1 The Epidemic as an Externality

Consider a static economy where households choose consumption c and leisure ℓ. Leisure is a contact-intensive activity that can lead to an infection. The infection carries a disutility for the household. All households are equal.

Let preferences be:

$$u(c, \ell, I) = c + \alpha \log \ell - \beta I$$

where I is the probability of being infected and β is the disutility from infection. This probability is given by

$$I = \gamma \ell + \Gamma L.$$

It depends both on the individual choice of leisure ℓ (the more often the individual dines out, the more likely they are to contract the virus), and the total amount of leisure in the economy L (the more people dine out, the more likely is that an individual will contract the virus when they go out).

Note that the economy features an externality. The individual takes L (the aggregate amount of leisure) as given, but its own actions affect L (e.g., if there are $j = 1, 2, ..., J$ individuals, then $L = \ell_1 + \ell_2 + ... + \ell_J$). In other words the individual choice of ℓ enters in everyone else utility directly through I because I depends on the aggregate amount of leisure. The size of the externality is proportional to Γ.

A representative firm produces the final good (the numeraire of the economy) by hiring labor with linear technology

$$y = zn,$$

at the wage w. The firm distributes profits as dividends.

1.1 Competitive equilibrium

Let's state the household problem first:

$$\begin{aligned} & \max_{c,\ell} c + \alpha \log \ell - \beta I \\ & s.t. \\ c &= d + w(1 - \ell) \\ I &= \gamma \ell + \Gamma L \end{aligned}$$

The Lagrangean is

$$L(c, \ell, \lambda) = c + \alpha \log \ell - \beta [\gamma \ell + \Gamma L] + \lambda [d + w(1 - \ell) - c]$$

and the first-order conditions with respect to (c, ℓ, λ) respectively are

$$\begin{aligned} 1 &= \lambda \\ \frac{\alpha}{\ell} - \beta \gamma &= \lambda w \\ c &= d + w(1 - \ell) \end{aligned}$$

Rearranging the first two FOCs, we obtain:

$$\ell^* = \frac{\alpha}{w + \beta\gamma}.$$

Turning to the firm problem:

$$\max_n zn - wn$$

which yields $w^* = z$ and $d^* = 0$ (the firm makes no profits). Combining these last two equations, we arrive at:

$$\ell^* = \frac{\alpha}{z + \beta\gamma},$$

which is the equilibrium amount of leisure expressed only as a function of model parameters.

You should complete the calculations for (c^*, y^*, I^*) and formally define an equilibrium.

1.2 Planner problem

The social planner problem (with a measure 1 of identical households) solves:

$$\begin{aligned} & \max_{c,\ell} c + \alpha \log \ell - \beta I \\ & s.t. \\ c &= z(1-\ell) \\ I &= \gamma\ell + \Gamma L \end{aligned}$$

where $c = z(1-\ell)$ is the feasibility constraint.

The key difference between the individual and the planner is that the planner takes into account the effect of individual leisure choices on the aggregate level.

The planner's Lagrangean is:

$$L(c, \ell, \mu) = c + \alpha \log \ell - \beta [\gamma\ell + \Gamma L] + \mu [z(1-\ell) - c]$$

with first-order conditions with respect to (c, ℓ, μ) given by respectively:

$$\begin{aligned} 1 &= \mu \\ \frac{\alpha}{\ell} - \beta(\gamma + \Gamma) &= \mu z \\ c &= z(1-\ell) \end{aligned}$$

Combining the first two conditions:

$$\hat{\ell} = \frac{\alpha}{z + \beta\gamma + \beta\Gamma}.$$

Note that the planner wants less leisure than the equilibrium allocations because it recognizes that an additional unit of leisure can raise infections (and their disutility) for everyone else. The higher is Γ (the parameter that regulates the externality in the infection function) the stronger this effect.

1.3 Correcting the externality

How do we correct the externality in the equilibrium allocations and restore the social optimum? We have three ways. We can let the government: (i) limit the quantity of leisure households can enjoy, (ii) tax leisure, or (iii) set up a market for 'infection rights' at the appropriate price.

1.3.1 Quantity controls

Suppose the government sets the following limit on the amount of leisure each individual can consume.

$$\ell < \hat{\ell}.$$

We know that the optimal solution in equilibrium is $\ell^* > \hat{\ell}$, therefore the constraint will bind and the corner solution will be $\ell^* = \hat{\ell}$. This is akin to a mitigation or containment policy, i.e. a partial lockdown. The government says, e.g., you can dine, but not indoor, you can go running outside, but not at the gym, etc.

1.3.2 Taxes on leisure

Suppose the government sets a tax τ on leisure (the contact-intensive activity). The household problem becomes:

$$\begin{aligned} & \max_{c,\ell} c + \alpha \log \ell - \beta I \\ & s.t. \\ c &= d + w(1-\ell) - \tau\ell \\ I &= \gamma\ell + \Gamma L \end{aligned}$$

with first-order conditions with respect to (c, ℓ):

$$\begin{aligned} 1 &= \lambda \\ \frac{\alpha}{\ell} - \beta z \gamma &= \lambda(w + \tau) \end{aligned}$$

which yield:

$$\ell^* = \frac{\alpha}{z + \beta\gamma + \tau}$$

So, comparing this equation to the one for $\hat{\ell}$, it is immediate that the tax that implements the efficient allocation is $\tau = \beta\Gamma$, i.e. it is rising with the externality parameter Γ and the disutility from infections β. A tax of this type, which corrects an externality, is called *a* Pigouvian tax, from the English economist Arthur Pigou (1877-1959).

1.3.3 Market for infection rights

Suppose the government lets you enjoy as much leisure as you like, but for every infection you cause you pay a price p. The household problem becomes:

$$\begin{aligned} & \max_{c,\ell,I} c + \alpha \log \ell - \beta I \\ & s.t. \\ c + pI &= d + w(1-\ell) \\ I &= \gamma \ell + \Gamma L \end{aligned}$$

which we can simplify as:

$$\max_{\ell} d + w(1-\ell) + \alpha \log \ell - (\beta + p)(\gamma \ell + \Gamma L)$$

The first-order condition yields

$$\begin{aligned} \frac{\alpha}{\ell} - w - (\beta + p)\gamma &= 0 \\ \ell^* &= \frac{\alpha}{z + \beta\gamma + p\gamma} \end{aligned}$$

Which price does the government need to set to induce $\ell^* = \hat{\ell}$? The answer is:

$$\hat{p} = \frac{\beta \Gamma}{\gamma},$$

again, increasing in the size of the negative externality and the disutility from infections.