



ERSLab

F. Nunziata

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Lossless media

Lossy media

Fresnel reflection coefficients Examples

Electromagnetics
and
Remote Sensing Lab
(ERSLab)

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Reflection coefficient

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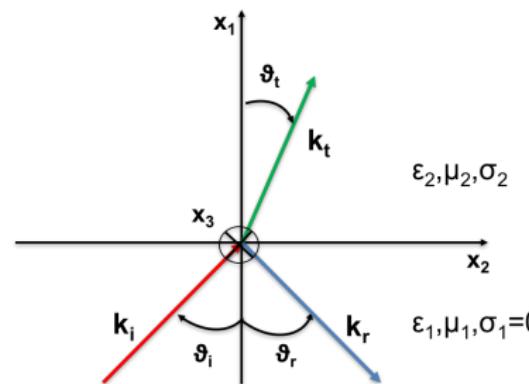
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The general formula for the reflection coefficient is given by:



$$R = \frac{Z_2(\hat{x}_1) - Z_1(\hat{x}_1)}{Z_2(\hat{x}_1) + Z_1(\hat{x}_1)} \quad (1)$$



Reflection coefficient

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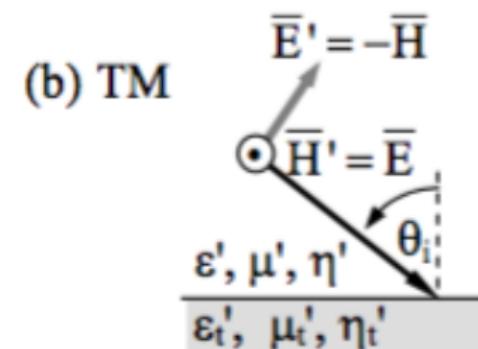
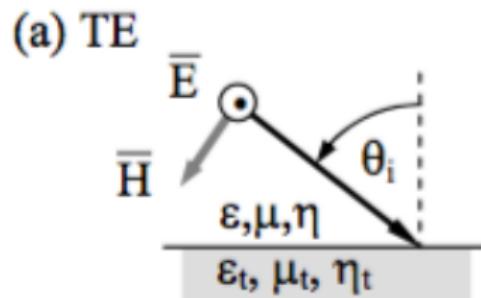
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$$Z_{TE} = \frac{Z}{\cos \vartheta} \quad Z_{TM} = Z \cos \vartheta \quad (2)$$



Reflection coefficient - TE

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$$\begin{aligned} R_{TE} &= \frac{\frac{Z_2}{\cos \vartheta_2} - \frac{Z_1}{\cos \vartheta_1}}{\frac{Z_2}{\cos \vartheta_2} + \frac{Z_1}{\cos \vartheta_1}} \\ &= \frac{\frac{Z_2 \cos \vartheta_1 - Z_1 \cos \vartheta_2}{\cos \vartheta_1 \cos \vartheta_2}}{\frac{Z_2 \cos \vartheta_1 + Z_1 \cos \vartheta_2}{\cos \vartheta_1 \cos \vartheta_2}} \end{aligned} \quad (3)$$

Reflection coefficient - TE

$$R_{TE} = \frac{Z_2 \cos \vartheta_1 - Z_1 \cos \vartheta_2}{Z_2 \cos \vartheta_1 + Z_1 \cos \vartheta_2} \quad (4)$$



Reflection coefficient - TM

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Reflection coefficient - TM

$$R_{TM} = \frac{Z_2 \cos \vartheta_2 - Z_1 \cos \vartheta_1}{Z_2 \cos \vartheta_2 + Z_1 \cos \vartheta_1} \quad (5)$$

- $Z = \sqrt{\frac{\mu}{\epsilon}}$
- When dealing with a non-magnetic medium $\mu = \mu_0$.
Hence:

$$Z = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}} = \frac{Z_0}{\sqrt{\epsilon_r}} = \frac{Z_0}{n} \quad (6)$$

- where n is the refractive index.



Reflection coefficients as function of incidence angle only

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- According to eq.(6), reflection coefficients can be rewritten as:

$$R_{TE} = \frac{n_1 \cos \vartheta_1 - n_2 \cos \vartheta_2}{n_1 \cos \vartheta_1 + n_2 \cos \vartheta_2} \quad (7)$$

$$R_{TM} = \frac{n_1 \cos \vartheta_2 - n_2 \cos \vartheta_1}{n_1 \cos \vartheta_2 + n_2 \cos \vartheta_1} \quad (8)$$

- Using Snell's law $n_1 \sin \vartheta_1 = n_2 \sin \vartheta_2$ and trigonometric formulas, one can write:

$$n_2 \cos \vartheta_2 = n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \vartheta_1 \right)^2} \quad (9)$$

$$n_1 \cos \vartheta_2 = n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \vartheta_1 \right)^2} \quad (10)$$



Reflection coefficients as function of incidence angle only

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Reflection coefficients

$$R_{TE} = \frac{n_1 \cos \vartheta_1 - n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \vartheta_1\right)^2}}{n_1 \cos \vartheta_1 + n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \vartheta_1\right)^2}} \quad (11)$$

$$R_{TM} = \frac{n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \vartheta_1\right)^2} - n_2 \cos \vartheta_1}{n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \vartheta_1\right)^2} + n_2 \cos \vartheta_1} \quad (12)$$



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TE and TM reflection coefficients

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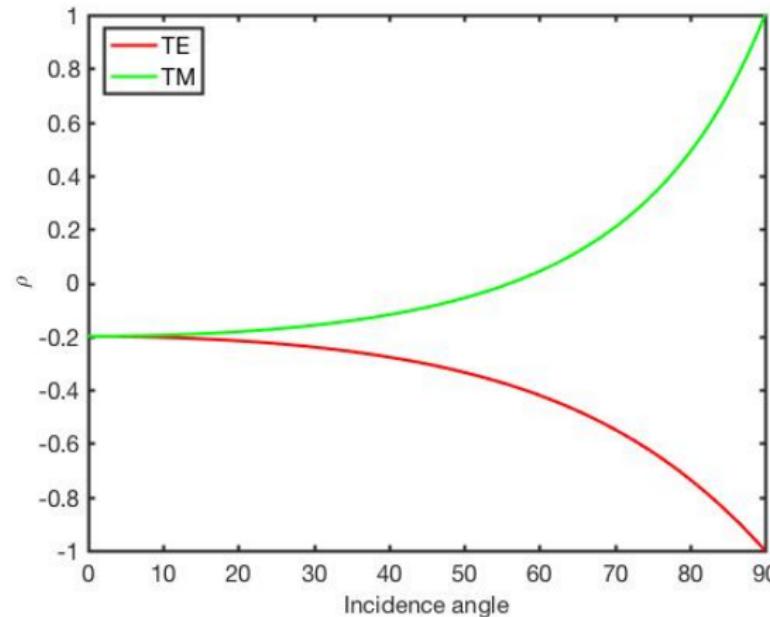
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$n_1=1, n_2=1.5$



Brewster angle

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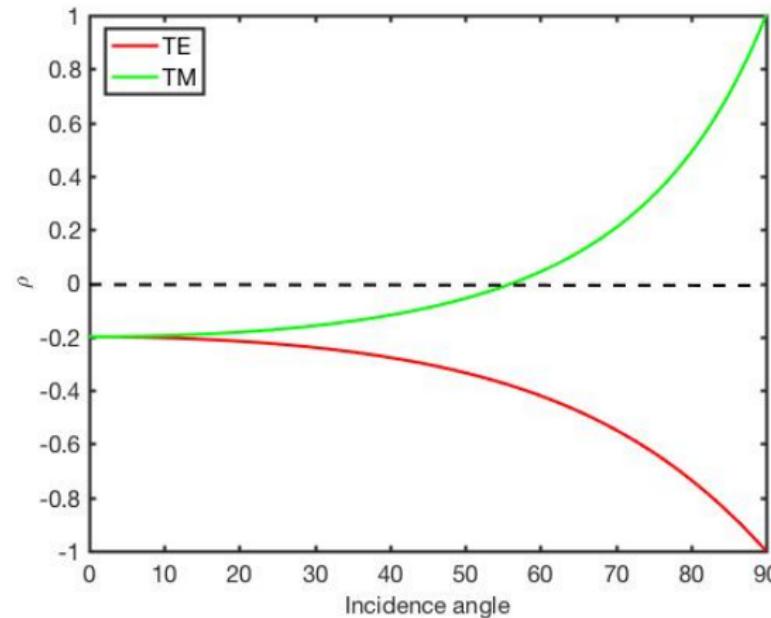
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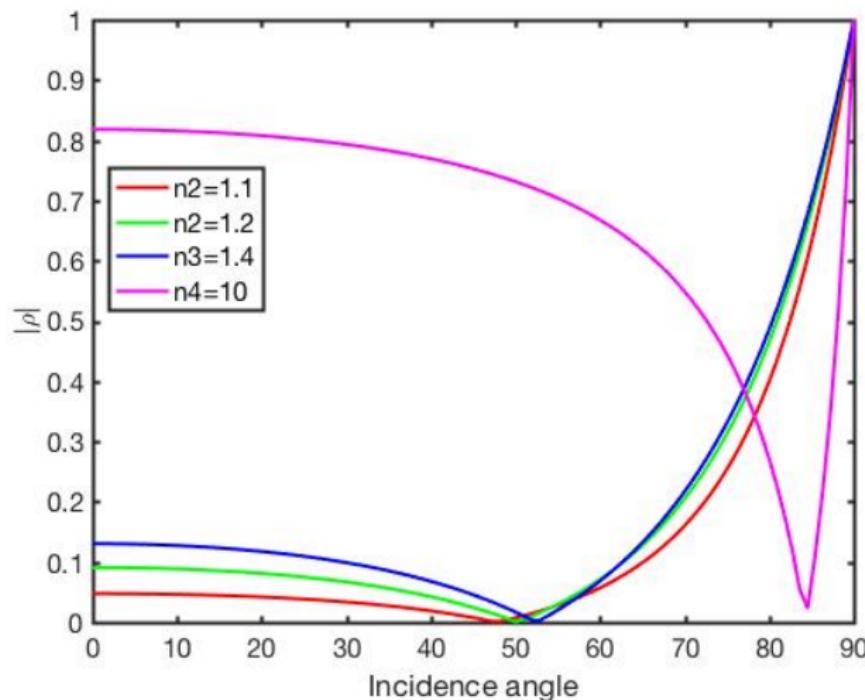
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$$\vartheta_B = \tan^{-1} \frac{n_2}{n_1} \quad (13)$$



TM reflection coefficient for varying n_2





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TE and TM reflection coefficients

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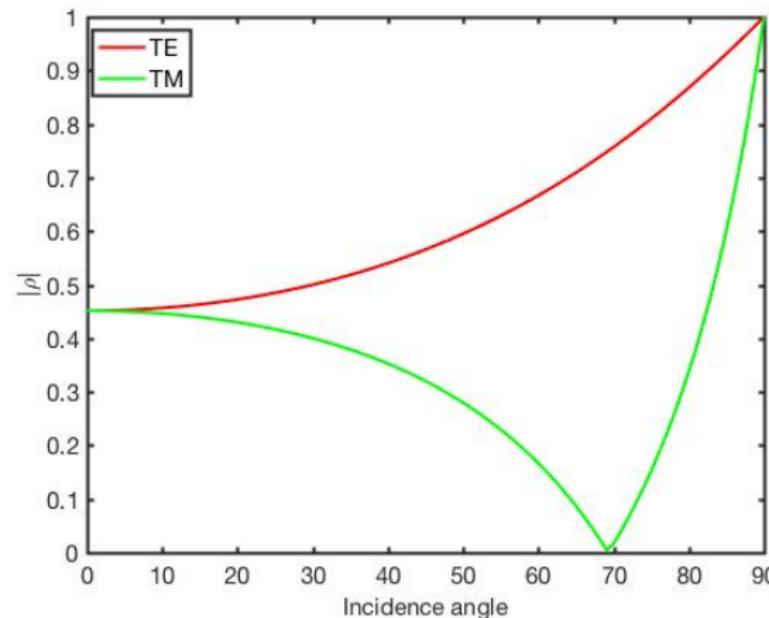
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$$n = \sqrt{\epsilon_c} = \sqrt{\left(\epsilon - j \frac{\sigma}{\omega \epsilon}\right)}$$

(14)

$\omega = 100MHz, \epsilon = 7, \sigma = 0.001$



TE and TM reflection coefficients

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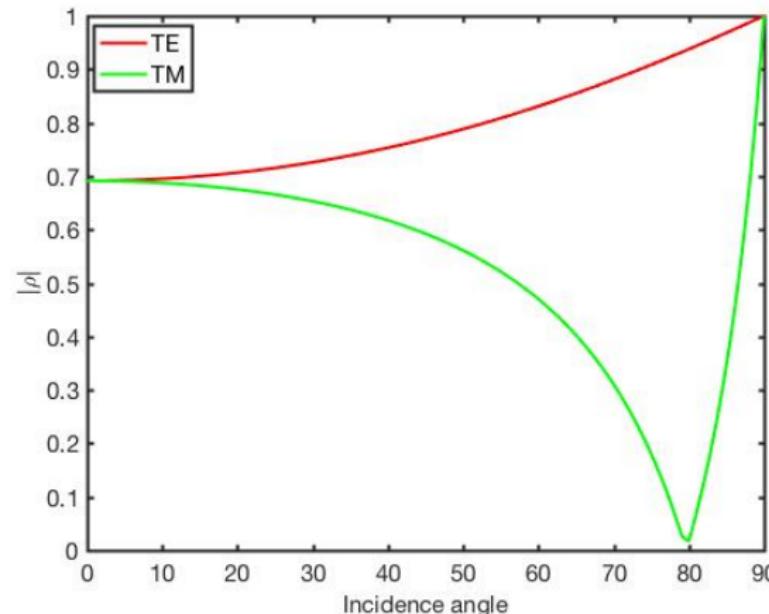
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$$\omega = 100 \text{ MHz}, \epsilon = 30, \sigma = 0.02$$