considering rings with small width Δr . Each ring has an area that is $2\pi r \Delta r$ and calls for a probability of coverage $p_e(r)$.

- The probability of coverage of the whole cell p_{cell} can be evaluated integrating all the probability related to the small rings and dividing by the area of the whole cell, i.e.; πr_{max}^2 :

$$p_{cell} = \frac{2}{r_{max}^2} \int_{r=0}^{r_{max}} r p_e(r) dr = \frac{1}{2} + \frac{1}{r_{max}^2} \int_0^{r_{max}} r \operatorname{erf} \left(\frac{L_{max} - L(r)}{\sigma_L \sqrt{2}} \right) dr. \quad (8)$$

- This integral can be solved numerically. Closed-form solutions exist when a power law model is assumed to describe $L(r)$.



The cellular concept

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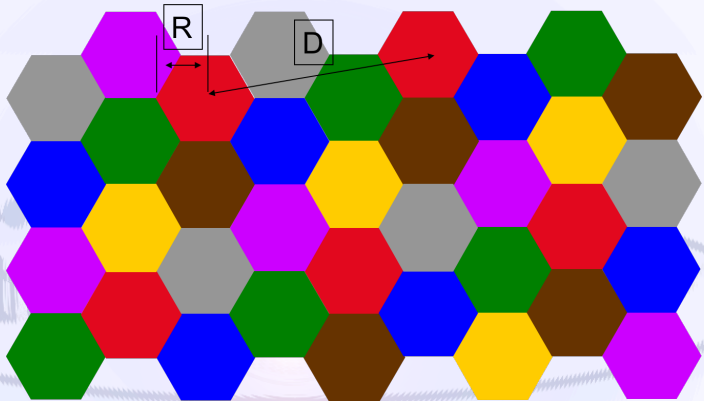
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Correlated shadowing: Why we do care

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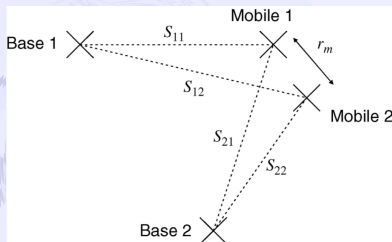
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Real-world shadowing losses on different links in a network are not independent. The correlations have both detrimental and beneficial impacts on networks.



The relationship between shadowing experienced by nearby paths needs to be accounted for.



Correlations

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Two TX stations are considered and two mobile locations (or - equivalently - two positions of a single mobile)

- Each path is marked with the value of the associated shadowing S , which is log-normally distributed (i.e.; its dB value is normally distributed).
- Since the four paths may include common obstacles, shadowing values are not independent of each other.

Two kind of correlations:

Autocorrelation - or serial correlation - accounts for the correlation between two mobile positions connected to the same TX station, e.g.; S_{11} , S_{12} or S_{22} S_{21} .

Cross-correlation - or site-to-site correlation - accounts for the correlation between two TX stations received by the same mobile position, e.g.; S_{11} , S_{21} or S_{22} S_{12} .



Correlated shadowing

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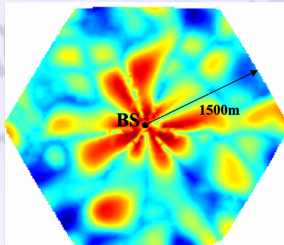
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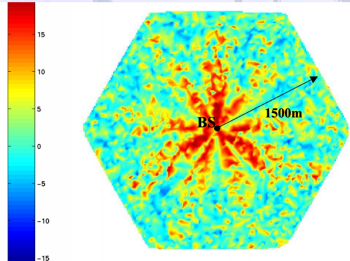
SERIAL

- It affects power control in code division multiple access (CDMA) systems



SITE TO SITE

- It affects the Carrier to Signal Interference (C/I) ratio





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Autocorrelation

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- It can be defined as follows:

$$\rho_m(r_m) = \frac{E(S_{11}S_{12})}{\sigma_{L1}\sigma_{L2}} \approx \frac{E(S_{11}S_{12})}{\sigma_L^2} \quad (9)$$

- It affects the time variability of the path loss experienced by the mobile when it moves around.

Effects on actual systems

It has a great importance in managing power-control processes, e.g.; network messages between TX station and mobile where the TX station instructs the mobile on how to adjust its transmit power.

Power estimations undertaken by the TX station depend on how rapidly the autocorrelation function reduces in time.



Autocorrelation

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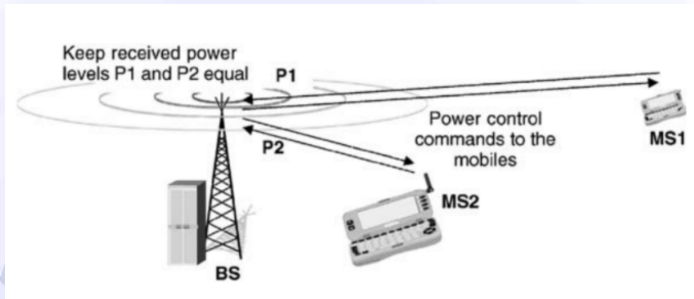
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Power management - CDMA near/far problem

Mobile stations MS1 and MS2 operate within the same frequency, separable at the base station only by their respective spreading codes. Tight and fast power control is perhaps the most important aspect in CDMA, in particular on the uplink.



Exponential type autocorrelation function

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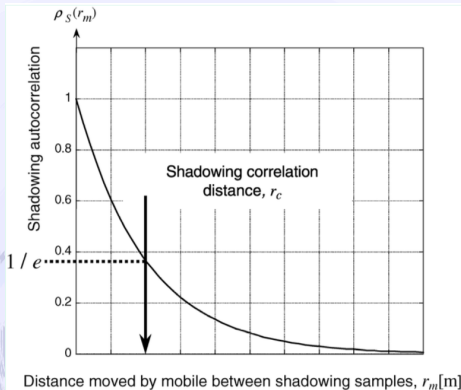
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The shadowing autocorrelation distance r_c is the distance where the normalized autocorrelation function reduces by 37%. It is larger at larger distances.



Generating a correlated shadowing process

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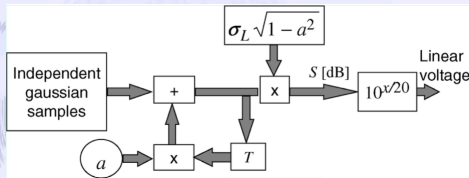
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Independent normally-distributed samples are generated at a sampling interval T . Individual samples are then delayed by T , multiplied by the coefficient a and then summed with the new samples. Finally, the filtered samples are multiplied by $\sigma_L \sqrt{1 - a^2}$.



$$a = e^{-\frac{vT}{rc}} \quad (10)$$

where v is the velocity of the mobile and T is the symbol's rate.



Do it yourself

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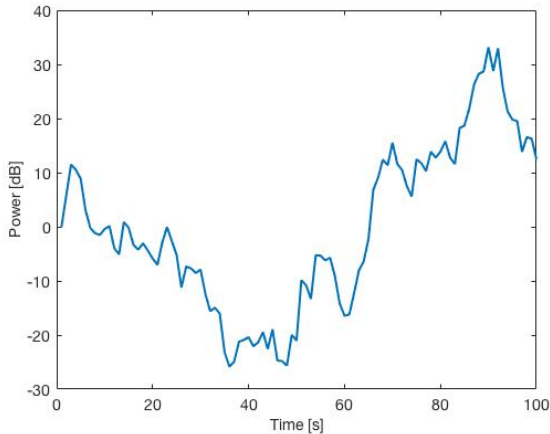
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$$\sigma_L = 8\text{dB}, v = 14\text{m/s}, r_c = 100\text{m}, T = 1\text{s}, N = 100 \text{ samples}$$



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Cross-correlation

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- It can be defined as follows:

$$\rho_c = \frac{E(S_{11}S_{21})}{\sigma_{L1}\sigma_{L2}} \quad (11)$$

- The paths involved may be widely separated; hence they call for different σ_L values.

Effects on actual systems

The two TX stations may be iso-channel; hence the station that is not connected to the mobile will contribute to worsen the carrier-to-interference ratio (C/I).

When ρ_c is large, the shadowing processes are correlated; hence C/I can be maintained to acceptable values. This is by far more complicated when ρ_c is low.



Carrier to Interference ratio

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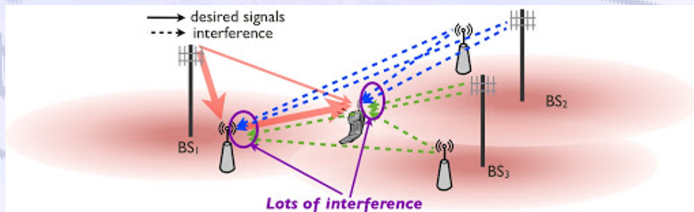
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Let's focus on a C/I in the downlink path dominated by only 2 stations whose distances from the mobile are r_1 and r_2 , respectively.

- A power law model with exponent n is assumed for the path loss;
- The C/I ratio (R) is itself Normal distributed with mean μ_R and standard deviation σ_R .

It can be shown that:

- The mean value:

$$\mu_R = 10n \log \left(\frac{r_2}{r_1} \right) \quad (12)$$

- The standard deviation (assuming $\sigma_1 \approx \sigma_2 = \sigma_L$):

$$\sigma_R = \sigma_L \sqrt{2(1 - \rho_c)} \quad (13)$$



Carrier to Interference ratio

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Probability of outage

It is the probability that the C/I ratio R is smaller than a system dependent threshold R_T .

$$Pr[R < R_T] = 1 - Q\left(\frac{R_T - \mu_R}{\sigma_R}\right) \quad (14)$$

- This formula gives the probability of outage for a cellular system dominated by only one interferer.
- For a probability of outage equal to 10%, the difference from $\rho_c = 0.8$ and $\rho_c = 0$ is around 7dB.

According to (12) with $\mu_R = 7$ and $n = 4$, the reuse distance r_2 should be increased by around 50%.



The effect of correlated shadowing on system's performance

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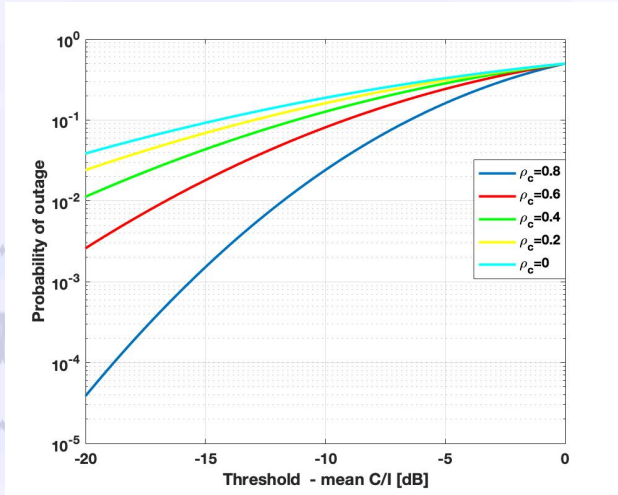
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Cross-correlation affects system design parameters

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- Optimum choice of antenna beamwidths for sectorisation.
- Performance of soft handover and site diversity.
- Design and performance of handover algorithms.
- Optimum frequency planning for minimized interference and hence maximized capacity.
- Adaptive antenna performance calculation.
- ...